SOFT α-COMPACTNESS VIA SOFT IDEALS

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Abstract:

In the present paper, we have continued to study the properties of soft topological spaces. We introduce new types of softcompactness based on the soft ideal Ĩ in a soft topological space (X, τ, E) namely, soft α_I-compactness, soft α_I-Ĩ-compactness, soft α-Ĩ-compactness, soft α-closed, soft α_I-closed,soft countablyα-Ĩ-compactness and soft countablyα_I-Ĩ-compactness. Also, several of their topological properties are investigated. Thebehavior of these concepts under various types of soft functions has obtained.

Keywords: Soft set; Soft topological space; Soft interior; Soft closure; Open soft; Closed soft; Soft α_I-compactness; Soft α-Ĩ-compactness; Soft α-closed; Soft α_I-closed,Soft countablyα-Ĩ-compactness; Soft countablyα_I-Ĩ-compactness.

1 Introduction

The concept of soft sets was first introduced by Molodtsov[10] in 1999 as a general mathematical tool for dealing with uncertain objects. Recently, in 2011, Shabir and Naz[11] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft neighborhood of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties.Hussain and Ahmad [3] investigated the properties of open (closed) soft, soft neighborhood and soft closure. They also defined and discussed the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces. Kandil et al. [6] introduced a unification of some types of different kinds of subsets of soft topological spaces using the notions of γ-operation. Kandil et al.[8] generalize this unification of some types of subsets of soft topological spaces using the notions of γ-operation to supra soft topological spaces. The notion of soft ideal is initiated for the first time by Kandil et al.[7]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal (X, τ, E, Ĩ). This paper, not only can form the theoretical basis for further applications of topology on soft sets, but also lead to the development of information system.

2 Preliminaries

In this section, we present the basic definitions and resultsof soft set theory which will be needed in the sequel.

Definition 2.1 [10]: Let X be an initial universe and E bea set of parameters. Let Ƥ(X) denote the power set of X and A be a non-empty subset of E. A pair (F, A) denotedby F_A is called a soft set over X, where F is a mapping given by F: A→ Ƥ(X). In other words, a soft set over Xis a parametrized family of subsets of the universe X. For a particular e ∈ A, F(e) may be considered the set of e-approximate elements of the soft set (F, A) and if e∉ A,then F(e) = Øi.e,
F_A ={F(e) : e∈ A⊆ E, F:A→ Ƥ(X)}. The family of all these soft sets over X denoted by $SS(X)_A$.

Definition 2.2 [9]: Let F_A, G_B∈$SS(X)_E$. Then F_A is softsubset of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if (1)A⊆ B, and (2)F(e) ⊆ G(e), for all e ∈ A. In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$.

Definition 2.3[9]: Two soft subsets F_A and G_B over acommon universe set X are said to be soft equal if F_A is asoft subset of G_B and G_B is a soft subset of F_A.

Definition 2.4 [2]: The complement of a soft set (F, A),denoted by$\tilde{X}$- (F, A), where $\tilde{X}$- (F, A) = (F′, A), F′: A → Ƥ(X) is a mapping given by F′(e) = X-F(e), for all e ∈ A. And F′ is called the soft complement function ofF, clearly (F′)′ is the same as F and $\tilde{X}$-($\tilde{X}$-(F, A))= (F, A).

Definition 2.5 [11]: The difference between two soft sets(F, E) and (G, E) over the common universe X, denoted by (F,E) -(G,E) is the soft set (H,E) where for all e∈E, H(e) = F(e) - G(e).

Definition 2.6 [11]:Let (F, E) be a soft set over X and x∈X. We say that x∈ (F, E) read as x belongs to the soft set (F, E) whenever x∈F(e) for all e∈ E.

Definition 2.7 [9]: A soft set (F, A) over X is said to be a NULL soft set denoted by $\widetilde{\emptyset}$ or $\emptyset_A$ if for all e∈ A, F(e) = {Ø}(null set).

Definition 2.8[9]**:** A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all e∈A, F(e) =X. Clearly we have $\widetilde{X}$- X_A = $\emptyset_A$ and $\widetilde{X}$- $\emptyset_A$ = X_A.

Definition 2.9 [9]: The union of two soft sets (F, A) and(G, B) over the common universe X is the soft set (H,C),where C= A ∪ B and for all e ∈ C,

$$H(e)=\begin{cases} F(e) & \text{if } e\in A-B,\\ G(e) & \text{if } e\in B-A,\\ F(e)\cup G(e) & \text{if } e\in A\cap B.\end{cases}$$

Definition 2.10 [9]: The intersection of two soft sets (F, A) and (G, B) over the common universe X is the softset (H, C), where C = A ∩ B and for all e ∈ C,
H(e) = F(e) ∩ G(e). Note that, in order to efficiently discuss, we consider only soft sets (F, E) over a universeX in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.

Definition 2.11 [12]: Let I be an arbitrary indexed set and L = {(F_i, E), i ∈I} be a subfamily of $SS(X)_E$.

i. The union of members ofL is the soft set (H, E), where H(e)=∪{F_i(e): for all e∈ E and for alli∈I}. We write $\widetilde{\cup}$ {(F_i, E): i∈I}= (H, E).
ii. The intersection of members of L is the soft set (M, E), where M(e) = ∩{F_i(e): for all e∈ E and for all i∈I}. We write $\widetilde{\cap}$ {(F_i, E): i∈I} = (M, E).

Definition 2.12[11]: Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then τ⊆$SS(X)_E$ is called a soft topology on X if

i. $\widetilde{X}$, $\widetilde{\emptyset}$∈τ, where $\widetilde{\emptyset}$(e) = {Ø} and $\widetilde{X}$(e) = X, for all e∈ E,
ii. the union of any number of soft sets in τ belongs to τ,
iii. the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.13[3]: Let(X, τ, E) be a soft topological space. The members of τ are said to be open soft sets inX. We denote the set of all open soft sets over X by OS(X, τ, E), or when there can be no confusion by OS(X).

Definition 2.14[3]: Let (X, τ, E) be a soft topologicalspace. A soft set (F, A) over X is said to be closed soft set in X, if its relative complement$\widetilde{X}$-(F, A) is an open soft set.We denote the set of all closed soft sets by CS(X,τ, E), or when there can be no confusion by CS(X).

Definition 2.15 [11]:Let (X, τ, E) be a soft topologicalspace and (F, E)∈$SS(X)_E$. The soft closure of (F, E),denoted by cl(F, E) is the intersection of all closed soft
supersets of (F, E). Clearly cl(F, E) is the smallest closedsoft set over X which contains (F, E), i.e. cl(F, E)= $\widetilde{\cap}$ {(H, E): (H, E) is closed soft set and (F, E)$\widetilde{\subseteq}$(H, E)}.

Definition 2.16 [12]:Let (X, τ, E) be a soft topological space and (F, E)∈$SS(X)_E$. The soft interior of (G, E),denoted by int(G, E) is the union of all open soft subsets of (G, E). Clearly int(G, E) is the largest open soft set over X which contained in (G, E), i.e. int(G, E)= $\widetilde{\cup}$ {(H, E): (H, E) is an open soft set and (H, E)$\widetilde{\subseteq}$ (G, E)}.

Definition 2.17 [12]:The soft set (F, E)∈ $SS(X)_E$ is called a soft point in X_E if there exist x∈ X and e∈ E such that F(e)= {x} and F(e′) = {Ø} for each e′∈ E- {e}, and the soft point (F, E) is denoted by x_e.

Definition 2.18[6]**:** Let (X, τ, E) be a soft topological space. A mapping γ: $SS(X)_E$ → $SS(X)_E$ is said to be anoperation on OS(X) if $F_E \widetilde{\subseteq} \gamma(F_E)$ for all F_E∈OS(X). The collection of all γ-open soft sets is denoted byOS(γ)={F_E: $F_E \widetilde{\subseteq} \gamma(F_E)$, F_E∈$SS(X)_E$}. Also, the complement of γ-open soft set is called γ-closed soft set,i.e.CS(γ)= {F'_E : F_E is a γ-open soft set, F_E∈ $SS(X)_E$} is the family of all γ-closed soft sets.

Definition 2.19[6]: Let (X, τ, E) be a soft topological space and γ be an operations on $SS(X)_E$. If γ= int(cl(int)), then γ is called α-open soft operator. We denote the set of all α-open soft sets by αOS(X, τ, E), or αOS(X) and the set of all α-closed soft sets by αCS(X, τ, E), or αCS(X).

Definition 2.20[1]: Let $SS(X)_A$ and $SS(Y)_B$ be two families of soft sets, u: X → Y and p: A → B be mappings. Let f_{pu}:$SS(X)_A$ → $SS(Y)_B$ be a mapping. Then;

i. If (F, A)∈$SS(X)_A$. Then the image of (F,A) under f_{pu},written as f_{pu}(F,A) = (f_{pu}(F), p(A)), is a soft set in $SS(Y)_B$ such that
$f_{pu}(F)(b)=\cup\{u(F(a)): a\in p^{-1}(b)\cap A\}$ if $p^{-1}(b)\cap A \neq \emptyset$

{Ø}otherwise for all b ∈ B.

ii. If (G,B)∈SS(Y)$_B$. Then the inverse image of (G,B) under f_{pu},written as f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B)), is a soft set in SS(X)$_A$ such that

f_{pu}^{-1}(G)(a)= u^{-1}(G(p(a))) if p(a)∈ B

{Ø} otherwise for all a∈ A.

The soft function $f_{pu}$is called surjective if p and u are surjective, also is said to be injective if p and u are injective.

Theorem 2.21 [1]: Let SS(X)$_A$ and SS(Y)$_B$ be families of soft sets. For the soft function f_{pu} :SS(X)$_A$ → SS(Y)$_B$,the following statements hold,

i. f_{pu}^{-1}(X̃-(G, B))= X̃- f_{pu}^{-1}((G, B)) for all (G,B)∈SS(Y)$_B$.
ii. f_{pu}(f_{pu}^{-1}((G, B)))⊑̃(G, B) for all (G,B)∈SS(Y)$_B$. If f_{pu} is surjective, then the equality holds.
iii. (F, A)⊑̃ f_{pu}^{-1}(f_{pu}((F, A))) for all (F,A)∈SS(X)$_A$. If $f_{pu}$is injective, then the equality holds.

Definition 2.22 [4]: A non-empty collection I of subsets of a set X is called an ideal on X, if it satisfies the following conditions;

i. A ∈I and B ∈I⇒ A ∪ B ∈I,
ii. A ∈I and B ⊆A⇒ B ∈I.
i.e. I is closed under finite unions and subsets.

Definition 2.23[7]: Let Ĩ be a non-null collection of softsets over a universe X with a fixed set of parameters E,then Ĩ⊆ SS(X)$_E$ is called a soft ideal on X with a fixed set E if it satisfies the following conditions;

i. (F, E) ∈Ĩ and (G, E) ∈Ĩ⇒ (F, E)⊔̃(G, E) ∈Ĩ,
ii. (F, E) ∈Ĩ and (G, E)⊑̃(F, E) ⇒ (G, E) ∈ Ĩ.
i.e. Ĩ is closed under finite soft unions and soft subsets.

Example 2.24[7]: Let X be a universe set. Then each ofthe following families is a soft ideal over X with the same set of parameters E,

i. Ĩ = {Ø̃},
ii. Ĩ=SS(X)$_E$ ={(F,E): (F,E); is a soft set over X with the fixed set of parameters E},
iii. Ĩ$_f$ ={(F, E) ∈SS(X)$_E$: (F, E) is finite}, called softideal of finite soft sets,
iv. Ĩ$_c$ ={(F,E)∈SS(X)$_E$: (F,E) is countable}, called soft ideal of countable soft sets,
v. Ĩ$_{(F,E)}$ ={(G, E)∈SS(X)$_E$: (G, E)⊑̃ (F, E)},
vi. Ĩ$_n$={(G, E)∈SS(X)$_E$: int(cl(G, E))=Ø̃}, called soft ideal of nowhere dense soft sets in (X, τ, E).

Definition 2.25[12]: A family ψ of soft sets is called a soft cover of a soft set (F, E) if (F, E)⊑̃⊔̃{(F_i, E): (F_i, E) ∈ ψ, i∈ I}.It is an open soft cover if each member of ψ is an opensoft set. A soft subcover of ψ is a subfamily of ψ which is also a soft cover of (F, E).

Definition 2.26[12]:A family ψ of soft sets is said to be have the finite intersection property (FIP for short) if the soft intersection of the members of each finite subfamily of ψ is not null soft set.

Definition 2.27[12]: A soft topological space (X, τ, E) is called soft compact space if each open soft cover of X̃ has a finite soft subcover.

3. Soft α-Ĩ-compact spaces

Definition 3.1: Let (X, τ, E, Ĩ) be a soft topological space with soft ideal Ĩand the same set of parameters E. A soft subset (F, E) of a space (X, τ, E, Ĩ) is called $α_I$-open soft set if there exists an open soft set (U, E) such that (U, E)-(F,E)∈Ĩ and (F, E)- int(cl((U,E)))∈Ĩ. The family of all $α_I$-open soft sets is denoted by $α_I$-S(X)$_E$.

Definition 3.2: A soft subset (F, E) of the space (X, τ, E, Ĩ) is called soft $α_I$-compact if every soft cover of (F, E) by $α_I$-open soft sets has a finite soft sub cover. The space(X, τ, E, Ĩ) is called soft $α_I$-compact ifX̃is soft $α_I$-compact as a soft subset.

Definition 3.3: A soft subset (F,E) of the space (X, τ, E, Ĩ) is called soft $α_I$-Ĩ-compact if for every soft cover {($G_λ$, E): λ∈Λ} of (F, E) by $α_I$-open soft setsthere is a finite subsetΛ$_0$ of Λ such that (F, E)-⊔̃{($G_λ$, E): λ∈Λ$_0$}∈Ĩ. The space(X, τ, E, Ĩ) is called soft $α_I$-Ĩ-compact if X̃is soft $α_I$-Ĩ-compact as a soft subset.

Remark 3.4: Let (X, τ, E, Ĩ) be a soft topological space with soft ideal Ĩwith the same set of parameters E. Then,

i. Every open soft set is α-open soft set, not conversely.
ii. Every α-open soft set is $α_I$-open soft set, not conversely.

Example 3.5: Let (X, τ, E, Ĩ) be a soft topological spacewith soft ideal Ĩwith the same set of parameters E, where X= E= ℕis the set of all natural numbers, Ĩ= $SS(X)_E$ and τ= {X̃, Ø̃}∪{(U_i,E): U_i:E → Ƥ(X) whereU_i(n)={1,2,…,i} for every i, n∈ ℕ}. Then;

i. The soft set (G,E), where G:E→ Ƥ(X) such that G(n)={1,2, 4, 6, …}for every n∈ℕ, is α-open soft set which is not open soft set.
ii. The soft set (G, E), where G:E→ Ƥ(X) such that G(n)={2, 4, 6, …} for every n∈ℕ, is $α_I$-open soft set which is not α-open soft set.

Remark 3.6: Let (X, τ, E, Ĩ) be a soft topological spacewith soft ideal Ĩwith the same set of parameters E. If Ĩ= {Ø̃}, then the concepts of α-open soft set and $α_I$-open soft set are equivalent.

Definition 3.7: A soft subset (F, E) of a soft topological space (X, τ, E) is called soft α-compact if every soft cover of (F, E) by α-open soft sets has a finite soft subcover. The soft topological space(X, τ, E) is called soft α-compact if X̃is soft α-compact as a soft subset.

Definition 3.8: A soft subset (F, E) of the space (X, τ, E, Ĩ) is called soft α-Ĩ-compact if for every soft cover {($G_λ$, E): λ∈ʌ} of (F, E) by α-open soft sets there is a finite subset$ʌ_0$ of ʌ such that (F, E)-Ũ{($G_λ$, E): λ∈$ʌ_0$}∈Ĩ. The soft topological space(X, τ, E, Ĩ) is called soft α-Ĩ-compact if X̃ is soft α-Ĩ-compact as a soft subset.

Remark 3.9: The following diagram shows the relations between the different types of compactness which introduced above;

soft□$_I$-compactness → soft α-compactness → soft compactness

↓ ↓ ↓

soft□$_I$-Ĩ-compactness → soft α-Ĩ-compactness → soft Ĩ-compactness

Diagram 1

The opposite direction of the implications in the above diagram may be false. For examples:

Example 3.10: See Example 3.5;sinceĨ= $SS(X)_E$ then, thesoft topological space(X, τ, E, Ĩ) is soft □$_I$-Ĩ-compact(respectively, soft α-Ĩ-compact and soft Ĩ-compact) spacewhich is not softcompact (respectively, not soft □-compact and not soft □$_I$-compact), since {(U_i, E): i∈ℕ}, where U_i(n)={1,2,…,i} for every i, n∈ℕ, is a cover of X̃by open (respectively, □-open and □$_I$-open) soft sets which has no finite subcover.

Example 3.11: Consider the soft topological space(X, τ, E, Ĩ)with soft ideal Ĩwith the same set of parameters E, where X= E= ℕis the set of all natural numbers, Ĩ={Ø̃}and τ= {X̃, Ø̃}∪{(U, E); U:E → Ƥ(X) such that U(n)={1} for every n∈E}. Then, (X, τ, E, Ĩ) is soft compact and soft Ĩ- compact. But it is not soft α-compact and not soft α-Ĩ-compact, since {(G_i, E): i∈ℕ}, whereG_i:E→ Ƥ(X) such that G_i(n)={1, i}for every i, n∈ℕ, is a cover of X̃by □-open soft sets which has no finite subcover.

Example 3.12: Consider the soft topological space(X, τ, E, Ĩ)with soft ideal Ĩwith the same set of parameters E, where X= E= ℕis the set of all natural numbers, Ĩ=$SS(X)_E$and τ = {X̃, Ø̃}. It is clear that □-S(X) = τ and □$_I$-S(X)=$SS(X)_E$. Hence, (X, τ, E, Ĩ) is soft α-compact which is not soft □$_I$-compact. Since, {(G_i, E): i ∈ℕ}, where G_i:E→Ƥ(X) such that G_i(n)={i}for every i, n∈ℕ, is a cover of X̃ by □$_I$-open soft sets which has no finite subcover.

Example 3.13: Consider the soft topological space(X, τ, E, Ĩ)with soft ideal Ĩwith the same set of parameters E, where X= E= ℕis the set of all natural numbers, Ĩ =$Ĩ_f$ and τ = {X̃,Ø̃}. Since □-S(X) = τ then, (X, τ, E, Ĩ) is soft □-Ĩ-compact space. But it is not soft □$_I$-Ĩ-compact since {(G_i, E): i∈ℕ}, where G_i:E→ Ƥ(X) such that G_i(n)={i}for every i, n∈ℕ, is a cover of X̃by □$_I$-open soft sets which has no finite subset $ℕ_0$ of ℕ such that X̃- Ũ{(G_i, E): i∈$ℕ_0$}∈Ĩ.

Remark 3.14: Let (X, τ, E, Ĩ) be a soft topological spacewith soft ideal Ĩwith the same set of parameters E. If Ĩ={Ø̃}, then the four concepts of soft α-compactness, soft $α_I$-compactness, soft α-Ĩ-compactness and soft $α_I$-Ĩ-compactness are equivalent.

Theorem 3.15: A soft topological space (X, τ, E) is soft α-compact (respectively, soft $α_{If}$ -compact) if and only if (X, τ, E, $Ĩ_f$) is soft α-$Ĩ_f$ -compact (respectively, soft$α_{If}$ -$Ĩ_f$ - compact), where $Ĩ_f$is the soft ideal of finite soft subset ofX̃.

Proof: Let (X, τ, E) is soft α-compact topological space and let {($G_λ$, E): λ∈ʌ} be a cover of X̃by α-open soft sets. Then there exists a finite subset $ʌ_0$ of ʌsuch thatX̃=Ũ{($G_λ$, E): λ∈$ʌ_0$}. It follows thatX̃- Ũ{($G_λ$, E): λ∈$ʌ_0$}=Ø̃∈$Ĩ_f$. Hence,(X, τ, E, $Ĩ_f$) is soft α-$Ĩ_f$ -compact.

Conversely, let (X, τ, E, Ĩ$_f$) be soft α-Ĩ$_f$ -compact and let {(G$_λ$, E): λ∈Λ} be a cover of X̃by α-open soft sets. Then there exists a finite subset Λ$_1$ of Λ such thatX̃- ⊔̃{(G$_λ$, E): λ∈Λ$_1$}=(F, E)∈Ĩ$_f$. So, there exists a finite subset Λ$_2$ of Λ such that (F, E)=⊔̃{(G$_λ$, E): λ∈Λ$_2$}. ThusX̃=⊔̃{(G$_λ$, E): λ∈Λ$_0$} where Λ$_0$= Λ$_1$∪Λ$_2$. Hence (X, τ, E) is soft α-compact.

In similar way we can prove that (X, τ, E) is soft α$_{If}$ -compact if and only if (X, τ, E, Ĩ$_f$) is softα$_{If}$ -Ĩ$_f$ -compact.

Proposition 3.16: If(X, τ, E, Ĩ) is soft □-Ĩ-compact space and J̃ is soft ideal with the same set of parameters E such that Ĩ⊆J̃, then (X, τ, E, J̃) is soft □-J̃-compact space.

Proof: Immediate.

The convers of the above proposition is not true in general, as the following example shows;

Example 3.17: Let (X, τ, E, Ĩ) be a soft topological spacewith soft ideal Ĩ, where X= E= ℕis the set of all natural numbers, Ĩ={∅̃} and τ= {X̃, ∅̃}∪{(U$_i$,E): U$_i$:E → Ƥ(X) such that U$_i$(n)={1,2,…,i} for every i, n∈ℕ} and letJ̃=SS(X)$_E$. Then (X, τ, E, J̃) is soft □-J̃-compact space, but (X, τ, E, Ĩ) is not soft □-Ĩ-compact. Since, {(U$_i$,E): i∈ℕ} is a cover of X̃ by α-open soft sets which has no finite subset ℕ$_0$ of ℕ such that X̃-⊔̃{(U$_i$,E): i∈ℕ$_0$}∈Ĩ.

Definition 3.18: A soft subset (F, E) of a soft topological space (X, τ, E, Ĩ) is called soft α-closed(respectively, soft α$_I$-closed) if every soft cover of (F, E) by α-open(respectively, α$_I$-open) soft sets has a finite subcollection whose soft closure of it is members covers (F, E). The space(X, τ, E, Ĩ) is called soft α-closed(respectively, soft α$_I$-closed) if X̃is soft α-closed(respectively, soft α$_I$-closed) as a soft subset.

Proposition 3.19: Let (X, τ, E, Ĩ) be a soft topological space with soft ideal Ĩwith the same set of parameters E. If Ĩ$_n$⊆Ĩ and (X, τ, E, Ĩ) is soft α-closed, then (X, τ, E, Ĩ) is soft α-Ĩ-compact.

Proof: Let {(G$_λ$, E): λ∈Λ} be a cover of X̃by α-open soft sets. Then there exists a finite subset Λ$_0$ of Λ such that X̃=⊔̃{cl(G$_λ$, E): λ∈Λ$_0$} and since (G$_λ$, E) is α-open soft setsfor every λ∈Λ$_0$, then X̃=⊔̃{cl(int(G$_λ$, E)): λ∈Λ$_0$}. Hence, X̃-⊔̃{cl(int(G$_λ$, E)): λ∈Λ$_0$}= ∅̃. So, int(cl(X̃- ⊔̃{(G$_λ$, E): λ∈Λ$_0$})= int(cl(X̃ ⊓̃(X̃-⊔̃{(G$_λ$, E): λ∈Λ$_0$})))⊆int(cl(X̃))⊓̃int(cl(X̃- ⊔̃{(G$_λ$, E): λ∈Λ$_0$}))⊆int(cl(X̃))⊓̃(⊓̃(int(cl({X̃- (G$_λ$, E): λ∈Λ$_0$}))))= X̃-⊔̃{cl(int(G$_λ$, E)): λ∈Λ$_0$}=∅̃. Therefore, X̃- ⊔̃{(G$_λ$, E): λ∈Λ$_0$}∈Ĩ$_n$⊆Ĩ. So, (X, τ, E, Ĩ) is soft α-Ĩ-compact.

The convers of Proposition 3.19 is not true, in general. As the following example shows;

Example 3.20: Consider the soft topological space (X, τ, E, Ĩ) where X= E= ℕis the set of all natural numbers andĨ=τ=SS(X)$_E$. It is clear that (X, τ, E, Ĩ) is soft α-Ĩ-compact and Ĩ$_n$={∅̃}⊆Ĩ. Now, let {(G$_i$, E): i∈ℕ}be a cover of X̃by □-open soft sets, where G$_i$:E→ Ƥ(X) such that G$_i$(n)={i}for every i, n∈ℕ. Since cl(G$_i$, E)= (G$_i$, E) for every i∈ℕ, then there is no finite subset ℕ$_0$ of ℕ such that X̃= ⊔̃{cl(G$_i$,E): i∈ℕ$_0$}. Therefore, (X, τ, E, Ĩ) is not soft α-closed.

Remark 3.21:

i. The conditions (X, τ, E, Ĩ)is soft α$_I$-closed and Ĩ$_n$⊆Ĩ are not sufficient to make (X, τ, E, Ĩ)soft α$_I$-Ĩ-compact. For example; Consider the soft topological space (X, τ, E, Ĩ) where X= E= ℕis the set of all natural numbers,Ĩ={∅̃}andτ={X̃,∅̃, (G$_1$, E), (G$_2$, E)}, where G$_1$:E→ Ƥ(X) such that G$_1$(n)={1} for every n∈ℕ and G$_2$:E→ Ƥ(X) such that G$_2$(n)=ℕ-{1} for every n∈ℕ. It is clear that Ĩ$_n$={∅̃} and (X, τ, E, Ĩ)is soft α$_I$-closedbut it is not soft α$_I$-Ĩ-compact since the cover {(G$_i$, E): i∈ℕ} of X̃ by α$_I$-open soft sets, where G$_i$:E→ Ƥ(X) such that G$_i$(n)={i}for every i, n∈ℕ, has no finite subcollection {(G$_i$, E): i∈ ℕ$_0$} such that X̃- ⊔̃{(G$_i$,E): i∈ℕ$_0$}∈Ĩ.
ii. The conditions (X, τ, E, Ĩ)is soft α$_I$-Ĩ-compact and Ĩ$_n$⊆Ĩ are not sufficient to make (X, τ, E, Ĩ)soft α$_I$-closed. For example; see Example 3.20.

Proposition 3.22: Let (X, τ, E) be a soft topological space. If (X, τ, E, Ĩ$_n$) is soft α-Ĩ$_n$-compact (respectively, soft α$_I$-Ĩ$_n$-compact), then (X, τ, E) is soft α-closed (respectively, soft α$_I$-closed).

Proof: Let {(G$_λ$,E): λ∈Λ} be a cover of X̃by α-open soft sets. Then there exists a finite subset Λ$_0$ of Λ such thatX̃-⊔̃{cl(G$_λ$, E): λ∈Λ$_0$}∈Ĩ$_n$. Implies, int(cl(X̃-⊔̃{cl(G$_λ$, E): λ∈Λ$_0$}))=∅̃. Now, X̃-⊔̃{cl(G$_λ$, E): λ∈Λ$_0$}= X̃⊓̃(X̃-(⊔̃{cl(G$_λ$, E): λ∈Λ$_0$})) = X̃⊓̃(⊓̃(X̃-({int(G$_λ$,E):λ∈Λ$_0$})))=int(X̃⊓̃(⊓̃(X̃-({(G$_λ$,E): λ∈Λ$_0$})))= int(X̃-(⊔̃{(G$_λ$,E):λ∈Λ$_0$}))⊆int(cl(X̃-(⊔̃{(G$_λ$, E): λ∈Λ$_0$})) = ∅̃. So, X̃=⊔̃{cl(G$_λ$, E): λ∈Λ$_0$}. Thus, (X, τ, E)is soft α-closed.

In similar way we can show that, if (X, τ, E, Ĩ$_n$)is soft α$_I$-Ĩ$_n$-compact then (X, τ, E)is soft α$_I$-closed.

Remark 3.23: The implication in proposition 3.22 is not reversible. See Example (i) of Remark 3.21.

Corollary 3.24: Let (X, τ, E) be a soft topological spacewith soft ideal Ĩwith the same set of parameters E. Then (X, τ, E, Ĩ$_n$)is soft α-Ĩ$_n$-compact if and only if (X, τ, E)is soft α-closed.

Theorem 3.25[5]: Let (X$_1$, τ$_1$, A, Ĩ) be a soft topological space with soft ideal, (X$_2$, τ$_2$, B) be a soft topological space and f$_{pu}$: (X$_1$, τ$_1$, A, Ĩ) → (X$_2$, τ$_2$, B)be a soft function. Then f$_{pu}$(Ĩ)= {f$_{pu}$((F, A)): (F, A)∈Ĩ} is a soft ideal on X$_2$.

Theorem 3.26 [5]: Let (X$_1$, τ$_1$, A) and (X$_2$, τ$_2$, B) are two soft topological spaces, f$_{pu}$: (X$_1$, τ$_1$, A) → (X$_2$, τ$_2$, B) be an injective soft function andJ̃ be a soft ideal on SS(X$_2$)$_B$. Then f_{pu}^{-1}(J̃) is a soft ideal on SS(X$_1$)$_A$.

Definition 3.27: Let (X, τ_1, A) and (Y, τ_2, B) are soft topological spaces and f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ be a function. Then, the function f_{pu} is called:

i. α-open (respectively, α_I-open) soft function if f_{pu}(G,A) is α-open (respectively, α_I-open) soft set in Y for each (G,A) open soft set in X.
ii. α-continuous (respectively, α_I-continuous) soft function if f_{pu}^{-1}(G,B) is α-open (respectively, α_I-open) soft set in X for each (G,B) open soft set in Y.
iii. α-irresolute (respectively, α_I-irresolute) soft function if f_{pu}^{-1}(G,B) is α-open (respectively, $\alpha_{f\text{-}1pu\ (I)}$-open) soft set in X for each (G,B) α-open (respectively, α_I-open) soft set inY.
iv. α-irresolute (respectively, α_I-irresolute) open soft function if f_{pu}(G,A) is α-open (respectively, $\alpha_{fpu(I)}$-open) soft set in Y for each (G,A) α-open (respectively, α_I-open) soft set inX.

Theorem 3.28: Let (X_1, τ_1, A, Ĩ) be a soft topological space with soft ideal, (X_2, τ_2, B) be a soft topological space and f_{pu}: (X_1, τ_1, A, Ĩ) → (X_2, τ_2, B)be anα-irresolute (respectively, α_I-irresolute)surjective soft function. If (X_1, τ_1, A) is soft α-Ĩ-compact (respectively, softα_I-Ĩ-compact), then (X_2, τ_2, B) is soft α-f_{pu}(Ĩ)-compact (respectively, soft $\alpha_{fpu(\tilde{I})}$-f_{pu}(Ĩ)-compact).

Proof: Let {(G_λ,B): λ∈Λ} be a cover of $\tilde{X}_2$by τ_2-α-open soft sets. Since f_{pu} is an α-irresolute function, then {f_{pu}^{-1}(G_λ,B): λ∈Λ} is a cover of $\tilde{X}_1$by τ_1-α-open soft sets. Implies, there exists a finite subset Λ_0 of Λ such that$\tilde{X}_1$- $\tilde{\sqcup}${f_{pu}^{-1}(G_λ, B): λ∈Λ_0}∈Ĩ. So, f_{pu}($\tilde{X}_1$- $\tilde{\sqcup}${f_{pu}^{-1}(G_λ, B): λ∈Λ_0})= $\tilde{X}_2$- $\tilde{\sqcup}${(G_λ, B): λ∈Λ_0}∈f_{pu}(Ĩ). Hence, (X_2, τ_2, B) is soft α-f_{pu}(Ĩ)-compact.

In similar way, we can show that if (X_1, τ_1, A) is soft α_I-Ĩ-compact, then (X_2, τ_2, B) is soft $\alpha_{fpu(\tilde{I})}$-f_{pu}(Ĩ)-compact.

Theorem 3.29: Let (X_1, τ_1, A) be a soft topological space, (X_2, τ_2, B,$\tilde{J}$) be a soft topological space with soft ideal and f_{pu}: (X_1, τ_1, A) → (X_2, τ_2, B,$\tilde{J}$) be a bijection α-irresolute (respectively, α_I-irresolute) open soft function. If (X_2, τ_2, B,$\tilde{J}$) is soft α-$\tilde{J}$-compact (respectively, soft α_J -$\tilde{J}$-compact), then (X_1, τ_1, A, $f_{pu}^{-1}(\tilde{J})$) is soft α-$f_{pu}^{-1}(\tilde{J})$- compact (respectively, soft $\alpha_{f^{-1}pu(\tilde{J})}$–$f_{pu}^{-1}(\tilde{J})$-compact).

Proof: Let {(G_λ,A): λ∈Λ} be a cover of $\tilde{X}_1$by τ_1-α-open soft sets. Since f_{pu} is bijection α-irresolute open soft function, then {f_{pu}(G_λ,A): λ∈Λ} is a cover of $\tilde{X}_2$by τ_2-α-open soft sets. Implies, there exists a finite subset Λ_0 of Λ such that$\tilde{X}_2$- $\tilde{\sqcup}${f_{pu}(G_λ, A): λ∈Λ_0}∈$\tilde{J}$. So, f_{pu}^{-1}($\tilde{X}_2$- $\tilde{\sqcup}${f_{pu}(G_λ, A): λ∈Λ_0})= $\tilde{X}_1$- $\tilde{\sqcup}${(G_λ, A): λ∈Λ_0}∈$f_{pu}^{-1}(\tilde{J})$. Hence, (X_1, τ_1, A, $f_{pu}^{-1}(\tilde{J})$) is soft α-$f_{pu}^{-1}(\tilde{J})$-compact.

In similar way, we can show that If (X_2, τ_2, B,$\tilde{J}$) is soft α_J -$\tilde{J}$-compact, then (X_1, τ_1, A, $f_{pu}^{-1}(\tilde{J})$) is soft $\alpha_{f^{-1}pu(\tilde{J})}$-$f_{pu}^{-1}(\tilde{J})$-compact.

Theorem 3.30: Let (X, τ, E, Ĩ) be a soft topological space with soft ideal, then the following are equivalent:

i. (X, τ, E) is soft α-Ĩ-compact (respectively, soft α_I-Ĩ-compact).
ii. For every family {(F_j, E): j∈ J} of α-closed soft (respectively, α_I-closed soft) subsets of $\tilde{X}$ for which $\tilde{\sqcap}${(F_j, E): j∈ J}=$\tilde{\emptyset}$, there exists finite subset J_0 of J such that $\tilde{\sqcap}${(F_j, E): j∈ J_0}∈Ĩ.

Proof: (i)→(ii): Let {(F_j, E): j∈ J} be a family of α-closed soft subsets of $\tilde{X}$for which $\tilde{\sqcap}${(F_j, E): j∈ J}=$\tilde{\emptyset}$, then {$\tilde{X}$ -(F_j, E): j∈ J} be a family of α-open soft sets covers $\tilde{X}$.Since (X, τ, E) is softα-Ĩ-compact, then there exists a finite subset J_0 of J such that$\tilde{X}$-$\tilde{\sqcup}${$\tilde{X}$-(F_j, E): j∈ J_0}∈Ĩ. Implies, $\tilde{\sqcap}${(F_j, E): j∈ J_0}∈Ĩ.

(ii)→(i): Let {(G_λ,E): λ∈Λ} be a cover of $\tilde{X}_1$by α-open soft sets. Then, {$\tilde{X}$-(G_λ, E): λ∈Λ} is a family of α-closed soft subsets of $\tilde{X}$for which $\tilde{\sqcap}${$\tilde{X}$-(G_λ, E): λ∈Λ}=$\tilde{\emptyset}$. Implies, there exists a finite subset Λ_0 of Λ such that $\tilde{\sqcap}${$\tilde{X}$-(G_λ, E): λ∈Λ_0}∈Ĩ. So, $\tilde{X}$-$\tilde{\sqcup}${(G_λ, E): λ∈Λ_0}∈Ĩ. Hence, (X, τ, E) is soft α-Ĩ-compact.

In similar way we can show that,(X, τ, E) is soft α_I-Ĩ-compact if and only if for every family {(F_j, E): j∈J} of α_I-closed soft subsets of $\tilde{X}$for which $\tilde{\sqcap}${(F_j,E): j∈ J}=$\tilde{\emptyset}$, there exists finite subset J_0 of J such that $\tilde{\sqcap}${(F_j, E): j∈ J_0}∈Ĩ.

4. Soft countably α-Ĩ-compactness spaces

Definition 4.1[5]**:** A soft subset (F, E) of the space (X, τ, E, Ĩ) is said to be soft countablyĨ-compact if and only if for every countable soft cover {(G_λ, E):λ∈Λ} of (F, E) by open soft set, there exists a finite subsetΛ_0 of Λ such that (F, E)-$\tilde{\sqcup}${(G_λ, E): λ∈Λ_0}∈Ĩ. The space (X, τ, E, Ĩ) is said to be soft countablyĨ-compact if $\tilde{X}$soft countablyĨ-compact as a soft subset.

Definition 4.2: A soft subset (F, E) of the space (X, τ, E, Ĩ) is said to be soft countablyα-Ĩ-compact (respectively, soft countablyα_I-Ĩ-compact) if and only if for every countable soft cover {(G_λ, E): λ∈Λ} of (F, E) by α-open (respectively, α_I-open) soft set, there exists a finite subsetΛ_0 of Λ such that (F, E)- $\tilde{\sqcup}${(G_λ, E): λ∈Λ_0}∈Ĩ.The space (X, τ, E, Ĩ) is said to be soft countablyα-Ĩ-compact (respectively, soft countablyα_I-Ĩ-compact) if $\tilde{X}$soft countablyα-Ĩ-compact (respectively, soft countablyα_I-Ĩ-compact) as a soft subset.

Remark 4.3: Let (F, E) be a soft subset of a soft topological space (X, τ, E, Ĩ). Then, the following statements are hold:

i. If (F, E) is soft countablyα_I-Ĩ-compactset, then it is soft countablyα-Ĩ-compactset. Not conversely, see Example 3.12.
ii. If (F, E) is soft countablyα-Ĩ-compactset, then it is soft countablyĨ-compactset. Not conversely, see Example 3.11.

Proof: Follows from Remark 3.4.

Remark 4.4: Let (X, τ, E, Ĩ) be a soft topological space with soft ideal Ĩ. Then every soft Ĩ-compact (respectively, soft α-Ĩ-compact and soft α_I-Ĩ-compact) subset of X̃is soft countablyĨ-compact (respectively, soft countably α-Ĩ-compact and soft countably α_I-Ĩ-compact) set.

The implication in Remark 4.4 is not reversible, in general, as the following example shows:

Example 4.5: Let X= $\mathbb{R}$ be the set of all real numbers, E= $\mathbb{N}$ be the set of all natural numbers, Irr be the set of irrational numbers and Q be the set of rational numbers.

Consider the soft topological space (X, τ, E,Ĩ) with the soft ideal Ĩ={Ø̃}and let **τ=**{X̃, Ø̃**,** SS(Irr)$_E$}∪{(G_i,E): i∈Irr} whereG_i:E→ Ƥ(X) such thatG_i(n)= Q∪A_i and A_i⊆Irr, for every n∈$\mathbb{N}$.

It is clear that (X, τ, E,Ĩ) is soft countablyĨ-compact (respectively, soft countably α-Ĩ-compact and soft countably α_I-Ĩ-compact) but it is not soft Ĩ-compact (respectively, soft α-Ĩ-compact and soft α_I-Ĩ-compact). Since {(G_i, E): i∈Irr}, such that G_i:E→ Ƥ(X); G_i(n)= Q∪{i} for all n∈$\mathbb{N}$ and for all i∈Irr, is a soft cover of X̃by soft open (respectively, soft α-open and soft α_I-open) sets which has no finite subcoverω̃ such thatX̃-ω̃∈Ĩ.

Proposition 4.6: Let (X, τ, E, Ĩ) be a soft topological space with soft ideal Ĩand J̃be a soft ideal with the same set of parameters E such thatĨ⊆J̃. If (X, τ, E, Ĩ) is soft countably α-Ĩ-compact (respectively, soft countably α_I-Ĩ-compact), then (X, τ, E, J̃) is soft countably α-J̃-compact (respectively, soft countably α_J-J̃-compact).

Proof: Immediate.

Proposition 4.7: Let (X_1, τ_1, A, Ĩ) be a soft topological space with soft ideal, (X_2, τ_2, B) be a soft topological space and f_{pu}: (X_1, τ_1, A, Ĩ) → (X_2, τ_2, B)be anα-irresolute (respectively, α_I-irresolute)surjective soft function. If (X_1, τ_1, A) is softcountablyα-Ĩ-compact (respectively, softcountablyα_I-Ĩ-compact), then (X_2, τ_2, B) is softcountablyα-f_{pu}(Ĩ)-compact (respectively, softcountably$\alpha_{f_{pu}(\tilde{I})}$-$f_{pu}$(Ĩ)-compact).

Proof: It is similar to the proof of Proposition 3.28.

Proposition 4.8: Let (X_1, τ_1, A) be a soft topological space, (X_2, τ_2, B,J̃) be a soft topological space with soft ideal and f_{pu}: (X_1, τ_1, A) → (X_2, τ_2, B,J̃) be a bijection α-f_{pu}^{-1}irresolute (respectively, α_I-irresolute) open soft function. If (X_2, τ_2, B,J̃) is soft α-J̃-compact (respectively, soft α_J -J̃-compact), then (X_1, τ_1, A, f_{pu}^{-1}(J̃)) is soft α-f_{pu}^{-1}(J̃)-compact (respectively, soft $\alpha_{f^{-1}{}_{pu(\tilde{J})}}$-$f_{pu}{}^{-1}$(J̃)-compact).

Proof: It is similar to the proof of Proposition 3.29.

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