ISSN 2347-1921

Designing an Appropriate Adaptive Controller for Synchronizing a Bi-Oscillator Heart Model with time Delay

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ABSTRACT

In this paper, synchronization of heart follower oscillator which has lower frequency AV with heart leader oscillator which has dominant frequency SA will be studied. It can be seen if two nodes SA and AV are not synchronized, different types of cardiac blocking arrhythmias occur. Thus, in this paper, beside putting voltage to node SA, by applying time delay τ in bi-oscillator model of heart system and designing appropriate controller via linear and adaptive methods, we try to prevent blocking arrhythmias of heart. Finally, we apply Lyapanuv stability theorem for ensuring convergence.

Keywords

Oscillators of heart pacemakers; Nonlinear oscillators; Time delay; Control signal; Synchronization.

Academic Discipline And Sub-Disciplines

Mathematical Modelling of Heart.

Mathematics Subject Classification

34H10; 37N35; 47N70; 49J15; 49J20; 49J21; 93C40.

Type (Method/Approach)

Synchronizing a bi-oscillator heart model with time delay.

Council for Innovative Research

Peer Review Research Publishing System

Journal: Journal of Advances in Mathematics

Vol 8, No 3
editor@cirjam.org
www.cirjam.com, www.cirworld.com

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INTRODUCTION

Synchronization problem is an important problem in nonlinear systems, in the control engineering. In general, the meaning of synchronization is to harmonize signal fluctuations of state in two or more systems or to harmonize some of their properties such as frequency. If without input control ($u = 0$) and by changing the coupling coefficients, systems do not synchronize then we encounter the problem of setting control law for synchronization. In this paper, we will study one of applications of synchronization problem in heart. Synchronization means that all the cardiac oscillators, which have different inherent frequencies, oscillate together with a unit frequency, and this is important, because arrhythmias of cardiac blocking occur in the absence of synchronization between SA and AV nodes. For example, when the frequency of the sinus node increases, AV node can't withstand the frequency imposed by the SA and this causes creation of atrioventricular blocking arrhythmia and gradually, it leads to complete blocking. Our main purpose in this paper is to synchronize heart follower oscillator which has lower frequency (AV), with leading oscillator which has the predominant frequency (SA), by using linear and adaptive methods $[4,8]$.To reach this goal, in this paper synchronization of two systems of oscillators of Van der Pol type will be studied, by considering two-side coupling $[1,2]$ and to prevent arrhythmias, as it is specified in the models, at first we add a voltage with the range a_1 and frequency ω to SA node.

Furthermore, because even the small delays may change the dynamic of the system, therefore letting time-delay in differential equations can prevent arrhythmias $[6,10]$.So, with assumption $x_i = x_i(t)$ and $x_i^{\tau} = x_i(t-\tau)$, in which τ denotes time delay we examine two-oscillator model of the heart system with time delay and design a proper controller $[5]$.Then regarding to results of synchronization of the two oscillators, we compare the heart oscillator systems before and after the synchronization, and in order to ensure convergence, Lyapunov stability theorem will be used $[7]$. To clarify the problem of synchronization, consider the below systems of Van der Pol model

$$(SA)\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^2 - 1)x_2 - c_1 x_1 + a_1 \cos\omega t + R_{13}(x_1 - x_3) \end{cases}$$
$$(AV)\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^2 - 1)x_4 - c_2 x_3 + a_2 \cos\omega t + R_{24}(x_2 - x_4) \end{cases} \quad (1)$$

In which the first system is considered as leader system and second system as a follower system $[9,10]$ and pairs (x_1, x_2) and (x_3, x_4), respectively show SA and AV oscillators **Table 1**. Now, with applying a proper control on AV node, the state variables of pursuant system will converge to the state variables of leader system after a transient time $[3,11,12]$. In fact, the second oscillator has to follow the behavior of the first oscillator, which has dominant frequency.

Hence the error of synchronization is considered as follow $[4]$.

$$\begin{aligned} e_1 &= x_1 - x_3 \\ e_2 &= x_2 - x_4 \end{aligned} \quad (2)$$

The purpose of synchronization is to vanish the error.

To choose appropriate parameters from the Van der Pol system which are close to the system of the heart, we use trying and error test[7] .Consider two properties of the cardiac conduction system.

1. To the form of potential in action of SA and AV nodes which depend on d_1 and d_2. 2. To the period of T_1 and T_2 of two oscillators, when they are not coupled. Which depends on c_1 and c_2.

According to above remarks, the appropriate parameters are as follow **Table 1**.

Table 1. Appropriate parameters to synchronize bi-oscillator system

Assumptions	Definition	Bi-oscillator system
c_1	SA frequency	1
c_2	AV frequency	1
a_1	SA Voltage range	3
a_2	AV Voltage range	40
d_1	SA Damping Coefficient	1
d_2	AV Damping Coefficient	1
ω	frequency	$2.2 \le \omega < 2.3$
R_{ij}	Coupling coefficients between x_i and x_j	
R_{13}		1
R_{24}		10
x_1	SA membrane flow	
x_2	SA membrane voltage	
x_3	AV membrane flow	
x_4	AV membrane voltage	

Also, in this paper for two non-equal Van der Pol systems with known parameters, we use the linearization method and in the case that parameters are unknown we apply adaptive method. The most important criterions to select above methods for synchronization of oscillators are simplicity, accuracy of synchronization method and computations amount in calculating an appropriate control signal.

Synchronization of model with time delay by the linearization method

Feedback linearization is a method for designing nonlinear control in which dynamics of nonlinear system becomes linear completely or partially. Generally, in this method the original system model is transformed to the equivalent model, which has a simpler form, finally, the asymptotic stability of the system is also checked. Since even small delays may change the dynamics of the system, including of time-delay in differential equations can cause drastic changes and chaos in a system that has regular behavior [6, 10]. Therefore, in this section we discuss on the behavior of the system according to intensity of different couples and different values of the time delay. For this purpose first the system equations are divided into linear and nonlinear parts appropriately, finally by adding delay to the coupling new equations of system obtain and then we discuss on the synchronization. Thus, the system (1) is changed as below in which τ represents time delay and $x_i^{\tau} = x_i(t-\tau)$.

$$SA:\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^{\ 2}-1)x_2 - c_1x_1 + a_1\cos\omega t + R_{13}(x_1 - x_3^{\tau_1})\end{cases}$$
$$AV:\begin{cases}\dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^{\ 2}-1)x_4 - c_2x_3 + a_2\cos\omega t + R_{24}(x_2 - x_4^{\tau_2})\end{cases} \quad (3)$$

By substituting we will have

$$SA:\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^{\ 2}-1)\,x_2 - c_1x_1 + a_1\cos\omega t + R_{13}\,(x_1 - x_3(t-\tau_1)) \end{cases}$$
$$AV:\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^{\ 2}-1)\,x_4 - c_2x_3 + a_2\cos\omega t + R_{24}\,(x_2 - x_4(t-\tau_2)\ ) \end{cases} \quad (4)$$

Now with applying the appropriate controller on it which is as below

$$SA:\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^{\ 2}-1)\,x_2 - c_1x_1 + a_1\cos\omega t + R_{13}\,(x_1 - x_3(t-\tau_1)) \end{cases}$$
$$AV:\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^{\ 2}-1)\,x_4 - c_2x_3 + a_2\cos\omega t + R_{24}\,(x_2 - x_4(t-\tau_2)) + u \end{cases} \quad (5)$$

The state variables of slave systems will be converged to state variables of master system, after a transient time. So synchronization error is considered as below [4].

$$\begin{aligned} e_1(t) &= x_1(t) - x_3(t-\tau_1) \\ e_2(t) &= x_2(t) - x_4(t-\tau_2) \end{aligned} \quad (6)$$

The purpose of the synchronization is to annihilate errors. So chosen control functions to vanish errors as $\dot{e}_i = 0,\ i = 1,2,3,4$ that are measured as below

$$\begin{aligned} \dot{e}_1(t) &= \dot{x}_1(t) - \dot{x}_3(t-\tau_1) \\ \dot{e}_2(t) &= \dot{x}_2(t) - \dot{x}_4(t-\tau_2) \end{aligned} \quad (7)$$

By substituting we will have

$$\begin{aligned} &\dot{e}_1(t) = \dot{x}_1(t) - \dot{x}_3(t-\tau_1) \\ &\dot{e}_2(t) = \dot{x}_2(t) - \dot{x}_4(t-\tau_2) = -d_1(x_1^{\ 2}-1)x_2 - c_1x_1 + a_1\cos\omega t \\ &+ R_{13}(x_1 - x_3(t-\tau_1)) + d_2(x_3^{\ 2}-1)x_4 + c_2x_3 - a_2\cos\omega t - R_{24}(x_2 - x_4(t-\tau_2)) - u \end{aligned} \quad (8)$$

With adding and subtracting terms, d_1x_4 and c_1x_3 to $\dot{e}_2$ we have

$$\begin{aligned} &\dot{e}_1 = e_2 \\ &\dot{e}_2 = -d_1x_1^{\ 2}x_2 + a_1\cos\omega t + d_2x_3^{\ 2}x_4 - a_2\cos\omega t - c_1e_1 + R_{13}e_1 + (c_2 - c_1)x_3 \\ &\qquad + (d_1 - d_2)x_4 + d_1e_2 - R_{24}e_2 - u \end{aligned} \quad (9)$$

Thus by choosing control rules as below

$$\begin{aligned} u = &-d_1x_1^{\ 2}x_2 + a_1\cos\omega t + d_2x_3^{\ 2}x_4 - a_2\cos\omega t + (c_2 - c_1)x_3 \\ &+ (d_1 - d_2)x_4 + k_1e_1 + k_2e_2 \end{aligned} \quad (10)$$

we have

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= (R_{13} - c_1 - k_1)e_1 + (d_1 - R_{24} - k_2)e_2 \end{aligned} \quad (11)$$

Matrix form of error equations is as $\dot{e} = Ae$ which is a linear system. Necessary and sufficient condition for asymptotic stability of the error is that the matrix of coefficients was Horvytz (it means that the coefficients of the characteristic
We have polynomial of second degree exist and have similar sign).

$$
\begin{aligned}
&A=\begin{bmatrix} 0 & 1 \\ R_{13}-c_1-k_1 & d_1-R_{24}-k_2 \end{bmatrix}\\
&A-\lambda I=0\\
&\lambda^2-(d_1-R_{24}-k_2)\lambda-\left(R_{13}-c_1-k_1\right)=0\\
&d_1-R_{24}-k_2<0\ ,\ k_2>d_1-R_{24}\ \ for\ \ \lambda_1,\lambda_2<0\\
&R_{13}-c_1-k_1<0\ ,\ k_1>c_1-R_{13}
\end{aligned}
\qquad (12)
$$

in which λ_1 and λ_2 are the eigenvalues of the matrix A. With changing eigenvalues so that the dynamic of the error still is stable one can change speed of synchronization. However how much the eigenvalues are selected larger, the synchronous speed may be higher and synchronization is done on lesser time, but eigenvalues have to be selected such that control signal needs less range for synchronization. By applying control u and the results of simulations we will see that x_1 , x_3 and x_2 , x_4 track each other [9], and have

$$\lim_{t\to\infty}\|e\|=0 \qquad (13)$$

The result of this section is that two-side coupling, physiologically, means that oscillator AV effects on oscillator SA , but the this effect is low. At first R_1 was selected about 0.01 times of value of R_2 , then we increased it. Calculations and simulation results showed that two-side coupling and increasing R_1 do not effect on time synchronization. These results according to adaption with heart physiology which states that oscillator AV has little effect on oscillator SA are correct.

Simulation diagrams related to synchronization of oscillators with time delay by linearization method is as below

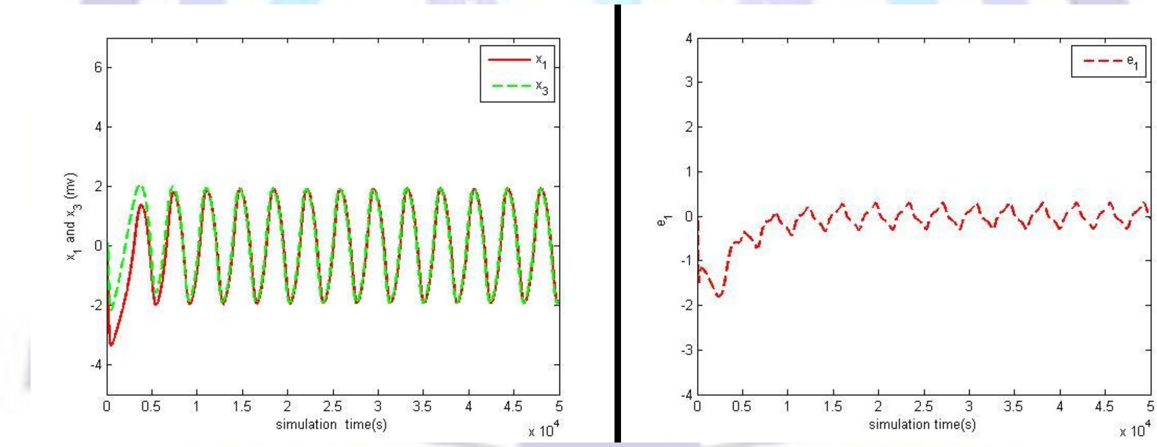


Figure1. Synchronization of curves x_1 and x_3 and error curve x_1 and x_3 with time delay before controller

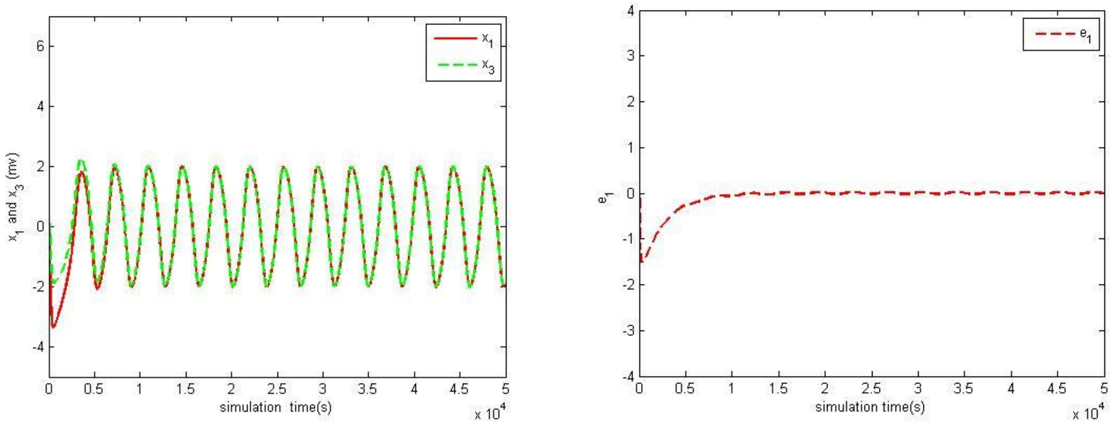


Figure 2. Synchronization of curves x_1 and x_3 and error curve x_1 and x_3 with time delay after controller

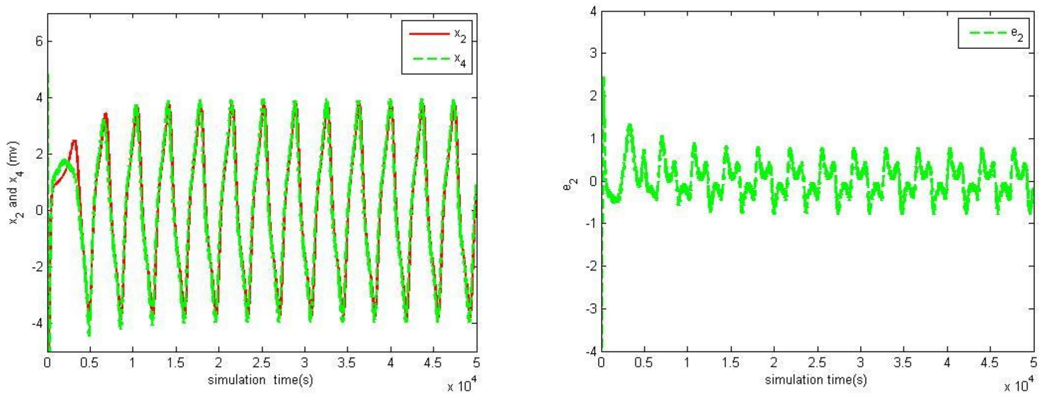


Figure 3. Synchronization of curves x_2 and x_4 and error curve x_2 and x_4 with time delay before controller

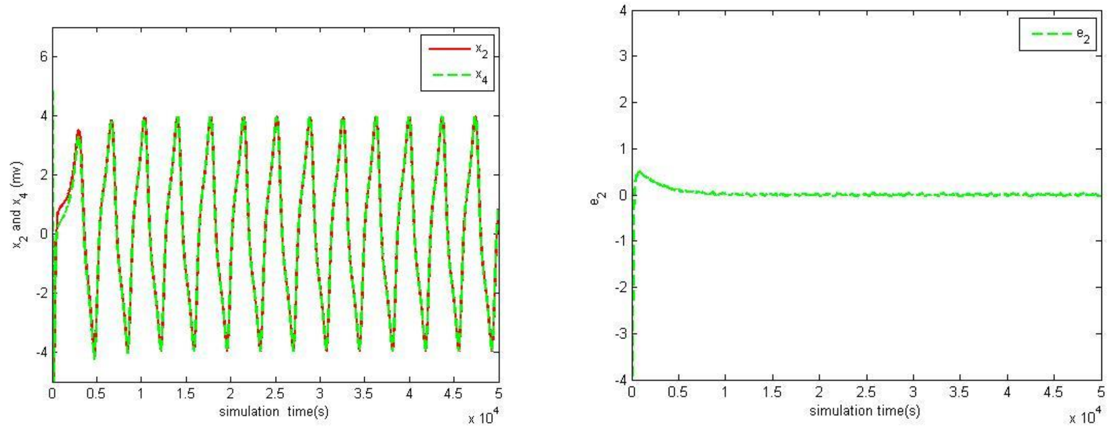


Figure 4. Synchronization of curves x_2 and x_4 and error curve x_2 and x_4 with time delay after controller

Synchronization in case of time delay with adaptive control (AC) method

During the synchronization of the two systems, it is possible that the system parameters be unknown. Therefore in this case for synchronization of two systems, we use adaptive methods. Generally, in this method, the aim is to find a controller and also a law for updating the parameters, so that the states of follower and leader systems are synchronized with each other globally and asymptotically. In this method the controller is designed and is added to the follower system such that non-linear parts of error dynamic between leader and follower systems are eliminated. Also the Lyapunov stability theorem is used to check system stability. In what follows, we study synchronization of oscillator SA and AV with different initial conditions in case of time delay and that one of its parameters is unknown $[6,13,14,15]$. Oscillators of SA and AV nodes define as below

$$SA:\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^{\ 2}-1)x_2 - c_1x_1 + a_1\cos\omega t + R_{13}(x_1 - x_3^{\tau_1})\end{cases}$$
$$AV:\begin{cases}\dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^{\ 2}-1)x_4 - c_2x_3 + a_2\cos\omega t + R_{24}(x_2 - x_4^{\tau_2})\end{cases} \quad (14)$$

Assume that all parameters are known and d_2 is unknown, in this cases by applying u we have

$$SA:\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^{\ 2}-1)x_2 - c_1x_1 + a_1\cos\omega t + R_{13}(x_1 - x_3^{\tau_1})\end{cases}$$
$$AV:\begin{cases}\dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^{\ 2}-1)x_4 - c_2x_3 + a_2\cos\omega t + R_{24}(x_2 - x_4^{\tau_2}) + u\end{cases} \quad (15)$$

We consider parameter error and the synchronization error as below

$$\tilde{d}_2 = d_2 - \hat{d}_2$$
$$e_1(t) = x_1(t) - x_3(t-\tau_1)$$
$$e_2(t) = x_2(t) - x_4(t-\tau_2) \quad (16)$$
$$\dot{e}_1(t) = \dot{x}_1(t) - \dot{x}_3(t-\tau_1)$$
$$\dot{e}_2(t) = \dot{x}_2(t) - \dot{x}_4(t-\tau_2) = -d_1(x_1^{\ 2}-1)x_2 - c_1x_1 + a_1\cos\omega t$$
$$+R_{13}(x_1 - x_3(t-\tau_1)) + d_2(x_3^{\ 2}-1)x_4 + c_2x_3 - a_2\cos\omega t - R_{24}(x_2 - x_4(t-\tau_2)) - u$$

Control signal with using estimation of parameter d_2 is determined as

$$u = -d_1x_1^{\ 2}x_2 + a_1\cos\omega t + \hat{d}_2x_3^{\ 2}x_4 - a_2\cos\omega t + (c_2 - c_1)x_3$$
$$+(d_1 - \hat{d}_2)x_4 + k_1e_1 + k_2e_2 \quad (17)$$

To obtain the lows for updating parameter $\hat{d}_2$, the Lyapunov function method is used.

Consider Positive definite Lyapunov function as

$$v = \frac{e_2^{\ 2} + \tilde{d}_2^{\ 2}}{2}$$

$$\dot{v} = \dot{e}_2e_2 + \tilde{d}_2\dot{\tilde{d}}_2 = (\tilde{d}_2x_3^{\ 2}x_4 - \tilde{d}_2x_4 + (R_{13} - c_1 - k_1)e_1 + (d_1 - R_{24} - k_2)e_2)e_2 + \tilde{d}_2\dot{\tilde{d}}_2 \quad (18)$$

Thus by choosing

$$\dot{\tilde{d}}_2 = (x_4 - x_3^{\ 2}x_4)e_2\ ,\ \ k_1 > R_{13} - c_1\ ,\ k_2 > d_1 - R_{24} \quad (19)$$

$$\dot{v} < 0 \quad (20)$$

Where $\dot{v}$ is negative definite. Now, according to the Lyapunov stability theorem, one can conclude that two systems SA and AV which have an unknown parameter will be synchronized asymptotically with definite control function, and synchronization error approaches to zero asymptotically. Generally, in case of time delay one can obtain these results.

1. With reducing the coupling coefficients in oscillator SA, synchronization frequency increases that may cause sinus tachycardia disease.
2. Different delays cause system switching to different frequencies and as a result of increasing frequency, heart rate increases and this may cause tachycardia arrhythmia.
3. Too much delay may cause arrhythmias blocks of grade 1 in the heart.

Simulation diagrams related to synchronization of oscillators with time delay by adaptive method is as below

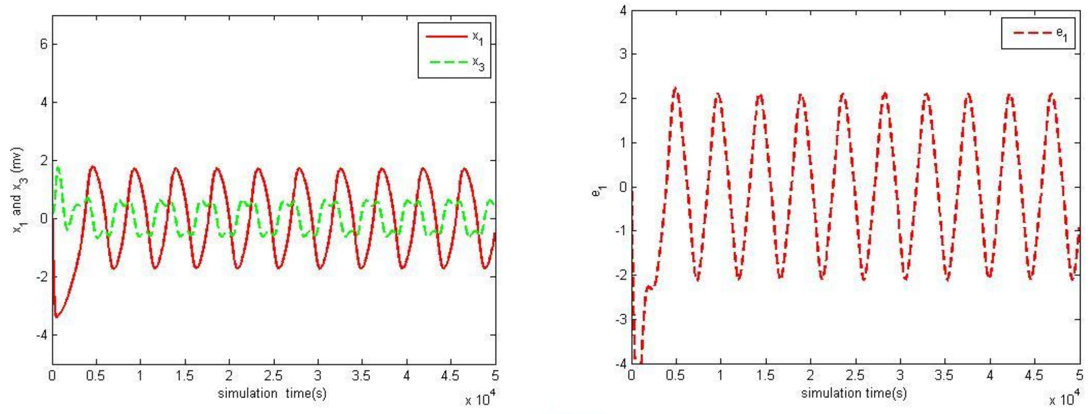


Figure 5. Synchronization of curves x_1 and x_3 and error curve x_1 and x_3 with time delay before AC controller

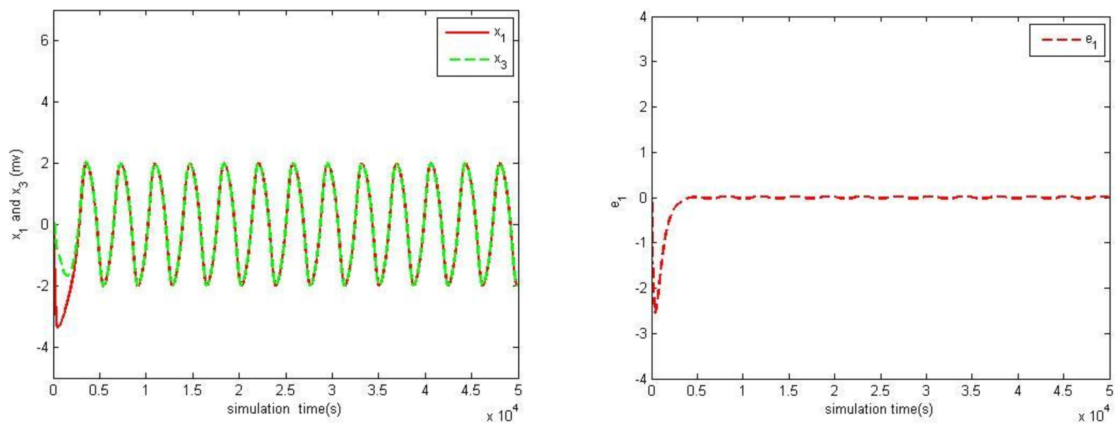


Figure 6. Synchronization of curves x_1 and x_3 and error curve x_1 and x_3 with time delay after AC controller

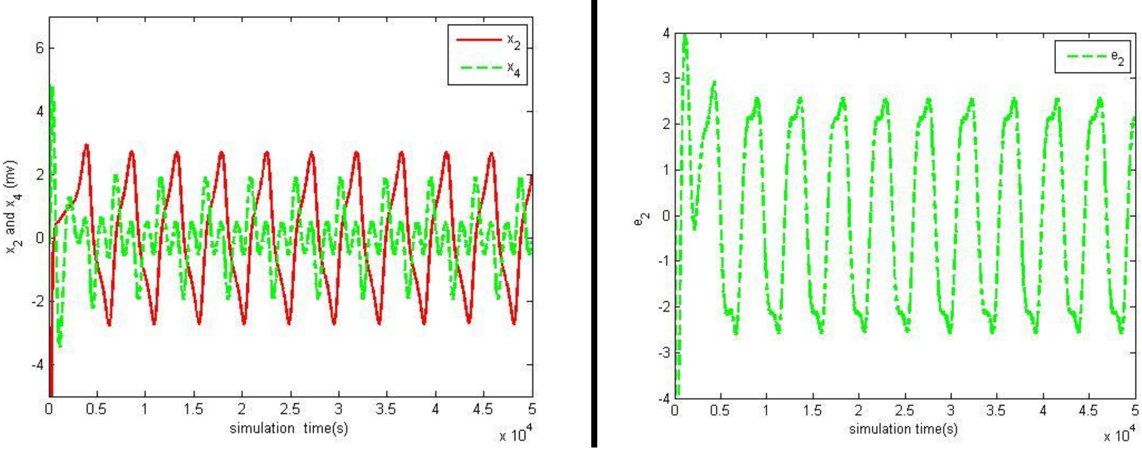
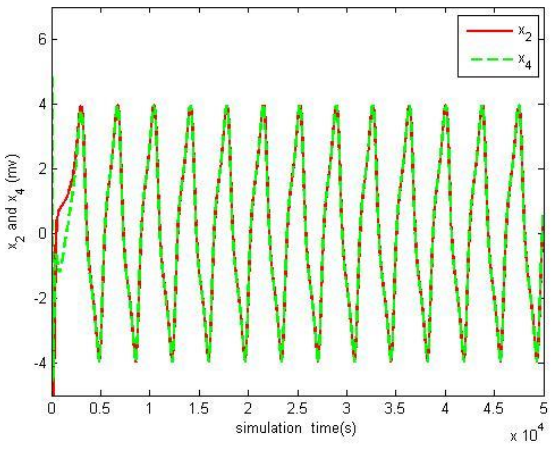


Figure 7. Synchronization of curves x_2 and x_4 and error curve x_2 and x_4 with time delay before AC controller



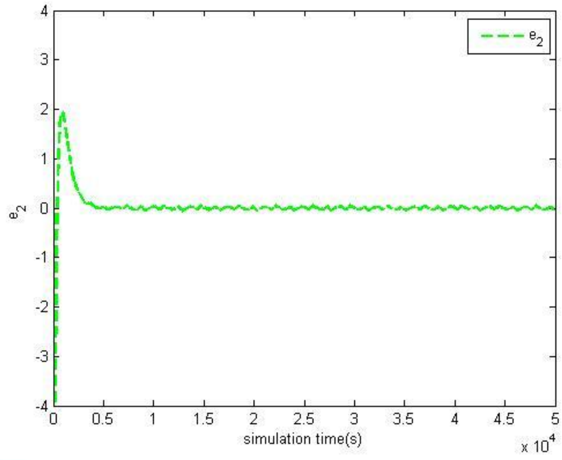


Figure 8. Synchronization of curves x_2 and x_4 and error curve x_2 and x_4 with time delay after AC controller

By choosing parameters as shown in table 1, we designed controller to the Model of two -oscillator system of the heart in case of time delay via linear and adaptive methods. Simulation result related to synchronization given in **Table 2**.

Table 2. Simulation results the Model of two -oscillator system of the heart in case of time delay

Definition	Two- oscillator in case of time delay with linear method after control	Two-oscillator in case of time delay with adaptive method befor control	Two- oscillator in case of time delay with adaptive method after control
Synchronization time of x_1 and x_3	0.5 (s)	0.5(s)	0.3(s)
Synchronization time of x_2 and x_4	0.4(s)	0.6(s)	0.3(s)

Conclusion

In this paper we saw that if two heart oscillator systems don't synchronize by changing parameters and coupling coefficients, it may cause different kinds of blocking arrhythmias and in order to prevent arrhythmia, from beginning, we added a voltage with the range a_1 and frequency ω ,to node SA, then we used time delay factor in oscillators and in order to design a proper controller for two non-equal Van der Pol systems, we applied the linearization method with known parameters and the adaptive method in case that the parameters are unknown. At last we applied Lyapanuv stability theorem to guaranty convergence of methods. Also regarding to the results of synchronization of two oscillators, we compared heart oscillator systems before and after the synchronization. It was seen that in adaptive control, synchronization is done in fewer time and dynamic of error converges to zero in about Less than 0.3 seconds where as in linearization method, dynamic of error converges to zero in about 0.5 seconds.

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