ISSN 2347-1921

Solvable subgroups of maximal order of some finite simple groups of Lie type

Abdullah A. Abduh and Falih. A.M Aldosray
Department of Mathematics - Umm Al-Qura University
Makkah P.O.Box 56199,Saudi Arabia
aaabduh@uqu.edu.sa fadosary@uqu.edu.sa

Abstract: The aim of this work is using the information in the ATLAS of Finite Groups and the GAP computational system , to determine the solvable subgroups of maximal orders in some finite non-abelian simple group of Lie type which have been appeared in the ATLAS. The (p,q,r)-generators , the structures and permutation representations of these subgroups have been found.

Mathematics Subject Classification: 20D05, 20F05, 20C33, 20C15.

Keywords: Simple groups; Presentations; Representations; Characters.

Council for Innovative Research
Peer Review Research Publishing System
Journal: Journal of Advances in Mathematics

Vol 8, No 2
editor@cirjam.org
www.cirworld.org, www.cirworld.com

ISSN 2347-1921

Introduction:

The classification of finite simple groups was declared accomplished in 1955 through 1983 by Daniel Gorenstein [8]. After announcement on the classification of finite simple groups investigation of known simple groups becomes one of the most important problem in finite group theory. In particular, subgroups structure of known finite simple group is of interest. The most important subgroups are maximal subgroups, maximal soluble subgroups, maximal nilpotent subgroups, and maximal abelian subgroups. In 2000 Vdovin, E. P. [12] devoted his work to abelian and nilpotent subgroups of maximal order of finite simple groups. In 1986 , Mann, A. [10] , found all solvable subgroups of maximal order in symmetric and alternating groups , and between 2006 to 2012 , Breuer,T., [2] , determined the orders of solvable subgroups of maximal orders in sporadic simple groups and their automorphism groups, using the information in the ATLAS of Finite Groups [14] and the GAP system , he also determined the structures and the conjugacy classes of these solvable subgroups in the big group. In 2013 [Master project] , Abduh and Alghawazi have determined , by using the information in the ATLAS of Finite Groups [14] and the GAP computational system , the solvable subgroups of maximal orders in the finite non-abelian simple group $L_2(p)=PSL_2(p)$, the projective special linear groups of dimension 2x2 on GF(p) , and in the finite non-abelian simple groups of orders less than 10^6

Here we are using the information in the ATLAS of Finite Groups and the GAP computational system , to determine the solvable subgroups of maximal orders S in the twisted simple groups of Lie type which have been appeared in the ATLAS such as Suzuki groups Suz(8) and Suz(32) , the Ree group Ree(27), and exceptional groups $^2F_4(2)'$ *and* $^3D_4(2)$ The (p,q,r)-generators , the structures and permutation representations of S have been found.

We use the axiom that any solvable subgroup of large order S of G is either one of the maximal subgroups of G or it is contained in one of them , so we deal with the maximal subgroups of G [2] and we get the following results

We summarized all results of our own GAP program in the following tables which list information about solvable subgroups of maximal order S in the twisted groups G . The first column in each table gives the names of twisted groups G , their orders and their presentations on their standard generators , their (p,q,r)-generators from their character tables their maximal subgroups up to isomorphisms. The second column gives the structure of the solvable subgroup of large order S in G , its **(p,q,r)-generators**, its permutation representations in G also it gives the fusion maps of the conjugacy classes of S into the conjugacy classes of G . The solvable subgroups of large order in the maximal subgroups in G and in their maximals are also computed.

ISSN 2347-1921

The GAP Program :

Define the group and compute its maximal subgroups	Compute the set of generators and the structure of S .		Compute fusions maps and permutation representations of S in G

Gap> g:=Group(a,b); (where a and b obtained from the first column of the groups tables below)
1-**Cases where the Character Table of G is available in GAP**
Gap> c := " The name of G ";
Gap> m=CharacterTable(c);
max:=Maxes(m);
for i in [1 .. Length(max)] do
k:=CharacterTable(max[i]);
ll:=IsSolvable(k);
if ll=true then
Display(max[i]);
Display(Size(k));
Display("-------------");
fi;od;
for i in [1 .. Length(max)] do
k:=CharacterTable(max[i]);
has:=HasMaxes(k);
if has=true then
Display(max[i]);
Display(Maxes(k));
fi;od;
1-**Cases where the Character Table of G is not available in GAP**
Gap>
max:=MaximalSubgroupClassReps(g);
sol:=List(max,IsSolvable);
List(max,Size);
	Gap> gen := [];;
	1-G=Suz(8) and S≅$\mathbf{2^{3+3}:7}$
	gen[1]:=a;;gen[2]:=b;;
	gen[3]:= gen[1]*gen[2];;
	gen[4]:= gen[3]*gen[2];;
	gen[5]:= gen[3]*gen[4];;
	gen[3]:= gen[4]*gen[5];;
	gen[5]:= gen[3]*gen[3];;
	gen[3]:= gen[5]^-1;;
	gen[6]:= gen[3]*gen[4];;
	gen[1]:= gen[6]*gen[5];;
	aa:=gen[1]; bb:=gen[2];
	2-G=Suz(32) and S≅2^{5+5}:31
	gen[1]:=a;;gen[2]:=b;;
	gen[3]:= gen[1]*gen[2];;
	gen[4]:= gen[3]*gen[2];;
	gen[5]:= gen[3]*gen[4];;
	gen[3]:= gen[5]*gen[2];;
	gen[7] := gen[4]^18;;
	gen[4] := gen[7]^-1;;
	gen[6]:= gen[4]*gen[2];;
	gen[1]:=gen[6] *gen[7];;
	gen[6] := gen[5]^14;;
	gen[5] := gen[6]^-1;;
	gen[4]:= gen[5]*gen[3];;
	gen[2]:=gen[4]*gen[6];;
	aa:=get[1];bb:= get[2];
	3-G=Ree(27) and S ≅3^{3+3+3}:26
	gen[1]:=a;;gen[2]:= b;;
	gen[3]:=gen[1]* gen[2];;
	gen[4]:=gen[3]*gen[2];;
	gen[5]:=gen[3]*gen[4];;
	gen[6]:=gen[3]*gen[5];;
	gen[7]:=gen[6]*gen[3];;
		gen[7] := gen[6]^-1;;
		gen[8] := gen[7] * gen[1];;
		gen[1] := gen[8] * gen[6];;
		gen[8] := gen[9]^23;;
		gen[7] := gen[8]^-1;;
		gen[4] := gen[7] * gen[5];;
		gen[2] := gen[4] * gen[8];;
		aa:=gen[1];bb:=gen[2];
		4- G=$^2F_4(2)'$ and S≅2.[2^8].5.4
		gen[1]:=a; gen[2]:=b;
		gen[3] := gen[1] * gen[2];;
		gen[4] := gen[2] * gen[3];;
		gen[5] := gen[4] * gen[1];;
		gen[6] := gen[5]^-1;;
		gen[5] := gen[1] * gen[4];;
		gen[4] := gen[6] * gen[5];;
		gen[1] := gen[4] * gen[4];;
		gen[4] := gen[3] * gen[2];;
		gen[5] := gen[3] * gen[4];;
		gen[6] := gen[3] * gen[5];;
		gen[2] := gen[6]^3;;
		gen[5] := gen[4]^9;;
		gen[6] := gen[5]^-1;;
		gen[7] := gen[6] * gen[2];;
		gen[2] := gen[7] * gen[5];;
		gen[4] := gen[3]^3;;
		gen[5] := gen[4]^-1;;
		gen[6] := gen[5] * gen[1];;
		gen[1] := gen[6] * gen[4];;
		aa := gen[1];;bb := gen[2];;
		5- G=$^3D_4(2)$ and S=[2^{11}]:(7 × S_3)
		gen:=[];gen[1]:=a;gen[2]:=b;
		gen[3] := gen[1] * gen[2];;
		gen[4] := gen[3] * gen[2];;
		gen[5] := gen[3] * gen[4];;
		gen[6] := gen[5] * gen[2];;
		gen[7] := gen[6] * gen[2];;
		gen[2] := gen[7] * gen[7];;
		aa := gen[1];;bb := gen[2];;
		Order(aa);Order(bb);Order(aa*bb);
		k:=Group(aa,bb);
		x1:=Size(Centralizer(k,aa));
			a:=AllCharacterTableNames(IsSolvable , true);
			os:=The order of S ;
			s:="The name of the Group G";
			for i in [1 .. Number(a)] do
			c:=CharacterTable(a[i]);
			m:=CharacterTable(s); cc:=m;
			if Size(c)=os then
			fus:=PossibleClassFusions(c,m);;
			fus1:=RepresentativesFusions(c,fus,m);
			get:=fus1[1];
			Display(a[i]);
			cname:=ClassNames(c);;
			ccname:=ClassNames(cc);;
			ncc:=NrConjugacyClasses(cc);Display(" ");
			Print("The Fusion map of S into G is : ");
			Display(" ");Print(get);Display(" ");
			Display("Class in S");
			for aa in [1 .. Number(cname)] do
			Print(cname[aa]); Print(" ");od;
			Display(" ");
			Display("Fusion in G");
			for rw in [1 .. Number(get)] do
			tw:=get[rw];;
			qw:=ccname[tw];;
			Print(qw);Print(" "); od;
			p:=PermChars(m,Size(m)/Size(c));;
			perm:=PermCharInfo(m,p).ATLAS;
			Display(perm);
			Display(" ");
			fi; od;
			Display("Some properties of S : ");
			z1:=IsSimple(c);;

	gen[8] := gen[7] * gen[4];;
	gen[9]:=gen[3]* gen[8];;
	gen[6] := gen[8]^11;;
		x2:=Size(Centralizer(k,bb));
		x3:=Size(Centralizer(k,aa*bb));
		StructureDescription(k);
			z2:=IsAbelian(c);;
			z3:=IsNilpotent(c);
			z4:=IsSupersolvable(c);;

The results :

The twisted simple groups of Lie type [atlas]	**The Solvable subgroups of G=Suz(8)**
Suzuki group Suz(8) It is a subgroup of SL(4;8) of Order = 29120 = 2^6.5.7.13.and it is generated by $< a, b \mid a^2 = b^4 = (ab)^5 = (ab^2)^7 = [a, b]^{13} = (abab^{-1}ab^2)^7 = 1>$ a=(1,2)(3,4)(5,7)(6,9)(8,12) (10,13)(11,15)(14,19)(16,21) (17,23)(18,25)(20,28)(22,31) (24,33)(26,35)(27,32)(29,37) (30,39)(34,43)(36,46)(38,48) (41,51)(42,44)(45,55)(47,50) (49,58)(52,60)(53,61)(54,59) (56,62)(57,63)(64,65); b =(1,3,5,8)(4,6,10,14)(7,11,16,22) (9,12,17,24)(13,18,26,36)(15,20,29,38)(19,27,31,28)(21,30,40,50)(23,32,41,52) (25,34,44,54)(33,42,53,43)(35,45,56,63) (37,47,51,46)(39,49,59,60)(48,57,55,58) (61,64,62,65); From the character table of Suz(8) , we found that Sz(8) is (2A,4A,5A) -group A maximal subgroup M of Suz(8) is isomorphic to one of the following : 1- **2^{3+3}:7 of order 448** 2- **13:4 of order 52** 3- **D_{10} .2 of order 20** 4- **D_{14}, of order 14** By applying the GAP tester , we found that all maximal subgroups of **Suz(8)**	1- **The solvable subgroup of large order S and its repesentations in G = Suz(8):** S≅2^{3+3}:7 = < (abbababb)$^{-2}$ abb(abbababb) , b.> **of order 448** From the Character table of S , we found that S is (7A,4A,7B)-group The fusion map of S into **Suz(8) is** : **<u>Classes in S</u>** 1a 7a 7b 7c 7d 7e 7f 2a 4a 4b **<u>Fusions in Suz(8)</u>** 1a 7a 7b 7a 7b 7c 7c 2a 4a 4b **The permutation character induced from S to Suz(8) is :** 1a+64a **<u>Some properties of S :</u>** S is SIMPLE : false S is ABELIAN : false S is NILPOTENT : false S is SUPERSOLVABLE : false 2- **The Lattice of Solvable subgroups of large orders in the maximal subgroups of G =Suz(8) :**

H ≤G	**The Maximal Subgroups**			
G	**2^{3+3}:7**	**D_{26} .2**	**D_{10} .2**	**D_{14}**
2^{3+3}:7	**2^{3+3}**	**2^3 :7**		
D_{26} .2	**D_{26}**	**C_4**		
D_{10} .2	**D_{10}**	**C_4**		
D_{14}	**C_7**	**C_2**		

are solvables	By applying the GAP tester , we found that the shadowed ones are the solvable subgroups of the large orders
The twisted simple groups of Lie type [atlas]	**The Solvable subgroups of G=Suz(32)**
Suzuki group Suz(32) It is a subgroup of SL(4;32) of Order = 32537600= $2^{10}.5^2.31.41$..and it is generated by $< a, b \mid a^2 = b^4 = (ab)^5 = (ab^2)^7 = [\mathbf{ababb}]^{25} = (\mathbf{abababbababbabb})^{25} = 1 >$. **The standard generators of Suz(32) can be found in the section of the permutation representations of Suz(32) on 1025 points appears in the ATLAS(as b11 and b21) . Let a=b11 and b:=b21** From the character table of Suz(32) , we found that Sz(8) is (2A,4A,5A) -group A maximal subgroup M of Suz(8) is isomorphic to one of the following : 1- 2^{5+5}:31 of order $2^{10}.31$ 2- 41:4 of order 164 3- 25:4 **of order 100** 4- D_{62} **of order 62**	**1- The solvable subgroup of large order S and its repesentations in G = Suz(32) :** S≅ $2^{5+5}:31 =< (abb)^{-18}\, b(abb)^{18}, (ababb)^{-14}\, (ababbb)\, (ababb)^{14} >$ of order 31744 From the Character table of S , we found that S is (4A,31A,31y)-group The fusion map of S into **Suz(32) is** : **<u>Classes in S</u>** 1a 2a 4a 4b 31a 31b 31c 31d 31e 31f 31g 31h 31i 31j 31k 31l 31m 31n 31o 31p 31q 31r 31s 31t 31u 31v 31w 31x 31y 31z 31aa 31ab 31ac 31ad **<u>Fusiosn in Suz(32)</u>** 1a 2a 4a 4b 31a 31o 31j 31c 31l 31f 31e 31n 31h 31b 31k 31j 31d 31m 31g 31a 31o 31i 31c 31l 31f 31e 31n 31h 31b 31k 31j 31d 31m 31g **The permutation character induced from S to Suz(32) is:** 1a+1024a **<u>Some properties of S :</u>** S is SIMPLE : false S is ABELIAN : false S is NILPOTENT : false S is SUPERSOLVABLE : false

ISSN 2347-1921

By applying the GAP tester , we found that all maximal subgroups of **Suz(32)** are solvables

2- The Lattice of Solvable subgroups of large orders in the maximal subgroups of G ==Suz(32) :

H ≤G	**The Maximal Subgroups**			
G	$\mathbf{2^{5+5}:31}$	41:4	25:4	D_{62}
2^{5+5}:31	$\mathbf{2^{5+5}}$	2^5:31		
41:4	$\mathbf{D_{82}}$	$\mathbf{C_4}$		
25:4	$\mathbf{D_{50}}$	$\mathbf{D_{10}\ .2}$		
D_{62}	$\mathbf{C_{31}}$	$\mathbf{C_2}$		

By applying the GAP tester , we found that the shadowed ones are the solvable subgroups of the large orders

The twisted simple groups of Lie type

The Ree group Ree(27)

Of Order = 10073444472 = $2^3.3^9.7.13.19.37$. and it is generated by $< a, b \mid a^2 = b^3 = (ab)^{19} = 1 >$

The standard generators of Ree(27) can be found in the section of the permutation representations of Ree(27) on 19684 points appears in the ATLAS(as b11 and b21) . Let a=b11 and b:=b21

From the character table of Ree(27) , we found that Ree(27) is (2A,3A,19A)-group

A maximal subgroup M of Suz(8) is isomorphic to one of the following :

The Solvable subgroups of G= Ree(27)

1- The solvable subgroup of large order S and its repesentations in G = Ree(27) :

S≅3^{3+3+3}:26 = < $(abababbababb)^{-11}a(abababbababb)^{11}$, $(ababababbababb)^{14}(ababb)(ababababbababb)^{-14}$. > of order 511758
From the Character table of S , we found that S is (2A,3A,2A)-group .
The fusion map of S into **Ree(27) is** :
Class in S
1a 3a 3b 3c 9a 9b 9c 2a 6a 6b 13a 26a 13b 26b 13c 26c 13d 26d 13e 26e 13f 26f 13g 26g 13h 26h 13i 26i 13j 26j 13k 26k 13l 26l
Fusion in Ree(27)
1a 3a 3b 3c 9a 9b 9c 2a 6a 6b 13a 26a 13e 26e 13c 26b 13d 26d 13b 26a 13f 26f 13a 26a 13e 26e 13c 26b 13 26d 13b 26a 13f 26f

The permutation character induced from S to Ree(27) is: 1a+19683a
Some properties of S :

ISSN 2347-1921

1- 3^{3+3+3}:26 of order 511758 = $2.3^9.13$
2- 37:6 of order = 222 = 2.3.37
3- $(2^2 \times D_{14})$:3 of order=168=2^3.3.7
4- 19:6 of order 114 = 2.3.19
5- $2 \times L_2(27)$ of order=19656 = $2^3.3^3$.7.13.
6- $L_2(8)$:3 of order = 1512 = $2^3.3^3$.7

By applying the GAP tester , we found that all maximal subgroups of Ree(27) , except the fifth and the sixth ones , are solvables

S is SIMPLE : false
S is ABELIAN : false
S is NILPOTENT : false
S is SUPERSOLVABLE : false

2- The Lattice of Solvable subgroups of large orders in the maximal subgroups of G ==Ree(27) :

H ≤G	**The Maximal Subgroups**					
G	**3^{3+3+3}:26**	37:6	$(2^2 \times D_{14})$:3	19:6	$2 \times L_2(27)$	L_2(8
3^{3+3+3}:26	**3^{3+3+3}: C_{13}**	3^{3+3+3}: **S_3**	3^6 **:C_{13}:C_2**			
37:6	**37:3**	**D_{74}**	**C_6**			
$(2^2 \times D_{14})$:3	**2x14 : 3**	7:3**:2**	D_{28} **x 2**		A_4 **x 2**	
19:6	**19:3**	D_{38}	**C_6**	**A_4**		
$2 \times L_2(27)$	**3^3 : C_{13}**	D_{28}	D_{26}			
$L_2(8)$:3	**2^3:7:3**	$L_2(8)$	3^{1+2}:2		7:3**:2**	

By applying the GAP tester , we found that the shadowed ones are the solvable subgroups of the large orders

The twisted simple groups of Lie type

The Solvable subgroups of G

ISSN 2347-1921

The twisted group **$^2F_4(2)'$**

Of Order = 17971200 = $2^{11}.3^3.5^2.13$.. and it is generated by $\langle\, a, b \mid a^2 = b^3 = (ab)^{13} = [a, b]^5 = [a, bab]^4 = ((ab)^4ab^{-1})^6 = 1 \,\rangle$

The standard generators of **$^2F_4(2)'$** can be found in the section of the permutation representations of **$^2F_4(2)'$**on 1600 points appears in the ATLAS(as b11 and b21) . Let a=b11 and b:=b21

From the character table of $^2F_4(2)'$, we found that $^2F_4(2)'$ is (2A,3A,13A)-group

A maximal subgroup M of $^2F_4(2)'$ is isomorphic to one of the following :

1- $L_3(3):2$ of order 5616 = $2^4.3^3.13$
2- $L_3(3):2$ of order 5616 = $2^4.3^3.13$
3- $2.[2^8].5.4$ of order=10240=$2^{11}.5$
4- $L_2(25)$ of order 7800 = $2^3.3.5^2.13$
5- $2^2.[2^8].S_3$ of order=6144 = $2^{11}.3$.
6- $A_6.2^2$ of order = 2880= $2^6.3^2.5$
7- $A_6.2^2$ of order = 2880= $2^6.3^2.5$
8- $5^2:4A_4$ order = 1200= $2^4.3.5^2$

By applying the GAP tester , we found that all maximal subgroups of $^2F_4(2)'$, except the first and the second ones , are solvables

1- The solvable subgroup of large order S and its repesentations in G =$^2F_4(2)'$:

S≅$2.[2^8].5.4$ = < $((baba)^{-1}(abab))^2$, $(ab)^{-4}(b)(ab)^{-4}$.>
From the Character table of S , we found that S is (2A,4A,16A)-group .

<u>The fusion map of S into $^2F_4(2)'$ is :</u>

<u>Class in S</u>

1a 2a 2b 2c 2d 2e 2f 4a 4b 4c 4d 4e 4f 4g 4h 4i 5a 8a 8b 8c 8d 8e 8f 10a 16a 16b 16c 16d

<u>Fusion in G</u>

1a 2a 2a 2a 2b 2b 2b 4a 4a 4a 4b 4b 4c 4c 4c 4c 5a 8a 8a 8b 8b 8c 8d 10a 16a 16b 16d 16c

<u>The permutation character induced from S to $^2F_4(2)'$ is:</u>

1a+78a+351a+650a+675a

<u>Some properties of S :</u>

S is SIMPLE : false
S is ABELIAN : false
S is NILPOTENT : false
S is SUPERSOLVABLE : false

2- The Lattice of Solvable subgroups of large orders in the maximal subgroups of G =$^2F_4(2)'$:

H ≤G	The Maximal Subgroups					
G	$L_3(3):2$	$5^2:4A_4$	$2.[2^8].5.4$	$L_2(25)$	$2^2.[2^8].S_3$	$A_6.2^2$
$L_3(3):2$	$L_3(3)$	$3^{1+2}:D_8$	$2.S_4.2$	13 : 6	S_4 x 2	
$5^2:4A_4$	5^2: 2 .2 : 2 : 2	$5^2:2A_4$	5^2 : 3 :2 . 2	$Q_8.6$		
$2.[2^8].5.4$	2^2. 4 :2: 2 :2. 2 : 2 : 5 : 2	8:2x2 :2.2 :2.2 :2:2	2^2: 5 :2.2x2			
$L_2(25)$	5^2 : 3 : 2 .2	$A_5.2$	D_{26}	D_{24}		
$2^2.[2^8].S_3$	D_8 x.2^2 :2 .2 :2 : 2 . 2 : 3	8 :2.2 :2.2 : 2.2 :2 : 2	D_8 x.4 :2 .2 :2 :2 . S_3			
$A_6.2^2$	3 : S_3 .2 . 2 : 2	$A_6.2$	D_{10} .2 x2	M_{10}	$Q_8.\ 2^2$	

By applying the GAP tester , we found that the shadowed ones are the solvable subgroups of the large orders

ISSN 2347-1921

The twisted simple groups of Lie type	**The Solvable subgroups of G**
The twisted group **$^3D_4(2)$ Of Order = 211341312 = $2^{12}.3^4.7^2.13$... and it is generated by $\langle a, b \mid a^2 = b^9 = (ab)^{13} = (ab)^8 = 1 \rangle$** The standard generators of **$^3D_4(2)$** can be found in the section of the permutation representations of **$^3D_4(2)$** on 819 points appears in the ATLAS(as b11 and b21) . Let a=b11 and b:=b21 **From the character table of $^3D_4(2)$, we found that $^3D_4(2)$ is (2A,9A,13A)-group** **A maximal subgroup M of $^3D_4(2)$ is isomorphic to one of the following :** 1- 2^{1+8}:<u>$L_2(8)$</u>, of order 258048=$2^{12}.3^2.7$ 2- $[2^{11}]:(7 \times S_3)$, of order 86016=$2^{12}.3.7$. 3- <u>$U_3(3)$</u>:2, order 12096=$2^6.3^3.7$. 4- $S_3 \times$ <u>$L_2(8)$</u>, of order 3024= $2^4.3^3.7$. 5- $(7 \times$ <u>$L_2(7)$</u>$)$:2, of order 2352= $2^4.3.7^2$. 6- $3^{1+2}.2S_4$, of order 1296= $2^4.3^4$. 7- $7^2:2A_4$, of order 1176= $2^3.3.7^2$. 8- $3^2:2A_4$, of order 216= $2^3.3^3$ 9- 13:4, of order 52= $2^2.13$.	**1-The solvable subgroup of large order S and its repesentations in G =**$^3D_4(2)$ $S \cong [2^{11}]:(7 \times S_3)$ = < a , (abababbbb))² > From the Character table of S , we found that S is (2A,14A,21A)-group . **<u>The fusion map of S into $^2F_4(2)'$ is :</u>** **<u>Class in S</u>** 1a 2a 2b 4a 4b 2c 4c 3a 6a 2d 4d 4e 2e 8a 8b 7a 14a 21a 14b 28a 7b 14c 21b 14d 28b 7c 14e 21c 14f 28c 7d 14g 21d 14h 28d 7e 14i 21e 14j 28e 7f 14k 21f 14l 28f **<u>Fusion in G</u>** 1a 2a 2b 4a 4b 2b 4c 3b 6b 2a 4b 4a 2b 8b 8a 7c 14b 21b 14b 28c 7a 14c 21c 14c 28a 7b 14a 21a 14a 28b 7c 14b 21b 14b 28c 7a 14c 21c 14c 28a 7b 14a 21a 14a 28b **<u>The permutation character induced from S to $^2F_4(2)'$ is:</u>** 1a+324a+468a+1664a **<u>Some properties of S :</u>** S is SIMPLE : false S is ABELIAN : false S is NILPOTENT : false S is SUPERSOLVABLE : false

ISSN 2347-1921

2-The Lattice of Solvable subgroups of large orders in the maximal subgroups of G =$^2F_4(2)'$:

H ≤G	The Maximal Subgroups								
G	2^{1+8}:L$_2$(8)	$S_3 \times$ L$_2$(8)	[2^{11}]:($7 \times S_3$)	U$_3$(3):2	($7 \times$ L$_2$(7))	7^2:2A_4	3^2:2A_4	3^{1+2}.2S_4	13:4
2^{1+8}:L$_2$(8)	2^{1+8}:D_{14}	2^{1+8}:D_{18}	2^{1+8}:(2x2x2) :7						
$S_3 \times$ L$_2$(8)	$S_3 \times D_{14}$	$S_3 \times D_{18}$	$S_3 \times$ (2x2x2 :7						
[2^{11}]:($7 \times S_3$)	[2^{11}]:(7×2)	[2^{11}]:(7×3)	[2^{11}]:S_3						
U$_3$(3):2	(SL(2,3:4:2	((4 x4):3):2	(((3x 3)3):8):2	PSL(3,2)					
($7 \times$ L$_2$(7))	7 x (7 : 3)	7 x S_4	7 x S_4						
7^2:2A_4	7^2:2.(2 x 2)	7^2: (2.3)							
3^2:2A_4	3^2:2.(2 x 2)	3^2: (2.3)							
3^{1+2}.2S_4	3^{1+2}.2A_4	3^{1+2}.2S_3	3^{1+2}.2Q_8						
13:4	13 : 2	13	4						

By applying the GAP tester , we found that the shadowed ones are the solvable subgroups of the large orders

Acknowledgements: The authors would like to thank Institute of Scientific Research and Revival of Islamic Heritage at Umm Al-Qura University (43305029) for the financial support)

References

[1] J. Bray , D. Hot and M. Colva "The maximal subgroups of the low-dimensional finite classical groups" .London mathematical society ,407 . , 2013

[2] Breuer,T., " Solvable Subgroups of Maximal Order in Sporadic Simple Groups" , Lehrstuhl Dfur Mathematik RWTH, 52056 Aachen, Germany ,version(1) in 2006 and version(2) in 2012 .

[3] J. H. Conway, R. T. Curtis, S. P. Norton, R. A. Parker, and R. A.Wilson. "An ATLAS of Finite Groups". Oxford University Press, 1985.

[4] Vahid Dabbaghian-Abdoly. "An algorithm to construct representations of finite groups".PhD thesis, Carleton University, 2003.

[5] John D. Dixon. "Constructing representations of finite groups". In Groups and computation(New Brunswick, NJ, 1991), volume 11 of DIMACS Ser. Discrete Math. Theoret. Comput. Sci., pages 105–112. Amer. Math. Soc., Providence, RI, 1993.

[6] The GAP Group, GAP { Groups, Algorithms, and Programming, Version 4.4.10; 2007.(http://www.gap-system.org)

[7] D. Gorenstein, "Finite Groups", Harper & Row New York, Evanston and London, Harper's Series in Modern Mathematics, 1968.

[8] D Gorenstein. , "*The classification of finite simple groups". Vol. 1. Groups of non characteristic 2 type , 1983*

[9] L. C. Grove, "Groups and Characters", John Wiley & Sons, New York, 1997.

[10] Mann, A. , " Soluble subgroups of symmetric and alternating groups " ,Israel Journal of Mathematics , Volume 55, Number 2, 1986.

[11] Simon Jon Nickerson , "An Atlas of Characteristic Zero Representations" , Ph.D Thesis , University of Birmingham , 2006

[12] Gordon James and Martin Liebeck. "Representations and characters of groups". Cambridge Mathematical Textbooks. Cambridge University Press, Cambridge, 1993.

[13] Vdovin, E. P. , " Abelian and Nilpotent Subgroups of Maximal Orders of Finite Simple Groups " , Ph.d thesis SB RAS, Institute of Mathematics ,2000

[14] Robert Wilson, Peter Walsh, Jonathan Tripp, Ibrahim Suleiman, Stephen Rogers, Richard Parker, Simon Norton, Simon Nickerson, Steve Linton, John Bray, and Rachel Abbott. "Atlas of Finite Group Representations". Birmingham University , 2012