ISSN 2347-1921
Volume 12 Number 4
Journal of Advances in Mathematics

The equivalent Cauchy sequences in partial metric spaces

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Abstract: In this paper we prove some conditions for equivalent Cauchy sequences in partial metric spaces. These conditions are necessary and sufficient for 0-equivalent 0-Cauchy sequences in partial metric spaces. Some examples are given to illustrate the observed results.

Keywords: Partial metric space; equivalent Cauchy sequences; 0-equivalent 0-Cauchy sequences.

Academic Discipline And Sub-Disciplines

Mathematics, Functional Analysis.

SUBJECT CLASSIFICATION

Functional Analysis

1. Introduction.

The notion of a partial metric space was introduced by G.S. Metthews [7, 8] in 1992. The partial metric space is a generalization of the usual metric spaces in which the distance of a point from itself may not be zero. Recently, many authors have been focused on the partial metric spaces and its topological properties. [1, 9,10].They show that partial metric spaces have many applications both in mathematics and computer science [5, 10]. The concept of Cauchy sequences and equivalent Cauchy sequences are very important in functional analysis and especially in fixed point theory.

In 1983 Leader [6] obtained a sufficient and necessary condition as a characterization of equivalent Cauchy sequences.

In 2001 Bushati [2] has given some new conditions for two sequences to be equivalent Cauchy in metric spaces. In 2014,Hoxha at all [3] generalized these condition in dislocated metric spaces and quasi-dislocated metric spaces.

In this paper we will show some condition about equivalent sequences and equivalent Cauchy sequences and 0-equivalent 0-Cauchy sequences in partial metric spaces.

2. Preliminaries.

For convenience we start with the following definitions, lemmas, and theorems.

Definition 1. [7] A function $p: X \times X \to R^+$ is a partial metric on X if, for all $x, y, z \in X$, the following condition hold:

p1) $x = y$ if and only if $p(x,x) = p(x,y) = p(y,y)$,

p2) $p(x,x) \le p(x,y)$

p3) $p(x,y) = p(y,x)$,

p4) $p(x,y) \le p(x,z) + p(z,y) - p(z,z)$

In this case, the pair (X, p) is called a partial metric space.

It is clear that if $p(x,y) = 0$ then from (p1) and (p2), $x = y$. But, if $x = y$, $p(x,y)$ may not be 0. As example of partial metric space is, (R^+, p) where $p(x,y) = \max\{x,y\}$.

Each partial metric p on X generates a T_0-topology on X, which has as base the family of open p-balls $\{B_p(x,\varepsilon): x \in X, \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < \varepsilon + p(x,x)\}$ for all $x \in X$ and $\varepsilon > 0$

Definition 2. [7,8] A sequence $\{x_n\}$ in a partial metric space (X, p) is said to be:

(i) p-convergent to a point $x \in X$ if $\lim_{n\to\infty} p(x, x_n) = p(x,x)$;

(ii) p -Cauchy sequence if $\lim_{n,m\to\infty} p(x_m, x_n)$ exists and is finite.

Notice that the limit of sequence in partial metric space is not necessary unique.

Proposition 3. [8] Every partial metric p defines a metric d_p, where

$d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y)$ for all $x, y \in X$.

The metric d_p is called the metric associated with partial metric p .

Lemma 1. [7,8]

(1) A sequence $\{x_n\}$ is a p -Cauchy sequence in a partial metric space (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_p) .

Definition 4. The sequences (x_n) and (y_n) in a metric space (X, d) are called equivalent if $\lim_{n\to\infty} d(x_n, y_n) = 0$.

Definition 5. The sequences (x_n) and (y_n) in a partial metric space (X, p) are called equivalent if $\lim_{n\to\infty} p(x_n, y_n)$ exists and is finite.

Definition 6. The sequences (x_n) and (y_n) in a partial metric space (X, p) are called equivalent Cauchy if they are Cauchy and equivalent in (X, p) .

Definition 7. Let (X, p) be a partial metric space

i) A subset A in X is called bounded if there exists a real number M>0 such that $p(x, y) \le M$ for all $x, y \in A$;

ii) If A is bounded set of X, than the diameter of A is denoted by δ(A) and

$\delta(A) = \sup\{p(x, y); x, y \in A\}$

Lemma 2. [4]. Let (X, p) be a partial metric space. A sequence $\{x_n\}$ is a p -Cauchy sequence in a partial metric space (X, p) if and only if it satisfies the following condition:

(*) for each ε>0 there is $n_0 \in N$ such that $p(x_n, x_m) - p(x_n, x_n) < \varepsilon$ whenever $n_0 \le n \le m$

Definition 8.. Let (X, p) be a partial metric space. A sequence $\{x_n\}$ in X is called 0-Cauchy if $\lim_{n,m\to\infty} p(x_m, x_n) = 0$

Definition 9. The sequences (x_n) and (y_n) in a partial metric space (X, p) are called 0-equivalent if $\lim_{n\to\infty} p(x_n, y_n) = 0$.

Definition 10. The sequences (x_n) and (y_n) in a partial metric space (X, p) are called 0-equivalent 0-Cauchy if they are 0-Cauchy and 0-equivalent in (X, p) .

3. Main Rezults.

Theorem 1. If the sequences (x_n) and (y_n) are equivalent Cauchy in (X, d_p), than they are equivalent Cauchy in partial metric space (X, p) .

Proof. Since (x_n) dhe (y_n) are equivalent in metric space (X, d_p) than $\lim_{n\to\infty} d_p(x_n, y_n) = 0$.

So

$$\lim_{n\to\infty}[\ 2p(x_n,y_n)-p(x_n,x_n)-p(y_n,y_n)]=0\,. \qquad (1)$$

Since $p(x_n,x_n)\le p(x_n,y_n)$ and $p(y_n,y_n)\le p(x_n,y_n)$ and (1) holds, than we have

$$\lim_{n\to\infty}[p(x_n,y_n)-p(x_n,x_n)]=0 \qquad \lim_{n\to\infty}[p(x_n,y_n)-p(y_n,y_n)]=0 \qquad (2)$$

The sequences (x_n) and (y_n) are Cauchy in (X,d_p), than they are Cauchy in (X,p) .

From lemma 2 we have that the sequences (x_n) and (y_n) satisfy the condition (*) in Lemma 2.

So, in the same way as in the proof of lemma 2 in [4], we can proof that sequences $\{p(x_n,x_n\}$ and $\{p(y_n,y_n\}$ converges for the Euclidean metric on R^+ .

Let be $\lim_{n\to\infty} p(x_n,x_n)=a$

Note that $|p(x,x)-p(y,y)|\le d_p(x,y)$ for all $x,y\in X$. So, for $x=x_n$ dhe $y=y_n$ we have $|p(x_n,x_n)-p(y_n,y_n)|<d_p(x_n,y_n)$.

By the equivalence of the sequences (x_n) and (y_n) in (X,d_p), we have

$$\lim_{n\to\infty} p(x_n,x_n)=\lim_{n\to\infty} p(y_n,y_n)=a \qquad (3)$$

From (2) and (3) we have $\lim_{n\to\infty} p(x_n,y_n)=\lim_{n\to\infty} p(x_n,x_n)=\lim_{n\to\infty} p(y_n,y_n)=a$

We conclude that sequences (x_n) and (y_n) are equivalent Cauchy in (X,p).

Remark 2. The converse of the theorem 1, is not true. For this we can see the following example.

Example 3.

Let X=R^+ and define a mapping $p:RxR\to R^+$ by

$$p(x,y)=\max\{x,y\}$$

Then, *p* is partial metric and (X,p) is a partial metric space.

Take the sequences (x_n)= $\frac{1}{n}$ and (y_n)= $(\frac{1}{2}+\frac{1}{n})$

These sequences are Cauchy, because

$$p(x_n,x_m)=p(\frac{1}{n},\frac{1}{m})=\max\left\{\frac{1}{n},\frac{1}{m}\right\}\to 0 \text{ whenever } n,m\to\infty$$

$$p(y_n,y_m)=p(\frac{1}{2}+\frac{1}{n},\frac{1}{2}+\frac{1}{m})=\max\left\{\frac{1}{2}+\frac{1}{n},\frac{1}{2}+\frac{1}{m}\right\}\to\frac{1}{2} \text{ whenever } n,m\to\infty$$

They are and equivalent in (X,p) because

$$\lim_{n\to\infty} p(x_n,y_n)=\lim_{n\to\infty} p(\frac{1}{n},\frac{1}{2}+\frac{1}{n})=\lim_{n\to\infty}\max\{\frac{1}{n},\frac{1}{2}+\frac{1}{n}\}=\lim_{n\to\infty}\left(\frac{1}{2}+\frac{1}{n}\right)=\frac{1}{2}$$

But the sequences (x_n)= $\frac{1}{n}$ and (y_n)= $(\frac{1}{2}+\frac{1}{n})$ although are Cauchy in (X,d_p) by the lemma 1, they are not equivalent in (X,d_p) because

$p(x_n, x_n) = \max\left\{\frac{1}{n}, \frac{1}{n}\right\} = \frac{1}{n} \to 0$ $p(y_n, y_n) = \max\left\{\frac{1}{2}+\frac{1}{n}, \frac{1}{2}+\frac{1}{n}\right\} = \frac{1}{2}+\frac{1}{n} \to \frac{1}{2}$ whenever n→∞, and

$$\lim_{n\to\infty} d_p(x_n, y_n) = \lim_{n\to\infty}[2p(x_n, y_n) - p(x_n, x_n) - p(y_n, y_n)] = 2\cdot\frac{1}{2} - 0 - \frac{1}{2} = \frac{1}{2} \neq 0$$

Remark 4.1) If the Cauchy sequences (x_n) and (y_n) satisfy the condition

$\lim_{n\to\infty} p(x_n, y_n) = \lim_{n\to\infty} p(x_n, x_n) = \lim_{n\to\infty} p(y_n, y_n)$ then the converse of theorem1 is true.

2) If the sequences (x_n) and (y_n) are 0- equivalent 0-Cauchy, than they satisfy the condition

$\lim_{n\to\infty} p(x_n, y_n) = \lim_{n\to\infty} p(x_n, x_n) = \lim_{n\to\infty} p(y_n, y_n)$ =0 and then the converse of theorem1 is true.

Let (x_n) and (y_n) be the sequences in partial metric space (X, p). Define $\delta_{ij} = \sup\{p(x_m, y_k) : m \geq i, k \geq j\}$ $\forall (i, j) \in N^2$. (4)

Preposition 5. Let (X, p) be a partial metric space and (x_n), (y_n) two sequences in it. If one $\delta_{i_0 j_0}$ is finite than all δ_{ij} are finite

Proof. Denote $A = \max\{p(x_{m,} x_{i_0}), 1 \leq m \leq i_0\}$ and $B = \max\{p(y_k, y_{j_0}) 1 \leq k \leq j_0\}$

We first prove that δ_{11} is finite.

By (4) we have $p(x_m, y_k) < \delta_{i_0 j_0}$, for $m \geq i_0$ and $k \geq j_0$.

For $m \geq i_0$ $k \leq j_0$ we have

$$p(x_m, y_k) \leq p(x_m, y_{j_0}) + p(y_{j_0}, y_k) - p(y_{j_0}, y_{j_0}) \leq \delta_{i_0 j_0} + B - p(y_{j_0}, y_{j_0})$$

For $m \leq i_0$, $k \geq j_0$ we have

$$p(x_m, y_k) \leq p(x_{m,} x_{i_0}) + p(x_{i_0}, y_k) - p(x_{i_0}, x_{i_0}) \leq A + \delta_{i_0 j_0} - p(x_{i_0}, x_{i_0})$$

For $m \leq i_0$, $k \leq j_0$ we have

$$p(x_m, y_k) \leq p(x_{m,} x_{i_0}) + p(x_{i_0}, y_{j_0}) + p(y_{j_0}, y_k) - p(x_{i_0}, x_{i_0}) - p(y_{j_0}, y_{j_0})$$
$$\leq A + \delta_{i_0 j_0} + B - p(x_{i_0}, x_{i_0}) - p(y_{j_0}, y_{j_0}).$$

So, $\delta_{11} \leq A + \delta_{i_0 j_0} + B - p(x_{i_0}, x_{i_0}) - p(y_{j_0}, y_{j_0})$ is finite. But $\delta_{ij} \leq \delta_{11}$ for $i, j \in N$, so $\delta_{ij} < +\infty$.

Corollary 6. Let (X, p) be a partial metric space and (x_n), (y_n) two sequences in it.

The sequences (x_n) and (y_n) are bounded if and only if δ_{11} is finite.

Proof. Denote M_1, M_2 restrictive constants for (x_n) and (y_n). Then, for $i, j \in N$ we have

$p(x_i, y_j) \leq p(x_i, x_{i_0}) + p(x_{i_0}, y_{i_0}) + p(y_{i_0}, y_j) - p(x_{i_0}, x_{i_0}) - p(y_{i_0}, y_{i_0}) =$

$p(x_i, x_{i_0}) + (p(x_{i_0}, y_{i_0}) - p(x_{i_0}, x_{i_0})) + (p(y_{i_0}, y_j) - p(y_{i_0}, y_{i_0})) \leq M_1 + (p(x_{i_0}, y_{i_0}) - p(x_{i_0}, x_{i_0})) + M_2$

for a fixed $i_0 \in N$. So, $\delta_{11} < +\infty$.

Conversely, if $\delta_{11} <+\infty$, let us show the statement for (x_n).

By the definition 7, $\delta((x_n)) = \sup\{p(x_i, x_j 0 : i, j \in N\}$

$p(x_i, x_j) \le p(x_i, y_i) + p(y_i, x_j) - p(y_i, y_i)$

$p(y_i, y_i) \le p(y_i, x_j)$ for $j \in N$ and $p(y_i, y_i) \ge 0$

$p(y_i, x_j) \le \delta_{11}$ and $p(y_i, x_j) - p(y_i, y_i) < \delta_{11}$.

So, $p(x_i, x_j) \le \delta_{11} + \delta_{11} \le 2\,\delta_{11}$ and $\delta((x_n)) = \sup\{p(x_i, x_j) : i, j \in N\}$ is finite and the sequence (x_n) is bounded. In the same way we can show that the sequence (y_n) is bounded.

Theorem 7. Let (X, p) be a partial metric space and (x_n), (y_n) two sequences in it. If the sequences (x_n), (y_n) satisfy one of the following conditions, than the sequences (x_n), (y_n) are equivalent Cauchy in (X, p).

(1) The sequences (x_n) and (y_n) are bounded in (X, p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0, +\infty), \exists \varepsilon_0 \in (0, \varepsilon)$ such that $\delta_{ij} \le \varepsilon + \delta \Rightarrow p(x_{i+r}, y_{j+r}) \le \varepsilon_0$ whenever $i, j \in N$

(2) The sequences (x_n) and (y_n) are bounded in (X, p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0, +\infty)$ such that $\delta_{ij} \le \varepsilon + \delta \Rightarrow \delta_{i+r, j+r} < \varepsilon$, whenever $i, j \in N$

(3) The sequences (x_n) and (y_n) are bounded in (X, p) and

$\forall n \in N, \exists \alpha_n \in (0, +\infty), \exists r \in N$, such that $\delta_{ij} < \alpha_n \Rightarrow \delta_{i+r, j+r} < \dfrac{1}{n}$ whenever $i, j \in N$

(4) The sequences (x_n) and (y_n) are bounded in (X, p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0, +\infty), \exists \varepsilon_0 \in (0, \varepsilon)$ such that $\delta_{ij} \le \varepsilon + \delta \Rightarrow \delta_{i+r, j+r} \le \varepsilon_0$ whenever $i, j \in N$

Proof.

Let (x_n) and (y_n) be the sequences in (X, p) satisfying (1). Define

$\alpha_n = \delta_{n,n} = \sup\left\{p(x_i, y_j), i \ge n, j \ge n\right\}$

The sequences (α_n) is decreasing and positive. Hence it converges and $\lim\limits_{n\to\infty} a_n = \inf\left\{\alpha_n : n \in N\right\} = a \ge 0$

Suppose that $a > 0$. From the condition (1) for $\varepsilon = a > 0$ there are $r \in N,\ \varepsilon_0 \in (0, \varepsilon)$ and $\delta > 0$

such that $\delta_{ij} \le \varepsilon + \delta \Rightarrow p(x_{i+r}, y_{j+r}) \le \varepsilon_0$ whenever $i, j \in N$ (5)

For this $\delta > 0$ exists $p \in N$ such that for $n \ge p \Rightarrow \alpha_n < a + \delta = \varepsilon + \delta$

For $i \ge p, j \ge p$ we have $\delta_{ij} \le \alpha_p = \delta_{p,p} < \varepsilon + \delta$ .By (5) we have $p(x_{i+r}, y_{j+r}) \le \varepsilon_0$.

But it is obvious that $i + r = k \ge p + r, j + r = l \ge p + r$, so $p(x_k, y_l) \le \varepsilon_0 < \varepsilon = a$, which is a contradiction. Hence we have $\lim\limits_{n\to\infty} a_n = \inf\left\{\alpha_n : n \in N\right\} = 0$.

Since $p(x_n, y_n) \le \alpha_n$ and $\lim_{n\to\infty} a_n = 0$ hold, then $\lim_{n\to\infty} p(x_n, y_n) = 0$ and the sequences (x_n) and (y_n) are equivalent. Furthermore $p(x_i, y_j) \le a_{\min\{i,j\}}$ and consequently $\lim_{i,j\to\infty} p(x_i, y_j) = 0$ (6)

Now, we show that the sequences (x_n) e (y_n) are Cauchy. Since $p(x_n, x_n) \le p(x_n, y_n)$, $p(y_n, y_n) \le p(x_n, y_n)$ and $\lim_{n\to\infty} p(x_n, y_n) = 0$, than

$\lim_{n\to\infty} p(x_n, x_n) = 0$ and $\lim_{n\to\infty} p(y_n, y_n) = 0$. (7)

So,

$p(x_n, x_m) \le p(x_n, y_n) + p(y_n, x_m) - p(y_n, y_n)$ and

$p(y_n, y_m) \le p(y_n, x_n) + p(x_n, y_m) - p(x_n, x_n)$ (8)

By (6), (7) and (8) we have that $\lim_{n,m\to\infty} p(x_n, x_m) = 0$, $\lim_{n\to\infty} p(y_n, y_m) = 0$, than the sequences (x_n) e (y_n) are Cauchy in (X, p).

(2) Let (x_n) and (y_n) be the sequences in (X, p) satisfying (2).

We first shall prove that (2) $\Rightarrow$ (3).

For $n \in N$, take $\varepsilon = \frac{1}{n}$ and by (2) we have that there exists $r \in N,\ \delta > 0$ and $\alpha_n = \delta + \frac{1}{n}$ such that $\delta_{ij} \le \varepsilon + \delta$ = $\alpha_n \Rightarrow \delta_{ij} < \varepsilon = \frac{1}{n}$ for $i, j \in N$.

Now, suppose (x_n) and (y_n) satisfying (3).

As in (1) the sequence $a_n = \delta_{n,n} = \sup\{p(x_i, y_j), i \ge n, j \ge n\}$ is a convergent sequence and

$\lim_{n\to\infty} a_n = \inf\{a_n : n \in N\} = a \ge 0$

Suppose that $a > 0$. From the condition (3) for $\varepsilon = a > 0$ exists $n \in N$ such that $\frac{1}{n} < \varepsilon$. For $\frac{1}{n} > 0$ exists $P \in N$ such that for $n > P$ we have $\varepsilon < a_n < \varepsilon + \frac{1}{n}$. Take $\alpha_n = \varepsilon + \frac{1}{n}$ in (3) and we have $\delta_{i,j} \le \delta_{P,P} = a_P < \varepsilon + \frac{1}{n}$ = $\alpha_n \Rightarrow \delta_{i+r,j+r} < \frac{1}{n} < \varepsilon = a$. But,

$a_{\max\{i+r,j+r\}} = \delta_{\max\{i+r,j+r\}} \le \delta_{i+r,j+r} < \frac{1}{n} < \varepsilon = a$, which is a contradiction. Hence we have $\lim_{n\to\infty} a_n = 0$.

In the same way as in (1) we can show that the sequences (x_n) and (y_n) are equivalent Cauchy in (X, p).

(4). Let (x_n) and (y_n) be the sequences in (X, p) satisfying (4).

It is clear that (4)$\Rightarrow$(2) and by (2) immediately follows that the sequences (x_n) and (y_n) are equivalent Cauchy in (X, p).

Remark 8. The converse of the theorem 7, is not true. For this we can see again example 3 above.

Let X=R^+ and define a mapping $p: RxR \to R^+$ by $p(x, y) = \max\{x, y\}$ as a partial metric.

The sequences (x_n)= $\frac{1}{n}$ and (y_n)= $(\frac{1}{2}+\frac{1}{n})$ are equivalent Cauchy in (X, p). But, δ_{ij} = $\frac{1}{2}$ for $i, j \in N$ and for $\varepsilon = \frac{1}{2}$, for any $\delta > 0$ and $r > 0$, though δ_{ij} = $\frac{1}{2}$ < $\varepsilon + \delta$ we have $\delta_{i+r, j+r} = \frac{1}{2} \geq \varepsilon$.

So, the sequences (x_n)= $\frac{1}{n}$ and (y_n)= $(\frac{1}{2}+\frac{1}{n})$ do not satisfy the condition (2).

In the same way we can show that these sequences do not satisfy and the conditions (1), (3) and (4).

Theorem 9. Let (X, p) be a partial metric space and (x_n), (y_n) two sequences in it. The sequences (x_n), (y_n) are 0-equivalent 0-Cauchy in (X, p) if and only if they satisfy one of the following conditions:

(1) The sequences (x_n) and (y_n) are bounded in (X, p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0, +\infty), \exists \varepsilon_0 \in (0, \varepsilon)$ such that $\delta_{ij} \leq \varepsilon + \delta \Rightarrow p(x_{i+r}, y_{j+r}) \leq \varepsilon_0$ whenever $i, j \in N$

(2) The sequences (x_n) and (y_n) are bounded (X, p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0, +\infty)$ such that $\delta_{ij} \leq \varepsilon + \delta \Rightarrow \delta_{i+r, j+r} < \varepsilon$, whenever $i, j \in N$

(3) The sequences (x_n) and (y_n) are bounded (X, p) and

$\forall n \in N, \exists \alpha_n \in (0, +\infty), \exists r \in N,$ such that $\delta_{ij} < \alpha_n \Rightarrow \delta_{i+r, j+r} < \frac{1}{n}$ whenever $i, j \in N$

(4) The sequences (x_n) and (y_n) are bounded (X, p) and

$\forall \varepsilon > 0, \exists r \in N, \exists \delta \in (0, +\infty), \exists \varepsilon_0 \in (0, \varepsilon)$ such that $\delta_{ij} \leq \varepsilon + \delta \Rightarrow \delta_{i+r, j+r} \leq \varepsilon_0$ whenever $i, j \in N$

Proof. We firs prove the "if" part. Let (x_n), (y_n) be 0-equivalent 0-Cauchy in (X, p).By the remark 4, the sequences (x_n), (y_n) be equivalent Cauchy in metric spaces (X, d_p). By [2] the conditions (1), (2) and (4) are equivalent to being of sequences (x_n), (y_n) equivalent Cauchy in a metric space.

So, now we can prove that if the sequences (x_n), (y_n) are 0-equivalent 0-Cauchy in (X, p), than they satisfy the condition (3).

Indeed, by the definition 8 and 9 we have

$$\lim_{i \to \infty} p(x_i, y_i) = \lim_{i, j \to \infty} p(x_i, x_j) = \lim_{i, j \to \infty} p(y_i, y_j) = 0 \qquad (5)$$

So, $p(x_i, y_j) \leq p(x_i, x_j) + p(x_j, y_j) - p(x_j, x_j)$ and by (5) we have $\lim_{i, j \to \infty} p(x_i, y_j) = 0$.

Than, for $n \in N$ there is $P \in N$ such that for $i > P$, j>P we have $p(x_i, y_j) < \frac{1}{n}$ and so $\delta_{PP} < \frac{1}{n}$. So for $\alpha_n > \frac{1}{n}$, $r = P$ we have $\delta_{ij} < \alpha_n \Rightarrow \delta_{i+r, j+r} < \delta_{PP} < \frac{1}{n}$ whenever $i, j \in N$.

So (3) hold.

The converse follows from Theorem 7.

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International Conference on Recent Advances in Pure and Applied Mathematics (ICRAPAM 2014) 6-9 November 2014, Antalya, Turkey

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