



A Note On Soft Fuzzy Volterra Spaces

A.Haydar EŞ

Department of Mathematics Education, Başkent University, Bağlıca, 06490 Ankara, Turkey
haydares@baskent.edu.tr

ABSTRACT

In this paper, the concepts of soft fuzzy \mathcal{E}_r -Volterra spaces and soft fuzzy \mathcal{E}_p -Volterra spaces are introduced and studied. We will discuss several characterizations of those spaces.

Indexing terms/Keywords

Soft fuzzy topology; soft fuzzy Volterra spaces; soft fuzzy weakly Volterra spaces; soft fuzzy \mathcal{E}_r -Volterra spaces; soft fuzzy \mathcal{E}_p -Volterra spaces.

Academic Discipline And Sub-Disciplines

Mathematics; Topology.

SUBJECT CLASSIFICATION

Mathematics Subject Classification; 54A40, 03E72.

1. INTRODUCTION

Zadeh introduced the fundamental concepts of fuzzy sets in his classical paper [12]. Chang in [1] introduced and developed the concept of fuzzy topological spaces. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. The concept of Volterra spaces have been studied extensively in classical topology in [3,4]. The concepts of fuzzy Volterra spaces, fuzzy weakly Volterra spaces and generalized fuzzy Volterra spaces in fuzzy topological spaces are introduced and studied by the authors in [5,6]. The concept of soft fuzzy topological space is introduced by I.U.Tiryaki [10]. The concept of almost P-spaces and almost GP-spaces in soft fuzzy setting was introduced by Es [2]. In this paper, in section 3, the concepts of soft fuzzy \mathcal{E}_r -Volterra spaces and soft fuzzy \mathcal{E}_p -Volterra spaces are introduced and studied.

2. PRELIMINARIES

We introduce some basic notions and results that are used in the sequel.

Definition 2.1. [8] Let (X, τ) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy G_δ set if $\lambda = \bigwedge_{i=1}^{\infty} \mu_i$ where each μ_i is fuzzy open set. The complement of a fuzzy G_δ set is fuzzy F_σ .

Definition 2.2. [10] Let X be a set, μ be a fuzzy subset of X and $M \subseteq X$. Then the pair (μ, M) will be called a soft fuzzy subset of X . The set of all soft fuzzy subsets of X will be denoted by $SF(X)$.

Proposition 2.3. [10] If $(\mu_j, M_j)_{j \in J} \in SF(X)$, then the family $\{(\mu_j, M_j) | j \in J\}$ has a meet, that is greatest lower bound, in $(SF(X), \subseteq)$, denoted by $\prod_{j \in J} (\mu_j, M_j)$ such that $\prod_{j \in J} (\mu_j, M_j) = (\mu, M)$

where $\mu(x) = \bigwedge_{j \in J} \mu_j(x), \forall x \in X,$

$$M = \bigcap_{j \in J} M_j.$$

Definition 2.4. [10] Let X be a non-empty set and the soft fuzzy sets A and B in the form,

$$A = \{(\mu, M) | \mu(x) \in I^X, \forall x \in X, M \subseteq X\}$$

$$B = \{(\lambda, N) | \lambda(x) \in I^X, \forall x \in X, N \subseteq X\}$$

Then,

- (i) $A \subseteq B \Leftrightarrow \mu(x) \leq \lambda(x), \forall x \in X, M \subseteq N.$
- (ii) $A = B \Leftrightarrow \mu(x) = \lambda(x), \forall x \in X, M = N.$
- (iii) $A' \Leftrightarrow 1 - \mu(x), \forall x \in X, X \setminus M.$



(iv) $A \cap B \Leftrightarrow \mu(x) \wedge \lambda(x), \forall x \in X$ and $M \cap N$, for all $(\mu, M), (\lambda, N) \in SF(X)$.

(v) $A \cup B \Leftrightarrow \mu(x) \vee \lambda(x), \forall x \in X$ and $M \cup N$, for all $(\mu, M), (\lambda, N) \in SF(X)$.

Definition 2.5. [10]

$(0, \emptyset) = \{(\lambda, N) | \lambda = 0, N = \emptyset\}$

$(1, X) = \{(\lambda, N) | \lambda = 1, N = X\}$

Definition 2.6. [11] For $(\mu, M) \in SF(X)$ the soft fuzzy set

$$(\mu, M)' = (1 - \mu, X \setminus M) \text{ is called the complement of } (\mu, M).$$

Definition 2.7. [10] A subset $\tau \subseteq SF(X)$ is called an SF -topology on X if

(i) $(0, \emptyset)$ and $(1, X) \in \tau$

(ii) $(\mu_j, M_j) \in \tau, j = 1, 2, \dots, n \Rightarrow \prod_{j=1}^n (\mu_j, M_j) \in \tau$

(iii) $(\mu_j, M_j), j \in J \Rightarrow \prod_{j \in J} (\mu_j, M_j) \in \tau$. The elements of τ are called soft fuzzy open,

and those of $\tau' = \{(\mu, M) | (\mu, M)' \in \tau\}$ soft fuzzy closed.

If τ is SF -topology on X we call the pair (X, τ) SF -topological space (in short SFTS).

Definition 2.8. [10] The closure of a soft fuzzy set (μ, M) will be denoted by $\overline{(\mu, M)}$. It is given by

$$\overline{(\mu, M)} = \sqcap \{(\gamma, N) | (\mu, M) \sqsubseteq (\gamma, N), (\gamma, N) \in \tau\}.$$

Likewise the interior is given by

$$(\mu, M)^{\circ} = \sqcup \{(\gamma, N) | (\gamma, N) \in \tau, (\gamma, N) \sqsubseteq (\mu, M)\}.$$

Note: $\overline{(\mu, M)} = cl(\mu, M)$ and $(\mu, M)^{\circ} = int(\mu, M)$.

Definition 2.9. [11] Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be a soft fuzzy set in (X, τ) . Then

(i) (λ, N) is said to be soft fuzzy regular open if $(\lambda, N) = int(cl(\lambda, N))$.

(ii) (λ, N) is said to be soft fuzzy regular closed if $(\lambda, N) = cl(int(\lambda, N))$.

Definition 2.10. [7] A fuzzy topological space (X, τ) is called a fuzzy P-space if countable intersection of fuzzy open sets in (X, τ) is fuzzy open. That is, every non-zero fuzzy G_{δ} set in (X, τ) , is fuzzy open in (X, τ) .

Definition 2.11. [8] A fuzzy topological space (X, τ) is called a fuzzy almost P-space if for every non-zero fuzzy G_{δ} set λ in (X, τ) , $int(\lambda) \neq 0$ in (X, τ) .

Definition 2.12. [8] A fuzzy topological space (X, τ) is called a weak fuzzy P-space if the countable intersection fuzzy regular open sets in (X, τ) is a fuzzy regular open set in (X, τ) .

Definition 2.13. [2] A soft fuzzy topological space (X, τ) is called a soft fuzzy weak P-space if the countable intersection soft fuzzy regular open sets in (X, τ) is a soft fuzzy regular open set in (X, τ) . That is, $\prod_{i=1}^{\infty} (\lambda_i, M_i)$ is a soft fuzzy regular open in (X, τ) , where (λ_i, M_i) 's are soft fuzzy regular open sets in (X, τ) .

Definition 2.14. [2] A soft fuzzy topological space (X, τ) is called a soft fuzzy P-space if countable intersection of soft fuzzy open sets in (X, τ) is soft fuzzy open. That is, every non-zero soft fuzzy G_{δ} set in (X, τ) is soft fuzzy open in (X, τ) .

Definition 2.15. [2] A soft fuzzy topological space (X, τ) is called a soft fuzzy almost P-space if for every non-zero soft fuzzy G_{δ} set in (X, τ) , $int(\lambda, M) \neq (0, \emptyset)$ in (X, τ) .

Definition 2.16. [2] A soft fuzzy set (λ, M) in a soft fuzzy topological space (X, τ) is called a soft fuzzy nowhere dense if there exists no non-zero soft fuzzy open set (μ, N) in (X, τ) such that $(\mu, N) \sqsubseteq cl(\lambda, M)$. That is, $int(cl(\lambda, M)) = (0, \emptyset)$.

Definition 2.17. [2] A soft fuzzy set (λ, M) in a soft fuzzy topological space (X, τ) is called a soft fuzzy dense if there exists no soft fuzzy closed set (μ, N) in (X, τ) such that $(\lambda, M) \sqsubseteq (\mu, N) \sqsubseteq (1, X)$. That is, $cl(\lambda, M) = (1, X)$.

Definition 2.18. [2] A soft fuzzy topological space (X, τ) is called a soft fuzzy submaximal space if for each soft fuzzy set (λ, M) in (X, τ) such that $cl(\lambda, M) = (1, X)$, then (λ, M) in (X, τ) .



Definition 2.19. [2] A soft fuzzy topological space (X, τ) is called a soft fuzzy almost GP-space if $\text{int}(\lambda, M) \neq (0, \emptyset)$, for each non-zero soft fuzzy dense and soft fuzzy G_δ set (λ, M) in (X, τ) . That is, (X, τ) is a soft fuzzy almost GP-space if every non-zero soft fuzzy G_δ set in (X, τ) with $\text{cl}(\lambda, M) = (1, X)$, $\text{int}(\lambda, M) \neq (0, \emptyset)$.

Definition 2.20. [2] A soft fuzzy set (λ, M) in (X, τ) is called a soft fuzzy first category set if $(\lambda, M) = \cup_{i=1}^{\infty} (\lambda_i, M_i)$, where (λ_i, M_i) 's are soft fuzzy nowhere dense in (X, τ) .

3. ON SOFT FUZZY VOLTERRA SPACES

Definition 3.1. Let (λ, M) be a soft fuzzy first category set in soft fuzzy topological space (X, τ) . Then $(1, X) - (\lambda, M)$ is called a soft fuzzy residual set in (X, τ) .

Definition 3.2. A SFTS (X, τ) is called a soft fuzzy \mathcal{E}_r -Volterra space if $\text{cl}(\prod_{i=1}^n (\lambda_i, M_i)) = (1, X)$, where (λ_i, M_i) 's are soft fuzzy dense and soft fuzzy residual sets in (X, τ) .

Proposition 3.3. If the SFTS (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space, then

$\text{int}(\cup_{i=1}^n (\lambda_i, M_i)) = (0, \emptyset)$, where the soft fuzzy set (λ_i, M_i) 's are soft fuzzy first category sets such that $\text{int}(\lambda_i, M_i) = (0, \emptyset)$ in (X, τ) .

Proof. Let (λ_i, M_i) 's ($i=1, 2, \dots, n$) be soft fuzzy first category sets such that $\text{int}(\lambda_i, M_i) = (0, \emptyset)$ in (X, τ) . Then $((1, X) - (\lambda_i, M_i))$'s are soft fuzzy residual sets such that $\text{cl}((1, X) - (\lambda_i, M_i)) = (1, X)$ in (X, τ) . Since (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space,

$\text{cl}(\prod_{i=1}^n ((1, X) - (\lambda_i, M_i))) = (1, X)$. Then $\text{cl}((1, X) - \cup_{i=1}^n (\lambda_i, M_i)) = (1, X)$ and hence $(1, X) - \text{int}(\cup_{i=1}^n (\lambda_i, M_i)) = (1, X)$. Therefore, we have $\text{int}(\cup_{i=1}^n (\lambda_i, M_i)) = (0, \emptyset)$, where (λ_i, M_i) 's are soft fuzzy first category sets such that $\text{int}(\lambda_i, M_i) = (0, \emptyset)$.

Proposition 3.4. Let (X, τ) be a soft fuzzy \mathcal{E}_r -Volterra space. Then (X, τ) is a soft fuzzy Volterra space.

Proof. Let (λ_i, M_i) 's ($i=1, 2, \dots, n$) be soft fuzzy dense and soft fuzzy G_δ sets in (X, τ) . Then, (λ_i, M_i) 's are soft fuzzy residual sets in (X, τ) . This implies that (λ_i, M_i) 's are soft fuzzy dense and soft fuzzy residual sets in (X, τ) . Since (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space

$\text{cl}(\prod_{i=1}^n (\lambda_i, M_i)) = (1, X)$. Hence (X, τ) is a soft fuzzy Volterra space.

Proposition 3.5. If each soft fuzzy nowhere dense set is a soft fuzzy closed set in a soft fuzzy Volterra space (X, τ) , then (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

Proof. Let (λ_i, M_i) 's ($i=1, 2, \dots, n$) be soft fuzzy dense and soft fuzzy residual sets in (X, τ) . Since (λ_i, M_i) 's are soft fuzzy residual sets, $((1, X) - (\lambda_i, M_i))$'s are soft fuzzy first category sets in (X, τ) . Now $((1, X) - (\lambda_i, M_i)) = \cup_{j=1}^{\infty} (\mu_{ij}, N_{ij}) \in \tau$, where (μ_{ij}, N_{ij}) 's are soft fuzzy nowhere dense sets in (X, τ) . From the hypothesis, the soft fuzzy nowhere dense sets (μ_{ij}, N_{ij}) 's are soft fuzzy closed sets and hence $((1, X) - (\lambda_i, M_i))$'s are soft fuzzy F_σ sets in (X, τ) . Therefore (λ_i, M_i) 's are soft fuzzy G_δ sets in (X, τ) . Since (X, τ) is a soft fuzzy Volterra space, $\text{cl}(\prod_{i=1}^n (\lambda_i, M_i)) = (1, X)$. Hence (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

Definition 3.6. A SFTS (X, τ) is called a soft fuzzy nodec space if every non-zero soft fuzzy nowhere dense set (λ, M) is soft fuzzy closed in (X, τ) . That is, if (λ, M) is a soft fuzzy nowhere dense set in (X, τ) , then $((1, X) - (\lambda, M)) \in \tau$.

Proposition 3.7. If the SFTS (X, τ) is a soft fuzzy Volterra space and soft fuzzy nodec space, then SFTS (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

Proof. Let (X, τ) be a soft fuzzy Volterra space and soft fuzzy nodec space and (λ_i, M_i) 's ($i=1, 2, \dots, n$) be soft fuzzy dense and soft fuzzy residual sets in (X, τ) . Since (λ_i, M_i) 's are soft fuzzy residual sets, $((1, X) - (\lambda_i, M_i))$'s are soft fuzzy first category sets in (X, τ) . Now $((1, X) - (\lambda_i, M_i)) = \cup_{j=1}^{\infty} (\mu_{ij}, N_{ij})$, where (μ_{ij}, N_{ij}) 's are soft fuzzy nowhere dense sets in (X, τ) . Since (X, τ) is a soft fuzzy nodec space, soft fuzzy nowhere dense sets (μ_{ij}, N_{ij}) 's are soft fuzzy closed in (X, τ) . By the Proposition 3.5, (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

Proposition 3.8. If the SFTS (X, τ) is a soft fuzzy Volterra space and soft fuzzy submaximal space, then (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

Proof. Obvious.

Proposition 3.9. If the SFTS (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space and soft fuzzy D-Baire space, then $\text{int}(\cup_{i=1}^n (\lambda_i, M_i)) = (0, \emptyset)$, where the (λ_i, M_i) 's are soft fuzzy first category sets in (X, τ) .

Proof. Let (X, τ) be a soft fuzzy \mathcal{E}_r -Volterra space and soft fuzzy D-Baire space and (λ_i, M_i) 's ($i=1,2,3,\dots,n$) are soft fuzzy first category sets in (X, τ) . Since (X, τ) is soft fuzzy D-Baire space, the soft fuzzy first category sets (λ_i, M_i) 's are soft fuzzy nowhere dense sets in (X, τ) and hence $((1, X) - (\lambda_i, M_i))$'s are soft fuzzy dense sets in (X, τ) . Then $((1, X) - (\lambda_i, M_i))$'s are soft fuzzy dense and soft fuzzy residual sets in (X, τ) . By the hypothesis, $\text{cl}(\prod_{i=1}^n ((1, X) - (\lambda_i, M_i))) = (1, X)$. This implies that

$\text{cl}((1, X) - \cup_{i=1}^n (\lambda_i, M_i)) = (1, X) - \text{int}(\cup_{i=1}^n (\lambda_i, M_i)) = (1, X)$. Therefore $\text{int}(\cup_{i=1}^n (\lambda_i, M_i)) = (0, \emptyset)$, where (λ_i, M_i) 's are soft fuzzy first category sets in (X, τ) .

Definition 3.10. Let (X, τ) be a SFTS. Then (X, τ) is called a soft fuzzy Baire space if $\text{int}(\cup_{i=1}^{\infty} (\lambda_i, M_i)) = (0, \emptyset)$, where (λ_i, M_i) 's are soft fuzzy nowhere dense sets in (X, τ) .

Proposition 3.11. If $\cup_{i=1}^n (\lambda_i, M_i)$, where the (λ_i, M_i) 's are soft fuzzy nowhere dense sets, is a soft fuzzy nowhere dense set in a soft fuzzy Baire space (X, τ) , then (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

Proof. The proof is similar to Proposition 3.7.

Proposition 3.12. If each soft fuzzy first category set is a soft fuzzy closed set in a soft fuzzy Baire space (X, τ) , then (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

Proof. Let (X, τ) be a soft fuzzy Baire space and (λ_i, M_i) 's ($i=1,2,3,\dots,n$) are soft fuzzy dense and soft fuzzy residual sets in (X, τ) . Since (λ_i, M_i) 's are soft fuzzy residual sets, $((1, X) - (\lambda_i, M_i))$'s are soft fuzzy first category sets in (X, τ) . By the hypothesis, the soft fuzzy first category sets $((1, X) - (\lambda_i, M_i))$'s are soft fuzzy closed sets in (X, τ) and hence (λ_i, M_i) 's are soft fuzzy open sets in (X, τ) . Since (λ_i, M_i) 's are soft fuzzy dense and soft fuzzy open sets in (X, τ) , $((1, X) - (\lambda_i, M_i))$'s are soft fuzzy nowhere dense sets in (X, τ) . Since (X, τ) is a soft fuzzy Baire space, $\text{int}(\cup_{i=1}^n ((1, X) - (\lambda_i, M_i))) \subseteq \text{int}(\cup_{i=1}^{\infty} ((1, X) - (\lambda_i, M_i))) = (0, \emptyset)$. That is, $\text{int}(\cup_{i=1}^n ((1, X) - (\lambda_i, M_i))) = (0, \emptyset)$. Hence $\text{cl}(\prod_{i=1}^n (\lambda_i, M_i)) = (1, X)$, where (λ_i, M_i) 's are soft fuzzy dense and soft fuzzy residual sets in (X, τ) . Therefore (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

Definition 3.13. A SFTS (X, τ) is called a soft fuzzy \mathcal{E}_p -Volterra space if $\text{cl}(\prod_{i=1}^n (\lambda_i, M_i)) = (1, X)$, where (λ_i, M_i) 's are soft fuzzy pre-open and soft fuzzy G_δ sets in (X, τ) .

Proposition 3.14. If the SFTS (X, τ) is a soft fuzzy \mathcal{E}_p -Volterra space, then $\text{int}(\cup_{i=1}^n (\lambda_i, M_i)) = (0, \emptyset)$, where (λ_i, M_i) 's are soft fuzzy pre-closed and a soft fuzzy F_δ sets in (X, τ) .

Proof. Let (λ_i, M_i) 's ($i=1,2,3,\dots,n$) be soft fuzzy pre-closed and soft fuzzy F_δ sets in (X, τ) . Then $((1, X) - (\lambda_i, M_i))$'s are soft fuzzy pre-open and soft fuzzy G_δ sets in (X, τ) . By the hypothesis, $\text{cl}(\prod_{i=1}^n ((1, X) - (\lambda_i, M_i))) = (1, X)$. Then $\text{cl}((1, X) - \cup_{i=1}^n (\lambda_i, M_i)) = (1, X) - \text{int}(\cup_{i=1}^n (\lambda_i, M_i)) = (1, X)$. Therefore, we have $\text{int}(\cup_{i=1}^n (\lambda_i, M_i)) = (0, \emptyset)$ where (λ_i, M_i) 's ($i=1,2,3,\dots,n$) be soft fuzzy pre-closed and soft fuzzy F_δ sets in (X, τ) .

Proposition 3.15. If the SFTS (X, τ) is a soft fuzzy \mathcal{E}_p -Volterra space, then (X, τ) is a soft fuzzy Volterra space.

Proof. Let (λ_i, M_i) 's ($i=1,2,3,\dots,n$) be soft fuzzy dense and soft fuzzy G_δ sets in (X, τ) . Since (λ_i, M_i) 's are soft fuzzy dense sets, $\text{cl}(\lambda_i, M_i) = (1, X)$. Now $\text{int}(\text{cl}(\lambda_i, M_i)) = (1, X)$. Then $(\lambda_i, M_i) \subseteq \text{int}(\text{cl}(\lambda_i, M_i))$. Since (X, τ) is a soft fuzzy \mathcal{E}_p -Volterra space and (λ_i, M_i) 's are soft fuzzy pre-open and soft fuzzy G_δ sets in (X, τ) , $\text{cl}(\prod_{i=1}^n (\lambda_i, M_i)) = (1, X)$, where (λ_i, M_i) 's are soft fuzzy dense and soft fuzzy G_δ sets in (X, τ) . Hence (X, τ) is a soft fuzzy Volterra space.

Proposition 3.16. If $\text{preint}(\cup_{i=1}^n (\lambda_i, M_i)) = (0, \emptyset)$ where (λ_i, M_i) 's are soft fuzzy pre-closed sets in a soft fuzzy topological space (X, τ) , then (X, τ) is a soft fuzzy \mathcal{E}_p -Volterra space.

Proof. Let (λ_i, M_i) 's ($i=1,2,3,\dots,n$) be soft fuzzy pre-open and soft fuzzy G_δ sets in (X, τ) . Then $((1, X) - (\lambda_i, M_i))$'s are soft fuzzy pre-closed in (X, τ) . By the hypothesis, $\text{preint}(\cup_{i=1}^n ((1, X) - (\lambda_i, M_i))) = (0, \emptyset)$. This implies that $(1, X) - \text{precl}(\cup_{i=1}^n (\lambda_i, M_i)) = (0, \emptyset)$ and hence $\text{precl}(\prod_{i=1}^n (\lambda_i, M_i)) = (1, X)$. Since $\text{precl}(\prod_{i=1}^n (\lambda_i, M_i)) \subseteq \text{cl}(\prod_{i=1}^n (\lambda_i, M_i))$, we have $\text{cl}(\prod_{i=1}^n (\lambda_i, M_i)) = (1, X)$. Therefore $\text{cl}(\prod_{i=1}^n (\lambda_i, M_i)) = (1, X)$, where (λ_i, M_i) 's are soft fuzzy pre-open and soft fuzzy G_δ sets in (X, τ) . Hence (X, τ) is a soft fuzzy \mathcal{E}_p -Volterra space.



Proposition 3.17. If $\text{precl}(\prod_{i=1}^n (\lambda_i, M_i)) = (1, X)$, where (λ_i, M_i) 's are soft fuzzy pre-open sets in a soft fuzzy topological space (X, τ) , then (X, τ) is a soft fuzzy \mathcal{E}_p -Volterra space.

Proof. The proof is similar to Proposition 3.16.

Proposition 3.18. If $(\lambda, M) = \cup_{i=1}^n (\lambda_i, M_i)$, where (λ_i, M_i) 's are soft fuzzy pre-closed sets, is a soft fuzzy nowhere dense set in a soft fuzzy topological space (X, τ) , then (X, τ) is a soft fuzzy \mathcal{E}_p -Volterra space.

Proof. Suppose that $(\lambda, M) = \cup_{i=1}^n (\lambda_i, M_i)$, where (λ_i, M_i) 's are soft fuzzy pre-closed sets, and (λ, M) is a soft fuzzy nowhere dense sets in (X, τ) . Then $\text{preint}(\text{precl}(\lambda, M)) = (0, \emptyset)$. Since $\text{preint}(\lambda, M) \subseteq \text{preint}(\text{precl}(\lambda, M))$, we have $\text{preint}(\lambda, M) = (0, \emptyset)$. Then $\text{preint}(\cup_{i=1}^n (\lambda_i, M_i)) = (0, \emptyset)$, where (λ_i, M_i) 's are soft fuzzy pre-closed sets in (X, τ) . Hence by Proposition 3.16, (X, τ) is a soft fuzzy \mathcal{E}_p -Volterra space.

Proposition 3.19. If each soft fuzzy pre-closed set is a soft fuzzy nowhere dense sets in soft fuzzy \mathcal{E}_r -Volterra space (X, τ) , then (X, τ) is a soft fuzzy \mathcal{E}_p -Volterra space.

Proof. Obvious.

Proposition 3.20. If a soft fuzzy \mathcal{E}_p -Volterra space (X, τ) is a soft fuzzy submaximal space, then (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

Proof. Immediate from the definitions.

Proposition 3.21. If each soft fuzzy nowhere dense set is soft fuzzy closed set in a soft fuzzy \mathcal{E}_p -Volterra space (X, τ) , then (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

Proof. It is clear from the definitions.

Proposition 3.22. If a soft fuzzy \mathcal{E}_p -Volterra space (X, τ) is a soft fuzzy nodec space, then (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

Proof. Let (X, τ) be a soft fuzzy \mathcal{E}_p -Volterra space and a soft fuzzy nodec space. Since (X, τ) is a soft fuzzy nodec space, each soft fuzzy nowhere dense set is a soft fuzzy closed set in (X, τ) . By Proposition 3.21, (X, τ) is a soft fuzzy \mathcal{E}_r -Volterra space.

REFERENCES

1. Chang, C. L., Fuzzy topological spaces, J.Math.Anal.,Vol. 24, 1968, 182-190.
2. Eş,A.H., On soft fuzzy almost P-spaces , International Journal of Mathematical Archive,7(5)(2016), 1-5.
3. Gauld, D., Greenwood, S., Piotrowski,Z., On Volterra spaces II, Ann. New York Acad.Sci., 806(1996),169-173.
4. Gauld, D., Piotrowski, Z., On Volterra spaces, Far East J.Math.Sci., 1(1993),209-214.
5. Thangaraj, G., Soundararajan,S., Generalized fuzzy Volterra spaces, Annals of Fuzzy Mathematics and Informatics , 11(4)(2016),633-644.
6. Thangaraj, G., Soundararajan,S., On fuzzy Volterra spaces, J.Fuzzy Math., 21(4)(2013), 895-904.
7. Thangaraj, G., Anbazhagan, C., Some remarks on fuzzy P-spaces, Gen.Math.Notes, 26(1) (2015), 8-16.
8. Thangaraj, G., Anbazhagan, C., Vivakanandan, P., On fuzzy P-spaces, weak fuzzy P-spaces and fuzzy almost P-spaces, Gen.Math.Notes, 18(2) (2013), 128-139.
9. Thangaraj, G., Anbazhagan, C., On fuzzy almost GP-spaces, Annals of Fuzzy Mathematics and Informatics, 10(5) (2015), 727-736.
10. Tiryaki, I.U., Fuzzy sets over the posets I, Hacettepe Journal of Mathematics and Statistics, 37(2) (2008), 143-166.
11. Visalakshi, V., Uma, M.K., Roja, E., On soft fuzzy G_δ pre continuity in soft fuzzy topological space, Annals of Fuzzy Mathematics and Informatics, 8(6) (2014), 921-939.
12. Zadeh, L.A., Fuzzy sets, Information and Control, 8 (1965), 338-353.