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Adomian Decomposition Method of Fredholm Integral Equation of the Second kind using Maple

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ABSTRACT

In this paper, we will be find exact solution of Fredholm Integral Equation of the second kind through using Adomian Decomposition Method by using Maple 17 program, then we found that exact solution.

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Fredholm integral equation of the second kind; Adomian Decomposition Method.

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INTRODUCTION

In this paper, we consider the Fredholm integral equation of the second kind

$$y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)\,dt, \quad (1)$$

The unknown function $y(x)$, that will be determined, occurs inside and outside the integral sign. Th kernel $K(x,t)$ and the function $f(x)$ are given real-valued functions, and λ is aparameter.

In this paper, we present the computation of exact solution of Fredholm integral equation of the second kind using Maple17.

Adomian Decomposition Method

In this section, we use the technique of the Adomian Decomposition Method [4,9]. The Adomain Decomposition Method consists of decomposing the unknown function $y(x)$ of any equation into a sum of an infinite number of components defined by the decomposition series

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \quad (2)$$

Or equivalenty

$$y(x) = y_0(x) + y_1(x) + y_2(x) + y_3(x) + \cdots \quad (3)$$

Where the components $y_n(x), n \geq 0$ will be determined recurrently. The Adomain Decomposition Method concerns itself with finding the components $y_0(x), y_1(x), y_2(x), y_3(x), \cdots$ individually.

To establish the recurrence relation, we substitute (2) into the Fredholm integral equatin (1) to obtain

$$\sum_{n=0}^{\infty} y_n(x) = f(x) + \lambda \int_a^b K(x,t)\left(\sum_{n=0}^{\infty} y_n(t)\right) dt, \quad (4)$$

or equivalenty

$$y_0(x) + y_1(x) + y_2(x) + y_3(x) + \cdots = f(x) + \lambda \int_a^b K(x,t)\,[\,y_0(t) + y_1(t) + y_2(t) \cdots]dt \quad (5)$$

The zeroth component $y_0(x)$ is identified by all terms that are not included under the integral sign. (This means that the components $y_n(x), n \geq 0$ of the unknown function $y(x)$ are completely determined by setting the recurrence relation

$$y_0(x) = f(x), \qquad y_{n+1}(x) = \lambda \int_a^b K(x,t)y_n(t)\,dt, \quad n \geq 0 \quad (6)$$

or equivalenty

$$y_0(x) = f(x),$$

$$y_1(x) = \lambda \int_a^b K(x,t)y_0(t)\,dt$$

$$y_2(x) = \lambda \int_a^b K(x,t)y_1(t)\,dt$$

$$y_3(x) = \lambda \int_a^b K(x,t)y_2(t)\,dt$$

$$y_4(x) = \lambda \int_a^b K(x,t)y_3(t)\,dt \quad (7)$$

And so on for other comonents.

In view of (7), the components $y_0(x), y_1(x), y_2(x), y_3(x), \cdots$ are completely determined. As a result, the solution $y(x)$ of the Fredholm integral equation (1) is readily obtained in a series form by using the series as sumption in (2).

NUMERICAL EXAMPLES

In this section, we solve three examples are provided. These examples are considered to illustrate the Adomian Decomposition Method using Maple17.

Example1. Consider the Fredholm integral equation of second kind

$$y(x) = 1 + \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y(t)dt\,.$$

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Applying the Adomian Decomposition Method we find

$$\sum_{n=0}^{\infty} y_n(x) = 1 + \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} \sum_{n=0}^{\infty} y_n(t)\, dt\,.$$

To determine the components of $y(x)$, we use the recurrence relation

$$y_0(x) = 1, \qquad y_{n+1}(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_n(t)dt, \qquad n \geq 0.$$

This in turn gives

$y_0(x) = 1,$

$$y_1(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_0(t)dt = \frac{\pi}{4}\sin^2 x,$$

$$y_2(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_1(t)dt = \frac{\pi^2}{32}\sin^2 x,$$

$$y_3(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_2(t)dt = \frac{\pi^3}{256}\sin^2 x,$$

$$y_4(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_3(t)dt = \frac{\pi^4}{2048}\sin^2 x,$$

$$y_5(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_4(t)dt = \frac{\pi^5}{16384}\sin^2 x,$$

$$y_6(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_5(t)dt = \frac{\pi^6}{131072}\sin^2 x,$$

$$y_7(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_6(t)dt = \frac{\pi^7}{1048576}\sin^2 x$$

$$y_8(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_7(t)dt = \frac{\pi^8}{8388608}\sin^2 x,$$

$$y_9(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_8(t)dt = \frac{\pi^9}{67108864}\sin^2 x,$$

$$y_{10}(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_9(t)dt = \frac{\pi^{10}}{536870912}\sin^2 x,$$

$$y_{11}(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_{10}(t)dt = \frac{\pi^{11}}{4294967296}\sin^2 x,$$

$$y_{12}(x) = \frac{1}{2}\sin^2 x \int_0^{\frac{\pi}{2}} y_{11}(t)dt = \frac{\pi^{12}}{34359738368}\sin^2 x$$

And so on. Using (2) gives the series solution

$$y(x) = 1 + \frac{\pi}{4}\sin^2 x + \frac{\pi^2}{32}\sin^2 x + \frac{\pi^3}{256}\sin^2 x + \frac{\pi^4}{2048}\sin^2 x + \frac{\pi^5}{16384}\sin^2 x + \frac{\pi^6}{131072}\sin^2 x + \frac{\pi^7}{1048576}\sin^2 x + \frac{\pi^8}{8388608}\sin^2 x + \frac{\pi^9}{67108864}\sin^2 x + \frac{\pi^{10}}{536870912}\sin^2 x + \frac{\pi^{11}}{4294967296}\sin^2 x + \frac{\pi^{12}}{34359738368}\sin^2 x + \cdots$$

Then the gives the exact solution

$$y(x) = 1 + \frac{2\pi}{(8-\pi)}\sin^2 x$$

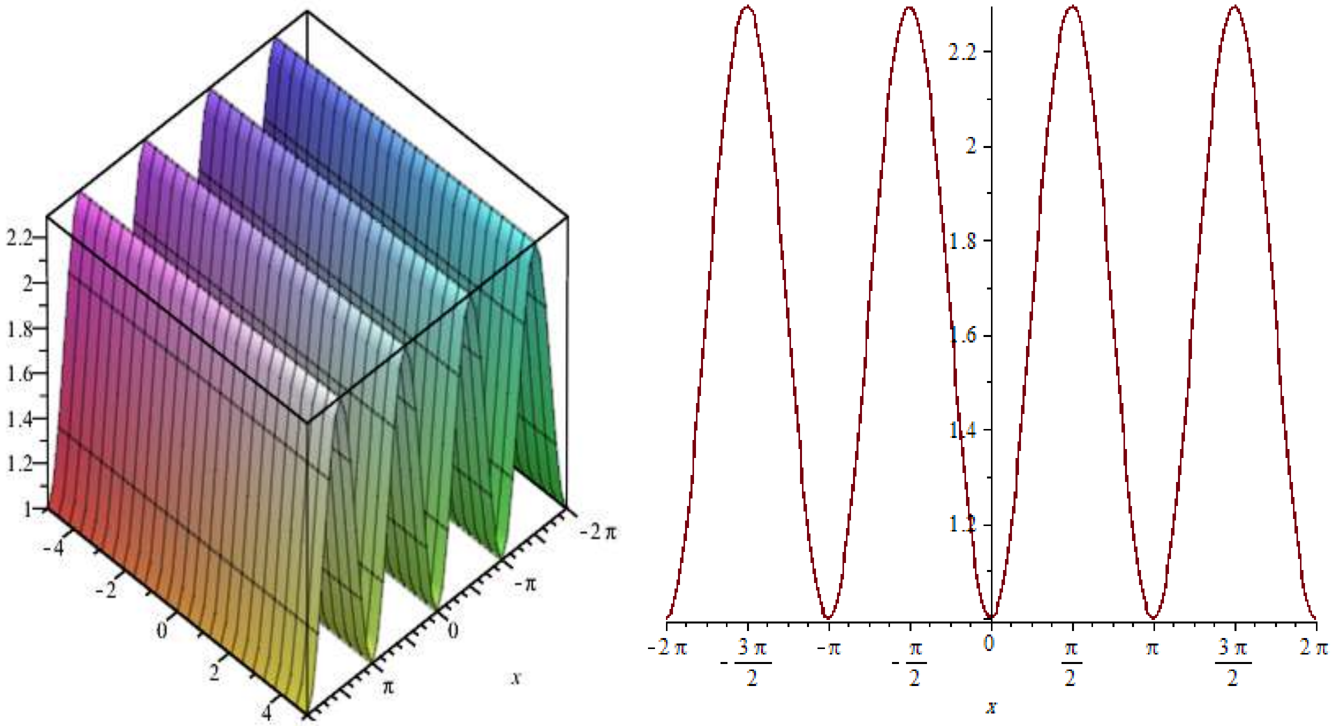

Fig. 1 Plot 3D and 2D of the exact solutions result Of Fredholm integral equation for example 1.

Example 2. Consider the Fredholm integral equation of second kind

$$y(x) = 1 - \frac{1}{15}x^2 + \int_{-1}^{1}(xy + x^2y^2)y(t)dt\,.$$

Applying the Adomian Decomposition Method we find

$$\sum_{n=0}^{\infty} y_n(x) = 1 - \frac{1}{15}x^2 + \int_{-1}^{1}(xy + x^2y^2)\sum_{n=0}^{\infty} y_n(t)\,dt.$$

To determine the components of $y(x)$, we use the recurrence relation

$$y_0(x) = 1 - \frac{1}{15}x^2, \qquad y_{n+1}(x) = \int_{-1}^{1}(xy + x^2y^2)\, y_n(t)dt, \qquad n \geq 0.$$

This in turn gives

$$y_0(x) = 1 - \frac{1}{15}x^2,$$

$$y_1(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_0(t)dt = \frac{16}{25}x^2$$

$$y_2(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_1(t)dt = \frac{32}{125}x^2$$

$$y_3(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_2(t)dt = \frac{64}{625}x^2,$$

$$y_4(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_3(t)dt = \frac{128}{3125}x^2,$$

$$y_5(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_4(t)dt = \frac{256}{15625}x^2$$

$$y_6(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_5(t)dt = \frac{512}{78125}x^2,$$

$$y_7(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_6(t)dt = \frac{1024}{390625}x^2,$$

$$y_8(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_7(t)dt = \frac{2048}{1953125}x^2,$$

$$y_9(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_8(t)dt = \frac{4096}{9765625}x^2,$$

$$y_{10}(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_9(t)dt = \frac{8192}{48828125}x^2,$$

$$y_{11}(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_{10}(t)dt = \frac{16384}{244140625}x^2,$$

$$y_{12}(x) = \int_{-1}^{1} (xy + x^2y^2)\, y_{11}(t)dt = \frac{32768}{1220703125}x^2$$

And so on. Using (2) gives the series solution

$$y(x) = 1 + \frac{16}{25}x^2 + \frac{32}{125}x^2 + \frac{64}{625}x^2 + \frac{128}{3125}x^2 + \frac{256}{15625}x^2 + \frac{512}{78125}x^2 + \frac{1024}{390625}x^2 + \frac{2048}{1953125}x^2 + \frac{4096}{9765625}x^2 + \frac{8192}{48828125}x^2 + \frac{16384}{244140625}x^2 + \frac{32768}{1220703125}x^2 + \cdots$$

Then the gives the exact solution

$$y(x) = 1 + \frac{3662043839}{3662109375}x^2$$

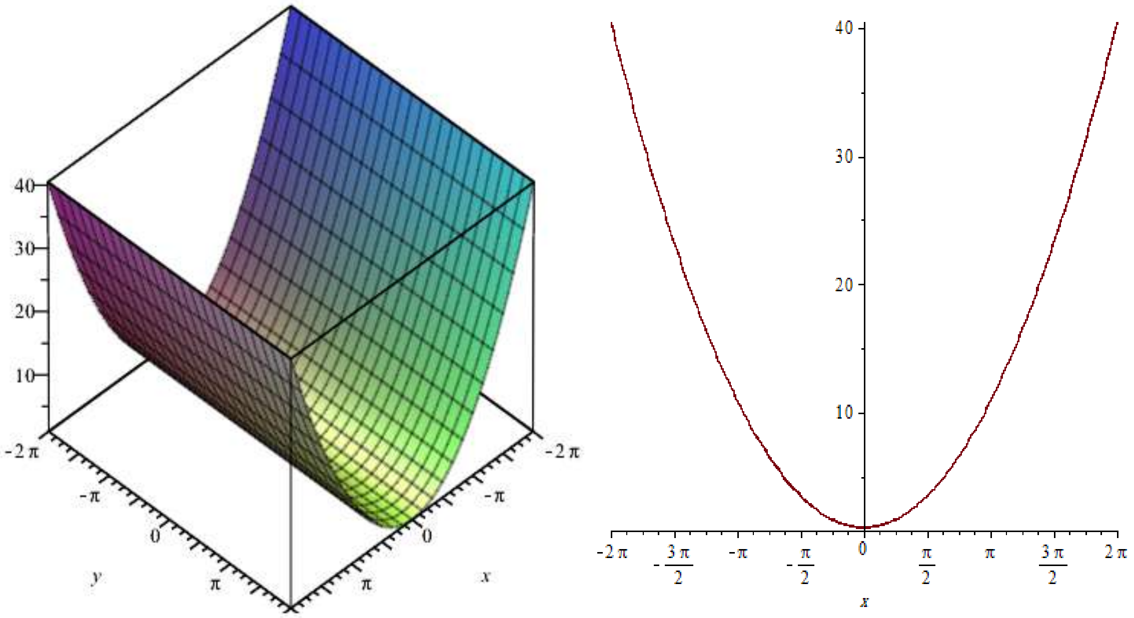

Fig. 2 Plot 3D and 2D of the exact solutions result Of Fredholm integral equation for example 2.

Example3. Consider the Fredholm integral equation of second kind

$$y(x) = \frac{\pi}{4} - sec^2 x - \int_0^{\frac{\pi}{4}} y(t)dt\,.$$

Applying the Adomian Decomposition Method we find

$$\sum_{n=0}^{\infty} y_n(x) = \frac{\pi}{4} - sec^2 x - \int_0^{\frac{\pi}{4}} \sum_{n=0}^{\infty} y_n(t)\, dt$$

To determine the components of $y(x)$, we use the recurrence relation

$$y_0(x) = \frac{\pi}{4} - sec^2 x, \qquad y_{n+1}(x) = -\int_0^{\frac{\pi}{4}} y_n(t)dt, \qquad n \geq 0.$$

This in turn gives

$$y_0(x) = \frac{\pi}{4} - sec^2 x,$$

$$y_1(x) = -\int_0^{\frac{\pi}{4}} y_0(t)dt = 1 - \frac{1}{16}\pi^2,$$

$$y_2(x) = -\int_0^{\frac{\pi}{4}} y_1(t)dt = \frac{1}{64}\pi^3 - \frac{1}{4}\pi,$$

$$y_3(x) = -\int_0^{\frac{\pi}{4}} y_2(t)dt = -\frac{1}{256}\pi^4 + \frac{1}{16}\pi^2,$$

$$y_4(x) = -\int_0^{\frac{\pi}{4}} y_3(t)dt = \frac{1}{1024}\pi^5 - \frac{1}{64}\pi^3,$$

$$y_5(x) = -\int_0^{\frac{\pi}{4}} y_4(t)dt = -\frac{1}{4096}\pi^6 + \frac{1}{256}\pi^4,$$

$$y_6(x) = -\int_0^{\frac{\pi}{4}} y_5(t)dt = +\frac{1}{16384}\pi^7 - \frac{1}{1024}\pi^5,$$

$$y_7(x) = -\int_0^{\frac{\pi}{4}} y_6(t)dt = \frac{1}{65536}\pi^8 + \frac{1}{4096}\pi^6,$$

$$y_8(x) = -\int_0^{\frac{\pi}{4}} y_7(t)dt = \frac{1}{262144}\pi^9 - \frac{1}{16384}\pi^7,$$

$$y_9(x) = -\int_0^{\frac{\pi}{4}} y_8(t)dt = -\frac{1}{1048576}\pi^{10} + \frac{1}{65536}\pi^8,$$

$$y_{10}(x) = -\int_0^{\frac{\pi}{4}} y_9(t)dt = \frac{1}{4194304}\pi^{11} - \frac{1}{262144}\pi^9,$$

$$y_{11}(x) = -\int_0^{\frac{\pi}{4}} y_{10}(t)dt = -\frac{1}{16777216}\pi^{12} + \frac{1}{1048576}\pi^{10},$$

$$y_{12}(x) = -\int_0^{\frac{\pi}{4}} y_{11}(t)dt = \frac{1}{67108864}\pi^{13} - \frac{1}{4194304}\pi^{11}$$

This in turn gives

$$y(x) = 1 - \frac{1}{16}\pi^2 + \frac{1}{64}\pi^3 - \frac{1}{4}\pi - \frac{1}{256}\pi^4 + \frac{1}{16}\pi^2 + \frac{1}{1024}\pi^5 - \frac{1}{64}\pi^3 - \frac{1}{4096}\pi^6 + \frac{1}{256}\pi^4 + \frac{1}{16384}\pi^7 - \frac{1}{1024}\pi^5 - \frac{1}{65536}\pi^8 + \frac{1}{4096}\pi^6 + \frac{1}{262144}\pi^9 - \frac{1}{16384}\pi^7 - \frac{1}{1048576}\pi^{10} + \frac{1}{65536}\pi^8 + \frac{1}{4194304}\pi^{11} - \frac{1}{262144}\pi^9 - \frac{1}{16777216}\pi^{12} + \frac{1}{1048576}\pi^{10} + \frac{1}{67108864}\pi^{13} - \frac{1}{4194304}\pi^{11} + \cdots$$

Then the gives the exact solution

$$y(x) = 1 - sec^2 x - \frac{1}{67108864}\pi^{13} - \frac{1}{16777216}\pi^{12}$$

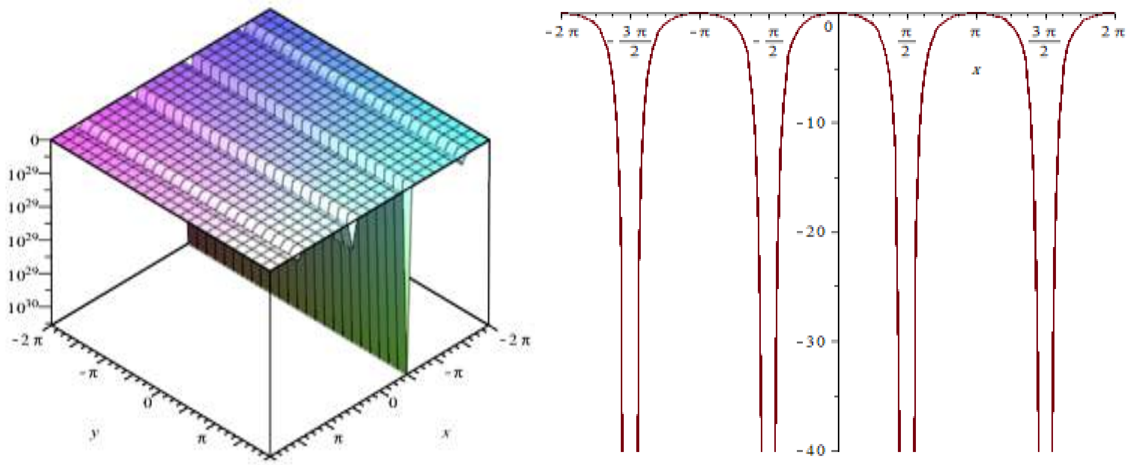

Fig. 3 Plot 3D and 2D of the exact solutions result Of Fredholm integral equation for example 3.

Conclusion

In this paper, Adomain Decomposition Method, for solving Fredholm integral equations of the second kind, is studied successfully. The computations associated with examples were performed using Maple 17.

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