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Some Remarks on Lukasiwicz disjunction and conjunction operators On Intuitionistic Fuzzy Matrices

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ABSTRACT

Some properties of two operations - conjunction and disjunction from Lukasiwicz type – over Intuitionistic Fuzzy Matrices are studied.

Keywords and Phrases: Intuitionistic Fuzzy Set (IFS); Intuitionistic Fuzzy Matrix (IFM).

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1 .INTRODUCTION:

After the introduction of fuzzy theory in [1], intuitionistic fuzzy theory [2] has been found to be highly meaningful to consider vagueness. In [3] krassmir T.Atanassov defined so many new operations over intuitionistic fuzzy sets and some operations deals with uncertainty case of the element also. Using the theory of IFS, Im et al[9] defined the notion of IFMs. X.Zhang[7] studied Intuitionistic Fuzzy Value and introduced the concept of composition two Intuitionistic Fuzzy matrices. Z.Xu [6], Ronald R.Yager [8], Meenakshi A.R and Gandhimathi[5] and several authors discussed IFMs. Here we give some inequalities contains Lukasiwicz fuzzy conjunction and disjunction operators and some properties in IFMs. The above said operators are introduced by Atanassov in [4] over IFSs and so many operations were defined by him in [3] also. In [10] we define the above operations over intuitionistic fuzzy matrix and this work is the extension of our previous research deals with conjunction and disjunction operators in [10] and [11].

2. PRELIMINARIES:

Definition 2.1[2,6,7]:

Let a set $X = \{x_1, x_2, \dots \dots x_n\}$ be fixed, then an intuitionistic fuzzy set (IFS) can be defined as $A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle \ / \ x_i \in X\}$ which assigns to each element x_i a membership degree $\mu_A(x_i)$ and a non membership degree $\nu_A(x_i)$ with the condition $0 \leq \mu_A(x_i) + \nu_A(x_i) \leq 1$ for all $x_i \in X$.

Definition 2.2.[6][7]:

The 2-tuple $\alpha(x_i) = (\mu_\alpha(x_i), \nu_\alpha(x_i))$ is called an intuitionistic fuzzy value (IFV), if $\mu_\alpha(x_i) \in [0,1]$, $\nu_\alpha(x_i) \in [0,1]$ and $\mu_\alpha(x_i) + \nu_\alpha(x_i) \leq 1$.

Definition 2.3[6]:

Let $A = \left(a_{ij}\right)$ be a matrix of order $m \times n$, if all $a_{ij}\,(i = 1,2,..m, j = 1,2,\dots n)$ are IFVs, then A is called an intuitionistic fuzzy matrix (IFM).

Definition 2.4[3]:

Let A and B be two $IFS\ s$ and α ∈ $[0,1]$ then

$(i) A \subseteq B$ if and only if for all $x_i \in X$, $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$.

$(ii) \Box A \ = \{\langle x_i, \mu_A(x_i), 1 - \mu_A(x_i)\rangle / x_i \in X\}$

$(iii) C(A) = \ \{\langle\, x_i, K, L\rangle \ / \ x_i \in X\}$ where $K =$ max. $\mu_A(x_i)$ and $L = \nu_A(x_i)$.

$(iv) D_\alpha(A) = \ \{\langle x_i, \mu_A(x_i) + \alpha\pi_A(x_i), \nu_A(x_i) + (1-\alpha)\pi_A(x_i)\ \rangle / x_i \in X\}$, where $\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i)$.

Definition2.5[12,13,14]:

In [12] and [13] $Norm(A)$ was defined as follows. Let X be a nonempty universal set. The normalization of an IFS A denoted by Norm (A) and defined by

$$Norm(A) = \left\{\langle x, \frac{\mu_A(x)}{sup\, \mu_A(x)}, \frac{\gamma_A(x) - inf\ \gamma_A(x)}{1 - inf\ \gamma_A(x)}\rangle\right\}.$$

Some remarks were given about the definition of Norm (A) in [14] such that the above definition will be valid only if $\pi_A(x)$ is negligible. Here we consider that case only.

Definition 2.6[10,11]:

Consider that two elements in IFS ⟨x, x’⟩ and ⟨y, y’⟩ such that 0≤ x+x’≤ 1 and 0 ≤ y+y’ ≤ 1

Now the Lukasiwicz conjunction and disjunction operators are defined as follows

$$\langle x, x'\rangle \oplus \langle y, y'\rangle = \langle (x+y) \wedge 1, (x' + y' - 1) \vee 0\rangle$$

$$\langle x, x'\rangle \odot \langle y, y'\rangle = \langle (x+y-1) \vee 0, (x' + y') \wedge 1\rangle$$

Now we define all the previous operations on IFMs as follows.

Let $A = \left[\langle a_{ij}, a'_{ij}\rangle\right]$ and $B = [\langle b_{ij}, b'_{ij}\rangle]$ be two IFMs of order m× n.Then the ij^{th} element of all the operations are given below.

i. $A \oplus B = \langle (a_{ij} + b_{ij}) \wedge 1, (\, a'_{ij} + b'_{ij} - 1) \vee 0\rangle$

ii. $A \odot B = \langle (\, a_{ij} + b_{ij} - 1) \vee 0, (\, a'_{ij} + b'_{ij}) \wedge 1\rangle$

iii. $A \leq B \Rightarrow a_{ij} \leq b_{ij}$ and $a'_{ij} \geq b'_{ij}$

iv. $\Box A = \langle a_{ij}, 1 - a_{ij}\rangle$

v. $D_\alpha(A) = \langle a_{ij} + \alpha a''_{ij}, a'_{ij} + (1-\alpha) a''_{ij}\rangle$ Where $a''_{ij} = 1 - a_{ij} - a'_{ij}$, the value of indertermacy.

vi. An IFM $J = [\langle 1,0\rangle]$ for all entries is known as Universal Matrix and $O = [\langle 0,1\rangle]$ for all entries is known as Zero matrix.

vii. $A^{[k+1]} = A^{[k]} \oplus A$ For all k=1, 2, ….

viii. $A^{(k+1)} = A^{k} \odot A$ For all k=1, 2, …

ix. $Norm(A) = \langle \frac{a_{ij}}{\max\ a_{ij}}, \frac{a_{ij}^{'} - min\, a_{ij}{'}}{1 - min\, a_{ij}{'}} \rangle$ for all i,j.

x. If A is reflexive the $A \geq I_n$ where I_n is the identity IFM contains ⟨1,0⟩ when i=j otherwise ⟨0,1⟩.

xi. If A is irreflexive then $\langle a_{ij}, a_{ij}^{'} \rangle = \langle 1,0 \rangle\ when\ i = j$.

xii. $c[A] = \langle K, L \rangle$ where $K = \max_{i,j} a_{ij}\ and\ L = \min_{i,j} a_{ij}{'}$.

Here after F_{mn} means set of all Intuitionistic Fuzzy Matrices of order $m \times n$.

3. MAIN RESULTS:

Theorem 3.1:

For any two IFMs A , $B \in F_{mn}$, the following inequalities are true.

i. $D_\alpha(A \oplus B) \leq D_\alpha(A) \oplus D_\alpha(B)$

ii. $D_\alpha(A \odot B) \geq D_\alpha(A) \odot D_\alpha(B)$ for some $\alpha \epsilon [0,1]$

Proof :

(i)From the Definition 2.5, we have

$A \oplus B = \langle (a_{ij} + b_{ij}) \wedge 1, (a_{ij}^{'} + b_{ij}^{'} - 1) \vee 0 \rangle$ and

$$D_\alpha[A \oplus B] = [(a_{ij} + b_{ij}) \wedge 1 + \alpha\{1 - (a_{ij} + b_{ij}) \wedge 1 - (a_{ij}^{'} + b_{ij}^{'} - 1) \vee 0\},$$

$$(a_{ij}^{'} + b_{ij}^{'} - 1) \vee 0 + (1 - \alpha)\{1 - (a_{ij} + b_{ij}) \wedge 1 - (a_{ij}^{'} + b_{ij}^{'} - 1) \vee 0\}] \text{ ---3.1}$$

Now consider $D_\alpha(A) = [(a_{ij} + \alpha a_{ij}^{''}, a_{ij}^{'} + (1 - \alpha) a_{ij}{''}]$, where $a_{ij}^{''} = 1 - a_{ij} - a_{ij}{'}$

$D_\alpha(B) = [\ (b_{ij} + \alpha b_{ij}^{''}, b_{ij}^{'} + (1 - \alpha) b a_{ij}{''}]$, where $b_{ij}^{''} = 1 - b_{ij} - b_{ij}^{'}$

Using the above two equations we have

$$D_\alpha(A) \oplus D_\alpha(B) = [\{a_{ij} + b_{ij} + \alpha(a_{ij}^{''} + b_{ij}^{''})\} \wedge 1, \{a_{ij}^{'} + b_{ij}^{'} + (1 - \alpha)(a_{ij}^{''} + b_{ij}^{''}) - 1\} \vee 0]$$

$$- - - - - 3.2$$

Case (i)

If $a_{ij} + b_{ij} \geq 1$ and $a_{ij}^{'} + b_{ij}^{'} - 1 \leq 0$ then

3.1 becomes $D_\alpha[A \oplus B] = [1,0]$

3.2 becomes $D_\alpha(A) \oplus D_\alpha(B) = [1,0]$

Hence in this case $D_\alpha[A \oplus B] = D_\alpha(A) \oplus D_\alpha(B)$

Case (ii)

If $a_{ij} + b_{ij} \leq 1$ and $a_{ij}^{'} + b_{ij}^{'} - 1 \geq 0$ then

$$D_\alpha[A \oplus B] = [\ a_{ij} + b_{ij} + \alpha\{1 - (a_{ij} + b_{ij}) - (a_{ij}^{'} + b_{ij}^{'} - 1)\}, (a_{ij}^{'} + b_{ij}^{'} - 1)$$

$$+ (1 - \alpha)\{1 - (a_{ij} + b_{ij}) - (a_{ij}^{'} + b_{ij}^{'} - 1)\}]$$

$$= [\ a_{ij} + b_{ij} + \alpha(a_{ij}^{''} + b_{ij}^{''}), (a_{ij}^{'} + b_{ij}^{'} - 1) + (1 - \alpha)(a_{ij}^{''} + b_{ij}^{''})]$$

$$= D_\alpha(A) \oplus D_\alpha(B)$$

Case (iii)

If $a_{ij} + b_{ij} \leq 1$ and $(a_{ij}^{'} + b_{ij}^{'} - 1) \leq 0$ then

$$D_\alpha[A \oplus B] = [\ a_{ij} + b_{ij} + \alpha\{1 - (a_{ij} + b_{ij})\}, (1 - \alpha)\{1 - (a_{ij} + b_{ij})\}]$$

And the membership value of

$$D_\alpha(A) \oplus D_\alpha(B) = [a_{ij} + b_{ij} + \alpha\{1 - (a_{ij} + b_{ij}) + 1 - (a_{ij}^{'} + b_{ij}^{'})\}] \wedge 1$$

Since $a_{ij}{}' + b_{ij}{}' \leq 1 \Rightarrow 1 - (a_{ij}{}' + b_{ij}{}')$ is positive which means the following

$$[1 - (a_{ij} + b_{ij})] + [1 - (a_{ij}^{'} + b_{ij}^{'})] \geq [1 - (a_{ij} + b_{ij})]$$

Gives $membership\ value\ of\ D_\alpha[A \oplus B] \leq membership\ value\ of\ D_\alpha(A) \oplus D_\alpha(B)$

Now consider the nonmember ship value of $D_\alpha(A) \oplus D_\alpha(B)$ as follows

$[(a_{ij}^{'} + b_{ij}^{'} - 1) + (1 - \alpha)\{1 - (a_{ij} + b_{ij}) + 1 - (a_{ij}^{'} + b_{ij}^{'})\}] \vee 0$ which can be written as

$$[1 - (a_{ij} + b_{ij}) - \alpha\{1 - (a_{ij} + b_{ij}) + 1 - (a_{ij}^{'} + b_{ij}^{'})\}] \vee 0 \leq$$

$$[1 - (a_{ij} + b_{ij}) - \alpha\{1 - (a_{ij} + b_{ij})\}]$$

Which is the nonmember ship value of $D_\alpha[A \oplus B]$

Therefore in this case $D_\alpha[A \oplus B] \leq D_\alpha(A) \oplus D_\alpha(B)$.

Case (iv)

$If\ a_{ij} + b_{ij} \geq 1\ and (a_{ij}^{'} + b_{ij}^{'} - 1) \geq 0$ then A and B are not an IFMs.

Hence from all the above cases we conclude $D_\alpha[A \oplus B] \leq D_\alpha(A) \oplus D_\alpha(B)$.

(ii) From the Definition 2.5

$A \odot B = [(a_{ij} + b_{ij} - 1) \vee 0, (a_{ij}^{'} + b_{ij}^{'}) \wedge 1]$

Now $[A \odot B] = [(a_{ij} + b_{ij} - 1) \vee 0 + \alpha\{1 - (a_{ij} + b_{ij} - 1) \vee 0 - (a_{ij}^{'} + b_{ij}^{'}) \wedge 1\}, (a_{ij}^{'} + b_{ij}^{'}) \wedge 1 +$

$$(1 - \alpha)\{1 - (a_{ij} + b_{ij} - 1) \vee 0 - (a_{ij}^{'} + b_{ij}^{'}) \wedge 1\}]$$

Case (i)

If $(a_{ij} + b_{ij} - 1) \geq 0\ and\ (a_{ij}^{'} + b_{ij}^{'}) \leq 1$ then

$$D_\alpha[A \odot B] = [(a_{ij} + b_{ij} - 1) + \alpha\{a_{ij}^{''} + b_{ij}^{''}\}, (a_{ij}^{'} + b_{ij}^{'}) + (1 - \alpha)\{a_{ij}^{''} + b_{ij}^{''}\}]$$

Where $a_{ij}^{''} = 1 - a_{ij} - a_{ij}^{'}\ and\ b_{ij}^{''} = 1 - b_{ij} - b_{ij}^{'}$

$$= [a_{ij} + \alpha a_{ij}^{''} + b_{ij} + b_{ij}^{''} - 1, a_{ij}^{'} + (1 - \alpha)a_{ij}^{''} + b_{ij}^{'} + (1 - \alpha)b_{ij}^{''}]$$

$$= D_\alpha[A] \odot D_\alpha(B).$$

Case (ii)

If $(a_{ij} + b_{ij} - 1) \leq 0\ and\ (a_{ij}^{'} + b_{ij}^{'}) \geq 1$ then $D_\alpha[A \odot B] = [0,1]$ and

$$D_\alpha[A] \odot D_\alpha(B) = [(a_{ij} + \alpha a_{ij}^{''} + b_{ij} + \alpha b_{ij}^{''} - 1) \vee 0, \{a_{ij}^{'} + (1 - \alpha)a_{ij}^{''} + b_{ij}^{'} + (1 - \alpha)b_{ij}^{''}\} \wedge 1]$$

Now consider the membership value of $D_\alpha[A] \odot D_\alpha(B)$ which can be written as

$$[(a_{ij} + b_{ij} - 1) + \alpha\{1 - (a_{ij} + b_{ij} - 1) - (a_{ij}^{'} + b_{ij}^{'})\}] \vee 0 =$$

$$[(1 - \alpha)(a_{ij} + b_{ij} - 1) + \alpha - \alpha(a_{ij}^{'} + b_{ij}^{'})] \vee 0 = 0$$

Since $(1 - \alpha)(a_{ij} + b_{ij} - 1) \leq 0\ and\ \alpha - \alpha(a_{ij}^{'} + b_{ij}^{'}) \leq 0$

Similarly the nonmember ship value $D_\alpha[A] \odot D_\alpha(B)$ can be considered as

$a_{ij} + b_{ij}^{'} + (1 - \alpha)(a_{ij}^{''} + b_{ij}^{''})] \wedge 1 = 1\ since\ (1 - \alpha)(a_{ij}^{''} + b_{ij}^{''}) \geq 0$

Hence $D_\alpha[A] \odot D_\alpha(B) = [0,1] = D_\alpha[A \odot B]$.

Case (iii)

If $(a_{ij} + b_{ij} - 1) \leq 0\ and\ (a_{ij}^{'} + b_{ij}^{'}) \leq 1$ then we have $D_\alpha[A \odot B] = [\alpha\{1 - (a_{ij}^{'} + b_{ij}^{'})\}, (a_{ij}^{'} + b_{ij}^{'}) + (1 - \alpha)(1 - a_{ij}^{'} - b_{ij}^{'})]$

and $D_\alpha[A] \odot D_\alpha[B] = [\{(a_{ij} + b_{ij} - 1) + \alpha(a_{ij}^{''} + b_{ij}^{''})\} \vee 0, \{(a_{ij}^{'} + b_{ij}^{'}) + (1 - \alpha)(a_{ij}^{''} + b_{ij}^{''})\} \wedge 1]$

Now rewrite the membership value of the above as follows

$[(a_{ij} + b_{ij} - 1)(1 - \alpha) + \alpha\{1 - (a_{ij}^{'} + b_{ij}^{'})\}] \vee 0 \leq \alpha\{1 - (a_{ij}^{'} + b_{ij}^{'})\}$ Which is the membership value of $D_\alpha[A \odot B]$.

Similarly we can prove the nonmember ship value of $D_\alpha[A] \odot D_\alpha(B)$ ≥Nonmembership of $D_\alpha[A \odot B]$

Hence from all above cases $D_\alpha[A \odot B] \geq D_\alpha[A] \odot D_\alpha(B)$

Theorem 4.2:

For any two IFMs A , $B \in F_{mn}$, the following inequalities are valid

$(i) c[A] \oplus c[B] \geq c[A \oplus B]$

(ii) $c[A] \odot c[B] \geq c[A \odot B$

$(iii) c[D_\alpha(A)] \leq D_\alpha[c(A)]$

Poof :

(i) Let $A = [a_{ij}, a_{ij}^{'}]$ and $B = [b_{ij}, b_{ij}^{'}]$ be two IFMs of order m× n.

From the Definition 2.6, $c[A] = [\max_{i,j} a_{ij}, \min_{i,j} a_{ij}'], c[B] = [\max_{i,j} b_{ij}, \min_{i,j} b_{ij}']$

Now $c[A] \oplus c[B] = [\{\max_{i,j} a_{ij} + \max_{i,j} b_{ij}\} \wedge 1, \{\min_{i,j} a_{ij}^{'} + \min_{i,j} b_{ij}^{'} - 1\} \vee 0]$

We know that $A \oplus B = [(a_{ij} + b_{ij}) \wedge 1, (a_{ij}^{'} + b_{ij}^{'} - 1) \vee 0]$

$c[A \oplus B] = [\max_{i,j}(a_{ij} + b_{ij}) \wedge 1, \min_{i,j}(a_{ij}^{'} + b_{ij}^{'} - 1) \vee 0]$

For all i, j it is clear that $a_{ij} + b_{ij} \leq \max_{i,j} a_{ij} + \max_{i,j} b_{ij}$

$$\Rightarrow \max_{i,j}\left(a_{ij} + b_{ij}\right) \wedge 1 \leq \{\max_{i,j} a_{ij} + \max_{i,j} b_{ij}\} \wedge 1 \ldots\ldots\ldots\ldots 3.3$$

Similarly for all i, j we have $a_{ij}' + b_{ij}' \geq \min_{i,j} a_{ij}' + \min_{i,j} b_{ij}'$

$$\Rightarrow \min_{i,j}\left(a_{ij}^{'} + b_{ij}^{'} - 1\right) \vee 0 \geq \{\min_{i,j} a_{ij}^{'} + \min_{i,j} b_{ij}^{'} - 1\} \vee 0] \ldots\ldots\ldots\ldots 3.4$$

From 3.3 and 3.4 we have the inequality $c[A] \oplus c[B] \geq c[A \oplus B]$

(ii) It is trivial from the above.

(iii) $D_\alpha[A] = [a_{ij} + \alpha a_{ij}^{''}, a_{ij}^{'} + (1-\alpha) a_{ij}^{''}]$

Now $c[D_\alpha(A)] = [\max_{i,j}(a_{ij} + \alpha a_{ij}^{''}), \min_{i,j}(a_{ij}^{'} + (1-\alpha) a_{ij}^{''})]$

And $D_\alpha[c(A)] = [\max_{i,j} a_{ij,} + \alpha\{1 - \max_{i,j} a_{ij} - \min_{i,j} a_{ij}'\},$

$$\min_{i,j} a_{ij}^{'} + (1-\alpha)\{1 - \max_{i,j} a_{ij} - \min_{i,j} a_{ij}'\}]$$

for all i, j it is clear that $(a_{ij} + \alpha\{1 - a_{ij} - a_{ij}^{'}\}) \leq \max_{i,j} a_{ij,} + \alpha\{1 - \max_{i,j} a_{ij} - \min_{i,j} a_{ij}'\}$

Hence $\max_{i,j}(a_{ij} + \alpha\{1 - a_{ij} - a_{ij}^{'}\}) \leq \max_{i,j} a_{ij,} + \alpha\{1 - \max_{i,j} a_{ij} - \min_{i,j} a_{ij}'\}$

Similarly we can prove that

$$\min_{i,j}\{a_{ij}^{'} + (1-\alpha)(1 - a_{ij} - a_{ij}^{'})\} \geq \min_{i,j} a_{ij}^{'} + (1-\alpha)\{1 - \max_{i,j} a_{ij} - \min_{i,j} a_{ij}'\}]$$

From the above two inequalities we conclude that $c[D_\alpha(A)] \leq D_\alpha[c(A)]$.

Theorem 3.3:

For any IFM $A, B \in F_{mn}$, the following statements are valid

$(i) A^{[k]} = U(the\ universal\ matrix)\ for\ some\ k = 1,2 \ldots$

$(ii) A^{(k)} = O(the\ zero\ matrix)\ for\ some\ k = 1,2, \ldots$

$(iii) (A \oplus B)^T = A^T \oplus B^T$

$(iv) (A \odot B)^T = A^T \odot B^T$

Proof:

It is clear From the Definition 2.6.

Theorem 3.4:

If A and B are two IFMs then we have the following

$(i) \square[Norm(A \oplus B)] \leq \square[Norm(A)] \oplus \square[Norm(B)]$

$(ii) If\ (a_{ij}, a_{ij}^{'}) = (1,0)\ for\ atleast\ one\ i,j\ then\ Norm[A] = A$

$(iii) If\ A\ is\ reflexive\ or\ irreflexive\ then\ it\ is\ same\ for\ Norm[A]\ also.$

Proof:

(i) Let $A = [a_{ij}, a_{ij}{}'] \; and \; B = [b_{ij}, b_{ij}']$, From the Definition 2.6

$$A \oplus B = [(a_{ij} + b_{ij}) \wedge 1, (a_{ij}' + b_{ij}' - 1) \vee 0] \; , Norm[A] = \left[\frac{a_{ij}}{\max a_{ij}}, \frac{a_{ij}' - \min a_{ij}{}'}{1 - \min a_{ij}{}'}\right]$$

$$Norm[B] = \left[\frac{b_{ij}}{\max b_{ij}}, \frac{b_{ij}' - \min b_{ij}{}'}{1 - \min b_{ij}{}'}\right], \square Norm[A] = \left[\frac{a_{ij}}{\max a_{ij}}, 1 - \frac{a_{ij}}{\max a_{ij}}\right], \; \square Norm[B] = \left[\frac{b_{ij}}{\max b_{ij}}, 1 - \frac{b_{ij}}{\max b_{ij}}\right]$$

$$Norm[A \oplus B] = \left[\frac{(a_{ij} + b_{ij}) \wedge 1}{max\,(a_{ij} + b_{ij}) \wedge 1}, \frac{(a_{ij}' + b_{ij}' - 1) \vee 0 - min\,(a_{ij}' + b_{ij}' - 1) \vee 0}{1 - min\left(a_{ij}' + b_{ij}' - 1\right) \vee 0}\right]$$

$$\square Norm[A \oplus B] = \left[\frac{(a_{ij} + b_{ij}) \wedge 1}{max\,(a_{ij} + b_{ij}) \wedge 1}, 1 - \frac{(a_{ij} + b_{ij}) \wedge 1}{max\,(a_{ij} + b_{ij}) \wedge 1}]\right]$$

$$Now \;\; \square[Norm(A)] \oplus \square[Norm(B) = \left[\{\frac{a_{ij}}{\max a_{ij}} + \frac{b_{ij}}{\max b_{ij}}\} \wedge 1, \; \{1 - \frac{a_{ij}}{\max a_{ij}} + 1 - \frac{b_{ij}}{\max b_{ij}} - 1\} \vee 0]\right]$$

Since $\frac{a_{ij}}{max\, a_{ij}} \geq \frac{a_{ij}}{max\,(a_{ij} + b_{ij})} \; and \; \frac{b_{ij}}{max\, b_{ij}} \geq \frac{b_{ij}}{max\,(a_{ij} + b_{ij})} \;\; gives \;\; \frac{a_{ij}}{max\, a_{ij}} + \frac{b_{ij}}{max\, b_{ij}} \geq \frac{a_{ij} + b_{ij}}{max\,(a_{ij} + b_{ij})} \ldots \ldots \ldots \ldots 3.5$

$$\left\{\frac{a_{ij}}{maxa_{ij}} + \frac{b_{ij}}{maxb_{ij}}\right\} \wedge 1 \geq \frac{(a_{ij} + b_{ij}) \wedge 1}{max(a_{ij} + b_{ij}) \wedge 1} \ldots 3.6$$

From equation 3.5 we have

$$1 - \left\{\frac{a_{ij}}{maxa_{ij}} + \frac{b_{ij}}{maxb_{ij}}\right\} \leq 1 - \frac{a_{ij} + b_{ij}}{max(a_{ij} + b_{ij})}$$

$$. \;\; \left\{1 - \frac{a_{ij}}{max\, a_{ij}} - \frac{b_{ij}}{max\, b_{ij}}\right\} \vee 0 \leq 1 - \frac{(a_{ij} + b_{ij}) \wedge 1}{max\,(a_{ij} + b_{ij}) \wedge 1} \ldots \ldots ..3.7$$

From equation 3.6 and 3.7 we prove $\square[Norm(A \oplus B)] \leq \square[Norm(A)] \oplus \square[Norm(B)]$.

(ii) If at least one $(a_{ij}, a_{ij}') = (1{,}0) \; then \; maxa_{ij} = 1 \;\; and \;\; mina_{ij}' = 0$

Therefore from the definition $Norm[A] = \left[\frac{a_{ij}}{1}, \frac{a_{ij}' - 0}{1 - 0}\right] = [a_{ij}, a_{ij}'] = A$

(iii) Suppose A is reflexive then $A \geq I_n$, that is $(a_{ij}, a_{ij}') = (1{,}0) when \; i = j$

In this case clearly $Norm[A] = A \; which \; is \; reflexive \; from \; (ii)$

Suppose A is irreflexive then $(a_{ij}, a_{ij}') = (0{,}1) \; when \; i = j$

$$when \;\; i = j, Norm[A] = \left[\frac{a_{ii}}{maxa_{ij}}, \frac{a_{ii}' - mina_{ij}{}'}{1 - mina_{ij}{}'}\right] = \left[\frac{0}{maxa_{ij}}, \frac{1 - mina_{ij}{}'}{1 - mina_{ij}{}'}\right] = [0{,}1]$$

Hence $Norm[A] \; is \; irreflexive.$

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