ISSN 2347-1921

Semiorthogonal B-spline Wavelet for Solving 2D- Nonlinear Fredholm-Hammerstein Integral Equations

[1]Eman Ali Hussain and [2]Lamyaa H. Ali
[1]Asst. Prof., Dr., Department of Mathematics, College of Science, University of AL- Mustansiriyah, Iraq
E-mail: dr_emansultan@yahoo.com
[2] Asst. Lec. Department of Mathematics, College of Science, University of AL- Mustansiriyah, Iraq
E-mail: lamyaa_h2@yahoo.com

Abstract

This work is concerned with the study of the second order (linear) semiorthogonal B-spline wavelet method to solve one-dimensional nonlinear Fredholm-Hammerstein integral equations of the second kind. Proof of the existence and uniqueness solution for the two-dimensional Fredholm-Hammerstein nonlinear integral equations of the second kind was introduced. Moreover, generalization the second order (linear) semiorthogonal B-spline wavelet method was achieved and then using it to solve two-dimensional nonlinear Fredholm-Hammerstein integral equations of the second kind. This method transform the one-dimensional and two-dimensional nonlinear Fredholm-Hammerstein integral equations of the second kind to a system of algebraic equations by expanding the unknown function as second order (linear) semiorthogonal B-spline wavelet with unknown coefficients. The properties of these wavelets functions are then utilized to evaluate the unknown coefficients. Also some of illustrative examples which show that the second order (linear) Semiorthogonal B-spline wavelet method give good agreement with the exact solutions.

Keywords: wavelet; spline; integral equation; Fredholm-Hammerstein.

Council for Innovative Research
Peer Review Research Publishing System
Journal: Journal of Advances in Mathematics

Vol 6, No. 1
editor@cirworld.com
www.cirworld.com, member.cirworld.com

INTRODUCTION

This work includes one of important numerical methods called linear two-dimensional semiorthogonal B-spline wavelets method, which is used to solve nonlinear two- dimensional Fredholm-Hammerstein integral equations of the second kind and prove the existence and uniqueness solution of the nonlinear two- dimensional Fredholm-Hammerstein integral equations of the second kind. The integral equations have important applications in many areas such as physics, engineering and finance and appear in studies involving elastic contact problems, fracture mechanics, airfoil theory and others. The theorems of the one-dimensional integral equations have various techniques and have increasing important applications in different areas of science [1]. There are basically two reasons for this interest. In first, some cases, as in the work of Abel on tautochrone curves, integral equations are the natural mathematical model for representing a physically interesting situation. The second, and perhaps more common reason, is that integral operators, transforms, and equations, are convenient tools for studying differential equations. Consequently, integral equation techniques are well known to classical analysis and many elegant and powerful results were developed by them [5].

Wavelet is now an old story for signal and image processing specialists. In 1982, that a French engineer working on seismological data for an oil company, Jean Morlet, proposed the concept of wavelet analysis to reach automatically the beast trade-off between time and frequency resolution [9]. This is probably because the original wavelets, which were widely used for signal processing, were primarily orthogonal. In signal processing applications, unlike integral equation methods, the wavelet itself is never constructed since only its scaling function and coefficients are needed. However, orthogonal wavelets either have infinite support or a nonsymmetrical, and in some cases fractal, nature. These properties can make them a poor choice for characterization of a function. In contrast, the semiorthogonal wavelets have finite support, both even and odd symmetry, and simple analytical expressions, ideal attributes of a basis function [3],[7]. In recent years, numerous works have been focusing on the development of more advanced and efficient methods for wavelet method such as in 1995 , Wagner and Chew, [10] solution of electromagnetic integral equations, in 2000, P.W. Zhang and Y. Zhang,[11] boundary integral equation, in 2004, M. Lakestani M. Razzaghi, and M. Dehghan, [3] solution of nonlinear Fredholm-Hammerstein integral equations by using semiorthogonal spline wavelets, in 2006, M. Lakestani, M. Razzaghi, and M. Dehghan, [4]semiorthogonal spline wavelets approximation for Fredholm integro-differential equations, in 2010, M. Rabbani, [8] Using two-dimensional multi-wavelet is constructed in terms of chebyshev polynomials for solving two-dimensional Fredholm integral equations, in 2012, H. Derili, S. Sohrabi and A. Arzhang, [2] apply two-dimensional Haar wavelets to solve linear two dimensional Fredholm integral equations.

1. GENERALIZATION OF TWO-DIMENSIONAL WAVELET AND SCALING FUNCTION:

This section extends the one-dimensional Semiorthogonal B-spline wavelet and scaling function results to multiple dimensions.

A two-dimensional wavelet basis is arrived at by computing the tensor product of two one-dimensional wavelet basis functions.

The two-dimensional approximation spaces ${V_j}^2$ and orthogonal complements ${W_j}^2$ are defined as follows [6]:

$$ {V_j}^2 = V_j \otimes V_j $$

$$ {W_j}^{2,1} = V_j \otimes W_j $$

$$ {W_j}^{2,2} = W_j \otimes V_j $$

$$ {W_j}^{2,3} = W_j \otimes W_j $$

The two-dimensional wavelets and scaling function are defined by:

$$ \phi^0_{j,k,l}(x,y) = \phi_{j,k}(x) \cdot \phi_{j,l}(y) $$

With 3 types of wavelets:

$$ \psi^1_{j,k,l}(x,y) = \phi_{j,k}(x) \cdot \psi_{j,l}(y) $$

$$ \psi^2_{j,k,l}(x,y) = \psi_{j,k}(x) \cdot \phi_{j,l}(y) $$

$$ \psi^3_{j,k,l}(x,y) = \psi_{j,k}(x) \cdot \psi_{j,l}(y) $$

If the one-dimensional relatives are semiorthogonal similar relationships follow for the two-dimensional setting, since for instance

$$\langle \psi_{j,k,l}(x,y), \psi_{j,k',l'}(x,y) \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \psi_{j,k,l}(x,y)\psi_{j,k',l'}(x,y)dxdy$$

$$= \left(\int_{-\infty}^{\infty} \psi_{j,k}(x)\cdot\psi_{j,k'}(x)dx\right)\cdot\left(\int_{-\infty}^{\infty} \psi_{j,l}(y)\cdot\psi_{j,l'}(y)dy\right)$$

$$= \langle \psi_{j,k}(x), \psi_{j,k'}(x) \rangle \cdot \langle \psi_{j,l}(y), \psi_{j,l'}(y) \rangle .$$

Thus one can immediately obtain the orthogonality relations between the subspaces [6]:

$$V_j^2 \perp W_j^{2,t}, \quad t = 1,2,3$$

$$W_j^{2,1} \perp W_j^{2,2}$$

$$W_j^{2,2} \perp W_j^{2,3}$$

$$W_j^{2,1} \perp W_j^{2,3}$$

$$W_j^{2,t} \perp W_i^{2,s}, \quad j \neq i, s,t = 1,2,3.$$

The two-dimensional second order (linear) B-splines scaling functions are given by:

$$\phi^0_{j,k,k'}(x,y) = \phi^2_{j,k}(x)\cdot\phi^2_{j,k'}(y)$$

The two-dimensional second order B-splines wavelets functions are defined with three types of wavelets.

The first types of wavelets are given by: $\psi^1_{j,k,k'}(x,y) = \phi^2_{j,k}(x)\cdot\psi^2_{j,k'}(y)$

The second types of wavelets are given by: $\psi^2_{j,k,k'}(x,y) = \psi^2_{j,k}(x)\phi^2_{j,k'}(y)$

The third types of wavelets are given by: $\psi^3_{j,k,k'}(x,y) = \psi^2_{j,k}(x)\cdot\psi^2_{j,k'}(y)$

For example, for $j = 2, k = -1, \quad k' = -1$ the scaling function $\phi^0_{2,-1,-1}(x,y)$ is obtained by putting:

$$\phi^0_{2,-1,-1}(x,y) = \begin{cases} (2-4x)(1-4y), & 0 \le x \le \frac{1}{4}, 0 \le y \le \frac{1}{4}, \\ 0, & otherwise, \end{cases}$$

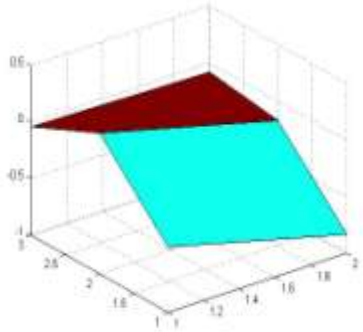

FIG (1) $\phi^0_{2,-1,-1}(x,y)$

For example, for $j = 2, k = -1, \quad k' = -1$ and $t = 1$ the wavelet function $\psi^1_{2,-1,-1}(x,y)$ is obtained by putting:

ISSN 2347-1921

$$\psi_{2,-1,-1}^{1}(x,y)=\frac{1}{6}\begin{cases}(1-4x)(-6+92y), & 0\le x\le\frac{1}{4},\ 0\le y\le\frac{1}{8},\\ (1-4x)(14-68y), & 0\le x\le\frac{1}{4},\ \frac{1}{8}\le y\le\frac{1}{4},\\ (1-4x)(-10+28y), & 0\le x\le\frac{1}{4},\ \frac{1}{4}\le y\le\frac{3}{8},\\ (1-4x)(2-4y), & 0\le x\le\frac{1}{4},\ \frac{3}{8}\le y\le\frac{1}{2},\\ 0, & otherwise,\end{cases}$$

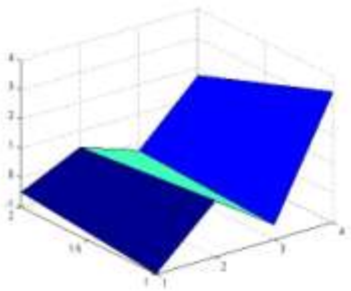

FIG (2) $\psi_{2,-1,-1}^{1}(x,y)$

For example, for $j=2, k=-1,\quad k'=-1$ and $t=2$ the wavelet function $\psi_{2,1,-1}^{2}(x,y)$ is obtained by putting:

$$\psi_{2,1,-1}^{2}(x,y)=\frac{1}{6}\begin{cases}(4x-1)(1-4y), & \frac{1}{4}\le x\le\frac{3}{8}, 0\le y\le\frac{1}{4},\\ (11-28x)(1-4y), & \frac{3}{8}\le x\le\frac{1}{2}, 0\le y\le\frac{1}{4},\\ (-35+64x)(1-4y), & \frac{1}{2}\le x\le\frac{5}{8}, 0\le y\le\frac{1}{4},\\ (45-64x)(1-4y), & \frac{5}{8}\le x\le\frac{3}{4}, 0\le y\le\frac{1}{4},\\ (-24+28x)(1-4y), & \frac{3}{4}\le x\le\frac{7}{8}, 0\le y\le\frac{1}{4},\\ (4-4x)(1-4y), & \frac{7}{8}\le x\le 1, 0\le y_j\le\frac{1}{4},\\ 0, & otherwise,\end{cases}$$

ISSN 2347-1921

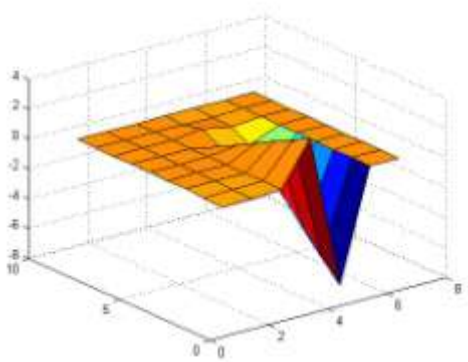

FIG (3) $\psi^{2}_{2,1,-1}(x,y)$

2. FUNCTION APPROXIMATION:

A function $f\,(x,y)$ defined over $[0,1]$ may be represented by B-spline wavelets and scaling functions as [2]:

$$f\,(x,y)=\sum_{k=-1}^{3}\sum_{k'=-1}^{3}c_{k,k'}\phi^{0}_{2,k,k'}(x,y)+\sum_{j=2}^{\infty}\left[\sum_{k=-1}^{3}\sum_{k'=-1}^{2^j-2}d^{1}_{j,k,k'}\psi^{1}_{j,k,k'}(x,y)+\sum_{k=-1}^{2^j-2}\sum_{k'=-1}^{3}d^{2}_{j,k,k'}\psi^{2}_{j,k,k'}(x,y)+\sum_{k=-1}^{2^j-2}\sum_{k'=-1}^{2^j-2}d^{3}_{j,k,k'}\psi^{3}_{j,k,k'}(x,y)\right] \quad \ldots (4.1)$$

Where $\phi^{0}_{2,k,k'}(x,y)$ and $\psi^{t}_{j,k,k'}(x,y)$, $t=1,2,3$ are two-dimensional scaling and wavelets functions, respectively.

If the infinite series in (4.1) is truncated, then (4.2) can be written as:

$$f\,(x,y)=\sum_{k=-1}^{3}\sum_{k'=-1}^{3}c_{k,k'}\phi^{0}_{2,k,k'}(x,y)+\sum_{j=2}^{M}\left[\sum_{k=-1}^{3}\sum_{k'=-1}^{2^j-2}d^{1}_{j,k,k'}\psi^{1}_{j,k,k'}(x,y)+\sum_{k=-1}^{2^j-2}\sum_{k'=-1}^{3}d^{2}_{j,k,k'}\psi^{2}_{j,k,k'}(x,y)+\sum_{k=-1}^{2^j-2}\sum_{k'=-1}^{2^j-2}d^{3}_{j,k,k'}\psi^{3}_{j,k,k'}(x,y)\right] \quad \ldots (4.2)$$

Where C and $\Psi(x,y)$ are $\left(2^{(M+1)}+1\right)^{2}\times 1$ vectors are given by:

$$C=\left[c_{-1,-1},c_{-1,0},\ldots,c_{3,3},d^{1}_{2,-1,-1},\ldots,d^{1}_{2,3,2},d^{2}_{2,-1,-1},\ldots,d^{2}_{2,2,3},d^{3}_{2,-1,-1},\ldots,d^{3}_{2,2,2},d^{1}_{3,-1,-1},\ldots,d^{1}_{3,3,6},\right.$$
$$\left.d^{2}_{3,-1,-1},\ldots,d^{2}_{3,6,3},d^{3}_{3,-1,-1},\ldots,d^{3}_{3,6,6},d^{1}_{M,-1,-1},\ldots,d^{1}_{M,3,2^M-2},d^{2}_{M,-1,-1},\ldots,d^{2}_{M,2^M-2,3},d^{3}_{M,-1,-1},\ldots,d^{3}_{M,2^M-2,2^M-2}\right] \quad \ldots (4.3)$$

$$\Psi=\left[\phi^{0}_{2,-1,-1}(x,y),\phi^{0}_{2,-1,0}(x,y),\ldots,\phi^{0}_{2,3,3}(x,y),\psi^{1}_{2,-1,-1}(x,y),\ldots,\psi^{1}_{2,3,2}(x,y),\right.$$
$$\psi^{2}_{2,-1,-1}(x,y),\ldots,\psi^{2}_{2,2,3}(x,y),\psi^{3}_{2,-1,-1}(x,y),\ldots,\psi^{3}_{2,2,2}(x,y),\psi^{1}_{3,-1,-1}(x,y),\ldots,$$
$$\psi^{1}_{3,3,6}(x,y),\psi^{2}_{3,-1,-1}(x,y),\ldots,\psi^{2}_{3,6,3}(x,y),\psi^{3}_{3,-1,-1}(x,y),\ldots,\psi^{3}_{3,6,6}(x,y),\psi^{1}_{M,-1,-1}(x,y)$$
$$\left.,\ldots,\psi^{1}_{M,3,2^M-2}(x,y),\psi^{2}_{M,-1,-1}(x,y),\ldots,\psi^{2}_{M,2^M-2,3}(x,y),\psi^{3}_{M,-1,-1}(x,y),\ldots,\psi^{3}_{M,2^M-2,2^M-2}(x,y)\right] \quad \ldots (4.4)$$

With

$$c_{k,k'} = \int_0^1\int_0^1 f(x,y)\tilde{\phi}^{0}_{2,k,k'}(x,y)dxdy, \quad k,k' = -1,0,...,3 \qquad \text{... (4.5)}$$

$$d^{1}_{k,k'} = \int_0^1\int_0^1 f(x,y)\tilde{\psi}^{1}_{j,k,k'}(x,y)dxdy, \quad j = 2,3,...,M,$$
$$k = -1,0,...,3, k' = -1,0,...,2^j - 2,$$

$$d^{2}_{k,k'} = \int_0^1\int_0^1 f(x,y)\tilde{\psi}^{2}_{j,k,k'}(x,y)dxdy, \quad j = 2,3,...,M, \qquad \text{... (4.6)}$$
$$k = -1,0,...,2^j - 2, k' = -1,0,...,3,$$

$$d^{3}_{k,k'} = \int_0^1\int_0^1 f(x,y)\tilde{\psi}^{3}_{j,k,k'}(x,y)dxdy, \quad j = 2,3,...,M,$$
$$k,k' = -1,0,...,2^j - 2,$$

Where $\tilde{\phi}^{0}_{2,k,k'}(x,y)$ and $\tilde{\psi}^{t}_{j,k,k'}(x,y)$, $t = 1,2,3$ are dual functions of $\phi^0_{2,k,k'}(x,y)$ and $\psi^t_{j,k,k'}(x,y)$, $t = 1,2,3$, respectively.

These can be obtained by linear combinations of the following:

$\phi^0_{2,k,k'}(x,y)$, $\quad k,k' = -1,0,...,3,$

$\psi^1_{j,k,k'}(x,y)$, $\quad j = 2,3,...,M, k = -1,0,...,3, k' = -1,0,...,2^M - 2,$

$\psi^2_{j,k,k'}(x,y)$, $\quad j = 2,3,...,M, k = -1,0,...,2^M - 2, k' = -1,0,...,3,$ and

$\psi^3_{j,k,k'}(x,y)$, $\quad j = 2,3,...,M, \ k,k' = -1,0,...,2^M - 2,$ as follows.

Let

$$\Phi^0 = \left[\phi^0_{2,-1,-1}(x,y), \phi^0_{2,-1,0}(x,y), ..., \phi^0_{2,3,3}(x,y)\right]^T \qquad \text{... (4.7)}$$

$$\Psi^1 = \left[\psi^1_{2,-1,-1}(x,y), \psi^1_{2,-1,0}(x,y), ..., \psi^1_{2,3,2^M-2}(x,y)\right]^T \qquad \text{... (4.8)}$$

$$\Psi^2 = \left[\psi^2_{2,-1,-1}(x,y), \psi^2_{2,-1,0}(x,y), ..., \psi^2_{2,2^M-2,3}(x,y)\right]^T \qquad \text{... (4.9)}$$

$$\Psi^3 = \left[\psi^3_{2,-1,-1}(x,y), \psi^3_{2,-1,0}(x,y), ..., \psi^3_{2,2^M-2,2^M-2}(x,y)\right]^T \qquad \text{... (4.10)}$$

Using (4.7) we get:

$$\int_0^1\int_0^1 \Phi^0(x,y)\left(\Phi^0(x,y)\right)^{\mathrm{T}}dxdy = p^0$$

$$p^0 = \begin{bmatrix} \frac{1}{12}\cdot p_1 & \frac{1}{24}\cdot p_1 & 0\cdot p_1 & 0\cdot p_1 & 0\cdot p_1 \\ \frac{1}{24}\cdot p_1 & \frac{1}{6}\cdot p_1 & \frac{1}{24}\cdot p_1 & 0\cdot p_1 & 0\cdot p_1 \\ 0\cdot p_1 & \frac{1}{24}\cdot p_1 & \frac{1}{6}\cdot p_1 & \frac{1}{24}\cdot p_1 & 0\cdot p_1 \\ 0\cdot p_1 & 0\cdot p_1 & \frac{1}{24}\cdot p_1 & \frac{1}{6}\cdot p_1 & \frac{1}{24}\cdot p_1 \\ 0\cdot p_1 & 0\cdot p_1 & \frac{1}{24}\cdot p_1 & \frac{1}{6}\cdot p_1 & \frac{1}{24}\cdot p_1 \end{bmatrix},$$

… (4.11)

Where $p_1 = \int_0^1 \Phi(x)\Phi^{\mathrm{T}}(x)dx$.

Using (4.8) we get:

$$\int_0^1\int_0^1 \Psi^1(x,y)\left(\Psi^1(x,y)\right)^{\mathrm{T}}dxdy = p^1 = \begin{bmatrix} \frac{1}{12}\cdot p_2 & \frac{1}{24}\cdot p_2 & 0\cdot p_2 & 0\cdot p_2 & 0\cdot p_2 \\ \frac{1}{24}\cdot p_2 & \frac{1}{6}\cdot p_2 & \frac{1}{24}\cdot p_2 & 0\cdot p_2 & 0\cdot p_2 \\ 0\cdot p_2 & \frac{1}{24}\cdot p_2 & \frac{1}{6}\cdot p_2 & \frac{1}{24}\cdot p_2 & 0\cdot p_2 \\ 0\cdot p_2 & 0\cdot p_2 & \frac{1}{24}\cdot p_2 & \frac{1}{6}\cdot p_2 & \frac{1}{24}\cdot p_2 \\ 0\cdot p_2 & 0\cdot p_2 & \frac{1}{24}\cdot p_2 & \frac{1}{6}\cdot p_2 & \frac{1}{24}\cdot p_2 \end{bmatrix},$$

… (4.12)

Where $p_2 = \int_0^1 \Psi(x)\Psi^{\mathrm{T}}(x)dx$.

Using (4.9) we get:

$$\int_0^1\int_0^1 \Psi^2(x,y)\left(\Psi^2(x,y)\right)^{\mathrm{T}} dxdy = p^2 = \begin{bmatrix} \frac{2}{27}\cdot p_1 & \frac{1}{96}\cdot p_1 & \frac{-1}{864}\cdot p_1 & 0\cdot p_1 & 0\cdot p_1 & \cdots & 0\cdot p_1 \\ \frac{1}{96}\cdot p_1 & \frac{1}{16}\cdot p_1 & \frac{5}{432}\cdot p_1 & \frac{-1}{864}\cdot p_1 & 0\cdot p_1 & \cdots & 0\cdot p_1 \\ \frac{-1}{864}\cdot p_1 & \frac{5}{432}\cdot p_1 & \frac{1}{16}\cdot p_1 & \frac{5}{432}\cdot p_1 & \frac{-1}{864}\cdot p_1 & \cdots & 0\cdot p_1 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0\cdot p_1 & \cdots & \frac{-1}{864}\cdot p_1 & \frac{5}{432}\cdot p_1 & \frac{1}{16}\cdot p_1 & \frac{1}{432}\cdot p_1 & \frac{-1}{864}\cdot p_1 \\ 0\cdot p_1 & \cdots & 0\cdot p_1 & \frac{-1}{864}\cdot p_1 & \frac{5}{432}\cdot p_1 & \frac{1}{16}\cdot p_1 & \frac{1}{96}\cdot p_1 \\ 0\cdot p_1 & \cdots & 0\cdot p_1 & 0\cdot p_1 & \frac{-1}{864}\cdot p_1 & \frac{1}{96}\cdot p_1 & \frac{2}{27}\cdot p_1 \end{bmatrix},$$

… (4.13)

Using (4.10) we get:

$$\int_0^1\int_0^1 \Psi^3(x,y)\left(\Psi^3(x,y)\right)^{\mathrm{T}} dxdy = p^3 = \begin{bmatrix} \frac{2}{27}\cdot p_2 & \frac{1}{96}\cdot p_2 & \frac{-1}{864}\cdot p_2 & 0\cdot p_2 & 0\cdot p_2 & \cdots & 0\cdot p_2 \\ \frac{1}{96}\cdot p_2 & \frac{1}{16}\cdot p_2 & \frac{5}{432}\cdot p_2 & \frac{-1}{864}\cdot p_2 & 0\cdot p_2 & \cdots & 0\cdot p_2 \\ \frac{-1}{864}\cdot p_2 & \frac{5}{432}\cdot p_2 & \frac{1}{16}\cdot p_2 & \frac{5}{432}\cdot p_2 & \frac{-1}{864}\cdot p_2 & \cdots & 0\cdot p_2 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0\cdot p_2 & \cdots & \frac{-1}{864}\cdot p_2 & \frac{5}{432}\cdot p_2 & \frac{1}{16}\cdot p_2 & \frac{1}{432}\cdot p_2 & \frac{-1}{864}\cdot p_2 \\ 0\cdot p_2 & \cdots & 0\cdot p_2 & \frac{-1}{864}\cdot p_2 & \frac{5}{432}\cdot p_2 & \frac{1}{16}\cdot p_2 & \frac{1}{96}\cdot p_2 \\ 0\cdot p_2 & \cdots & 0\cdot p_2 & 0\cdot p_2 & \frac{-1}{864}\cdot p_2 & \frac{1}{96}\cdot p_2 & \frac{2}{27}\cdot p_2 \end{bmatrix},$$

… (4.14)

Where p^0, p^1, p^2 and p^3 are 25×25, $\left(5\left(2^{M+1}-4\right)\right)\times\left(5\left(2^{M+1}-4\right)\right)$, $\left(5\left(2^{M+1}-4\right)\right)\times\left(5\left(2^{M+1}-4\right)\right)$ and $\left(2^{M+1}-4\right)^2\times\left(2^{M+1}-4\right)^2$ matrices, respectively.

Suppose $\tilde{\Phi}^0, \tilde{\Psi}^1, \tilde{\Psi}^2$ and $\tilde{\Psi}^3$ are the dual functions of Φ^0, Ψ^1, Ψ^2 and Ψ^3, respectively, given by:

$$\tilde{\Phi}^0 = \left[\tilde{\phi}^0_{2,-1,-1}(x,y), \tilde{\phi}^0_{2,-1,0}(x,y), \ldots, \tilde{\phi}^0_{2,3,3}(x,y)\right]^{\mathrm{T}}$$ … (4.15)

$$\tilde{\Psi}^1 = \left[\tilde{\psi}^1_{2,-1,-1}(x,y), \tilde{\psi}^1_{2,-1,0}(x,y), \ldots, \tilde{\psi}^1_{2,3,2^M-2}(x,y)\right]^{\mathrm{T}}$$ … (4.16)

$$\tilde{\Psi}^2 = \left[\tilde{\psi}^2_{2,-1,-1}(x,y), \tilde{\psi}^2_{2,-1,0}(x,y), \ldots, \tilde{\psi}^2_{2,2^M-2,3}(x,y)\right]^{\mathrm{T}}$$ … (4.17)

$$\tilde{\Psi}^3 = \left[\tilde{\psi}^3_{2,-1,-1}(x,y), \tilde{\psi}^3_{2,-1,0}(x,y), \ldots, \tilde{\psi}^3_{2,2^M-2,2^M-2}(x,y)\right]^{\mathrm{T}}$$ … (4.18)

Using (4.5)-(4.10), and (4.11)-(4.14) we have:

$$\int_0^1\int_0^1 \tilde{\Phi}^0 \left(\Phi^0\right)^{\mathrm{T}} dxdy = \mathrm{I}_0,$$
$$\int_0^1\int_0^1 \tilde{\Psi}^1 \left(\Psi^1\right)^{\mathrm{T}} dxdy = \mathrm{I}_1,$$
$$\int_0^1\int_0^1 \tilde{\Psi}^2 \left(\Psi^2\right)^{\mathrm{T}} dxdy = \mathrm{I}_2, \qquad \ldots (4.19)$$
$$\int_0^1\int_0^1 \tilde{\Psi}^3 \left(\Psi^3\right)^{\mathrm{T}} dxdy = \mathrm{I}_3,$$

Where $\mathrm{I}_0, \mathrm{I}_1, \mathrm{I}_2,$ and I_3 are 25×25, $\left(5\left(2^{M+1}-4\right)\right)\times\left(5\left(2^{M+1}-4\right)\right)$, $\left(5\left(2^{M+1}-4\right)\right)\times\left(5\left(2^{M+1}-4\right)\right)$ and $\left(2^{M+1}-4\right)^2\times\left(2^{M+1}-4\right)^2$ identity matrices, respectively.

Then (4.47)-(4.50), and (4.55) give:

$$\tilde{\Phi}^0 = \left(p^0\right)^{-1}\Phi^0,$$
$$\tilde{\Psi}^1 = \left(p^1\right)^{-1}\Psi^1, \qquad \ldots (4.20)$$
$$\tilde{\Psi}^2 = \left(p^2\right)^{-1}\Psi^2,$$
$$\tilde{\Psi}^3 = \left(p^3\right)^{-1}\Psi^3,$$

3. TECHNIQUE SOLUTION OF NONLINEAR TWO DIMENSIONAL FHIE's:

According to this technique one can solve nonlinear two-dimensional FHIE's of second kind of the form:

$$u(x,y) = f(x,y) + \int_0^1\int_0^1 k(x,y,t,s)\, g\left(t,s,u(t,s)\right) dtds, \qquad (x,y)\in D = [0,1]\times[0,1] \qquad \ldots. (5.1)$$

by using two-dimensional B-spline wavelet. For this purpose, we first assume:

$$z(x,y) = g\left(t,s,u(t,s)\right), \qquad (x,y)\in D = [0,1]\times[0,1] \qquad \ldots (5.2)$$

Now using (4.38) to approximate $u(x,y),\ \ z(x,y)$ as

$$u(x,y) = B^{\mathrm{T}}\Psi(x,y), \quad z(x,y) = E^{\mathrm{T}}\Psi(x,y) \qquad \ldots (5.3)$$

Where $\Psi(x,y)$ is defined in (4.5) and B, E are $\left(2^{(M+1)}+1\right)^2\times 1$ unknown vectors defined similarly to C in (4.3).

Also expand $f(x,y), k(x,y,t,s)$ by two-dimensional B-spline dual wavelets $\tilde{\Psi}$ defined as in (4.15)-(4.18) as:

$$f(x,y) = \Lambda^{\mathrm{T}}\tilde{\Psi}(x,y),\ \ k(x,y,t,s) = \tilde{\Psi}(t,s)\Theta\tilde{\Psi}(x,y), \qquad \ldots (5.4)$$

Where Λ is $\left(2^{(M+1)}+1\right)^2\times 1$ known vectors defined as:

$$\Lambda^{\mathrm{T}} = f(x,y)\Psi^{\mathrm{T}}(x,y) \qquad \ldots (5.5)$$

And Θ is $\left(2^{(M+1)}+1\right)^2 \times \left(2^{(M+1)}+1\right)^2$ known matrix given by:

$$\Theta_{(i,j)} = \int_0^1\int_0^1 \left[\int_0^1\int_0^1 k(x,y,t,s)\Psi_i(t,s)dtds \right] \Psi(x,y)_j \, dxdt \qquad \ldots (5.6)$$

From (5.2), (5.3), and (5.4) we get:

$$\begin{aligned} \int_0^1\int_0^1 k(x,y,t,s)\, g(t,s,u(t,s))dtds &= \int_0^1\int_0^1 E^{T}\Psi(t,s)\tilde{\Psi}^{T}(t,s)\Theta\Psi(x,\tilde{y})dtds \\ &= E^{T}\left[\int_0^1\int_0^1 \Psi(t,s)\tilde{\Psi}^{T}(t,s)dtds\right]\Theta\tilde{\Psi}(x,y) \\ &= E^{T}\Theta\tilde{\Psi}(x,y) \end{aligned}$$

… (5.7)

Applying (5.2)-(5.7) in (5.1), we get:

$$B^{T}\Psi(x,y) = \Lambda^{T}\tilde{\Psi}(x,y) + E^{T}\Theta\tilde{\Psi}(x,y); \qquad \ldots (5.8)$$

Multiplying (5.8) by $\Psi^{T}(x,y)$

$$B^{T}\Psi(x,y)\Psi^{T}(x,y) = \Lambda^{T}\tilde{\Psi}(x,y)\Psi^{T}(x,y) + E^{T}\Theta\tilde{\Psi}(x,y)\Psi^{T}(x,y)$$

Integrating from 0 to 1,

$$\int_0^1\int_0^1 B^{T}\Psi(x,y)\Psi^{T}(x,y)dxdy = \int_0^1\int_0^1 \Lambda^{T}\tilde{\Psi}(x,y)\Psi^{T}(x,y)dxdy + \int_0^1\int_0^1 E^{T}\Theta\tilde{\Psi}(x,y)\Psi^{T}(x,y)dxdy$$

$$B^{T}\int_0^1\int_0^1 \Psi(x,y)\Psi^{T}(x,y)dxdy = \Lambda^{T}\int_0^1\int_0^1 \tilde{\Psi}(x,y)\Psi^{T}(x,y)dxdy + E^{T}\Theta\int_0^1\int_0^1 \tilde{\Psi}(x,y)\Psi^{T}(x,y)dxdy$$

$$B^{T}\bar{p} = \Lambda^{T} + E^{T}\Theta$$

$$B^{T}\bar{p} - E^{T}\Theta = \Lambda^{T} \qquad \ldots (5.9)$$

In which $\bar{p}$ is a $\left(2^{(M+1)}+1\right)^2 \times \left(2^{(M+1)}+1\right)^2$ square matrix given by the following:

$$\bar{p} = \int_0^1\int_0^1 \Psi(x,y)\Psi^{T}(x,y)dxdy = \begin{bmatrix} p^0 & & & \\ & p^1 & & \\ & & p^2 & \\ & & & p^3 \end{bmatrix}. \qquad \ldots (5.10)$$

To find the solution $y(x,y)$ in (5.3), we first calculate the following equation in $x_i = i/2^{(M+1)}+1, \quad y_i = i/2^{(M+1)}+1$:

$$g\left(x,y,B^{T}\Psi(x,y)\right) = E^{T}\Psi(x,y) \qquad \ldots (5.11)$$

Equation (5.9) generates a set of $\left[2^{(M+1)}+1\right]^2$ algebraic equations. The total number of unknowns for vectors B and E in (5.3) is $2\left[2^{(M+1)}+1\right]^2$. these can be obtained by using (5.9) and (5.11).

4. ILLUSTRATIVE EXAMPLE:

Example:

Consider the equation

$$u(x,y)=x^2+\frac{1}{4}y\left(7+e^{-1}+e^{-2}-e^{-3}\right)-3+\int_0^1\int_0^1 ty\cdot e^{(u(t,s))}dtds,$$

Exact solution of this problem is $u(x,y)=x^2+2y-3.$ the solution for $u(x,y)$ is obtained by the method in section 5. The computational results for $M=2,$ and $M=4$ together with the exact solution $u(x,y)=x^2+2y-3$ are given in Table 6.1.

Table 6.1. Exact and approximate solutions.

(x,y)	Approximate M=2	Approximate M=4	Exact
(0,0)	-3.0100	-3.0000	-3.0000
(0,0.25)	-2.5784	-2.6001	-2.6000
(0,0.5)	-2.3287	-2.0001	-2.0000
(0,0.75)	-1.6909	-1.5998	-1.6000
(0,1)	-1.1761	-1.0000	-1.0000
(0.25,0)	-2.9203	-2.9600	-2.9600
(0.25,0.25)	-2.5692	-2.5602	-2.5600
(0.25,0.5)	-1.9332	-1.9601	-1.9600
(0.25,0.75)	-1.6265	-1.5599	-1.5600
(0.25,1)	-1.0565	-0.9602	-0.9600
(0.5,0)	-2.6710	-2.7499	-2.7500
(0.5,0.25)	-2.3061	-2.3501	-2.3500
(0.5,0.5)	-1.7276	-1.7502	-1.7500
(0.5,0.75)	-1.3716	-1.3502	-1.3500
(0.5,1)	-0.8098	-0.7499	-0.7500
(0.75,0)	-2.4010	-2.5099	-2.5100
(0.75,0.25)	-2.0281	-2.1101	-2.1100
(0.75,0.5)	-1.4696	-1.5101	-1.5100
(0.75,0.75)	-1.0965	-1.1099	-1.1100
(0.75,1)	-0.5376	-0.5102	-0.5100
(1,0)	-1.8477	-2.0001	-2.0000
(1,0.25)	-1.4762	-1.5998	-1.6000
(1,0.5)	-0.9140	-1.0002	-1.0000
(1,0.75)	-0.5441	-0.6000	-0.6000
(1,1)	0.0004	0	0

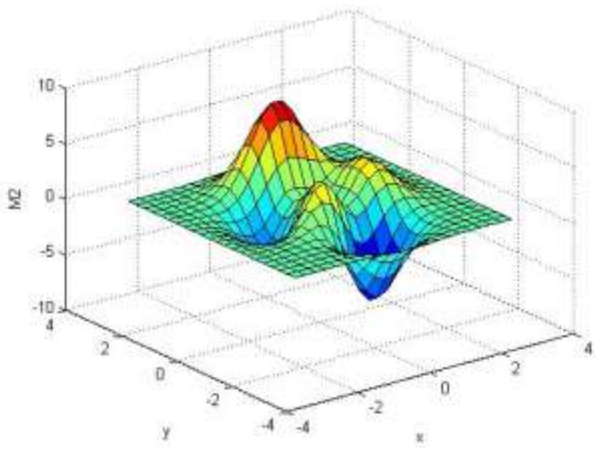

FIG (4) ***M2***

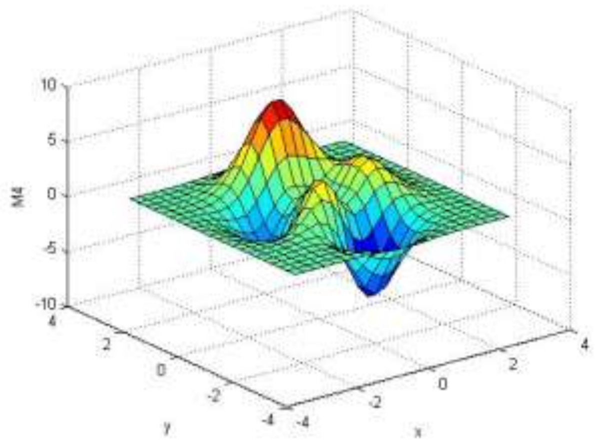

FIG (5) ***M***4

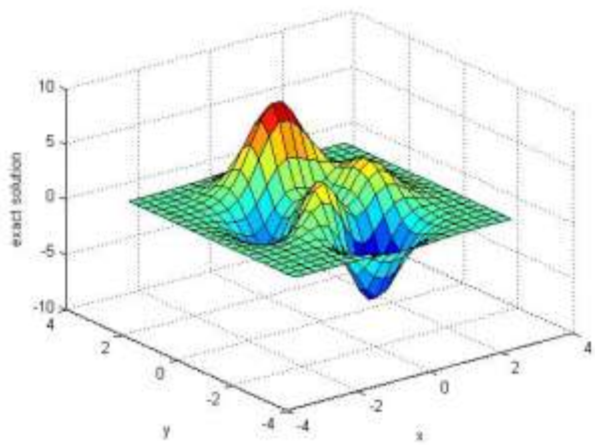

FIG (6) ***exact solution***

ISSN 2347-1921

CONCLUSION:

Generalize the two-dimensional second order (linear) semiorthogonal B-spline wavelet to solve two-dimensional nonlinear FHIE's of the second kind. Introduce numerical examples which give a good agreement with exact results.

REFERENCE:

[1] Abdou M. A. **''On Asymptotic Methods for Fredholm-Volterra Integral Equation of the Second Kind in Contact Problems''** J. Computational and Applied Mathematics, Vol. 154 (2003), pp. 431-446.

[2] Derili et al, **''Two-Dimensional wavelets numerical solution of Integral Equations''** Mathematical sciences 2012, 5:6.

[3] Lakestani M., Razzaghi M., and Dehghan M., **''Solution of Nonlinear Fredholm-Hammerstein Integral Equations by Using Semiortogonal Spline Wavelets''** Mathematical problems in Engineering (2005), pp. 113-121.

[4] Lakestani M., Razzaghi M., and Dehghan M., **''semiorthogonal spline wavelets approximation for Fredholm integro-differential equations''** Mathematical problems in Engineering (2006), pp. 1-12.

[5] Linz, P. **''Analytical and numerical Methods for Volterra Equations''** SIAM Philadelphia, 1985.

[6] Michael U, Philippe T, and Akram A. **''Shift-orthogonal Wavelet Bases''**, IEEE Transactions on signal processing, Vol. 46, No. 7, (1998), pp. 1827-1836.

[7] Nevels R. D., Goswami J. C., and Tehrani H., **''Semi-orthogonal versus orthogonal wavelet basis sets for solving integral equations''**, IEEE Trans. Antennas Propagate, vol. 45 (1997), no. 9, pp. 1332-1339.

[8] Rabbani M. **''Constructing Two-Dimensional Multi-Wavelet for Solving Two-Dimensional Fredholm Integral Equations''** Journalof Applied Mathematics, Islamic Azad University of Lahijan, Vol. 7, No. 4(27), winter 2011, pp 45-54.

[9] Su C. and Sarkar T. K. **''A Multiscale Moment Method for Solving Fredholm Integral Equation of the First kind''**,Progress In Electromagnetic Research, PIER 17, (1997), pp. 237-264.

[10] Wagner and Chew **''A Study of Wavelets for the Solution of Electromagnetic Integral Equations''**, IEEE Trans. Antennas Propagate, vol. 43 (1995), no. 8, pp. 802-810.

[11] Zhang P.W. and Zhang Y. **''Wavelet Method for Boundary Integral Equations''**, Journal of Computational Mathematics, vol. 18 (2000), no. 1, pp. 25-42.