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New Analytical and Empirical Expressions for the Percentage of the Star's Flux Through Transparent Atmosphere Between Two Wave Lengths

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ABSTRACT

In this paper, new analytical and empirical expressions for the percentage of the star's flux through transparent atmosphere between two wave lengths $\Delta\lambda = \lambda_2 - \lambda_1$ are developed. For the analytical developments, literal expression of the percentage, A, of the total emitted flux from a star of temperature T radiates as black body is established in terms of the polyogarithm functions, together with the rate of change of A, with respect to T.

For the computational developments, the maximum value of the percentage of the total flux MF in the region $\Delta\lambda$ was computed together with the corresponding temperature at the maximum percentage flux. The variations of the percentage of the total flux with temperature are displayed graphically for some values of $\Delta\lambda$ Linear correlation coefficients between different attributes are also given. Finally, new empirical relation was developed between MF and $\Delta\lambda$, with complete error estimates These error estimates are ,the variance of the fit ,the variance of the least squares solutions vector, the average square distance between the exact and the least-squares solutions. All the precision criteria of this relation are very satisfactory.

Keywords: Stellar flux through transparentatmosphere; meteorology; radiative transfer.

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1. Introduction

Radiation flux is a measure of the flow of radiation from a given radioactive source. For example Earth is heated by the Sun at a rate of heating depends on the flux of radiation energy that reaches the Earth from the Sun. Since the flux is the amount of energy that reaches the Earth per unit of area and per unit of time, then it is the power reaching per unit of area. Moreover, any redistribution of UV flux to the visible, and vice versa will modify the transfer of solar radiation through the atmosphere (Vardavas 1987), and, hence, alter the Earth's radiation budget. On the other hand, solar ultraviolet B flux (UV-B) radiation (280–320 nm) has been associated with reduced risk of cancer of the breast, colon, ovary, and prostate, as well as non-Hodgkin's lymphoma (Grant2003)

The above discussion is very much brief, but in fact their exist countless texts and reaches papers on the scientific and human being roles that played by the fluxes arrive at the Earth's surface whatever their sources may be.

So, the calculations of the percentage of total emitted flux arrives the Earth's surface from the star (Sun, or other radiating source) is in fact desperately needed, for this aims the present work is devoted.

In the present paper, new analytical and empirical expressions for the percentage of the star's flux through transparent atmosphere between two wave lengths $\Delta\lambda=\lambda_2-\lambda_1$ are developed. For the analytical developments, literal expression of the percentage, A, of the total emitted flux from a star of temperature T radiates as black body is established in terms of the polyogarithm functions, together with the rate of change of A, with respect to T.

For the computational developments, the maximum value of the percentage of the total flux MF in the region $\Delta\lambda$ is computed together with the corresponding temperature at the maximum percentage flux. The variations of the percentage of the total flux with temperature are displayed graphically for some values of $\Delta\lambda$. Linear correlation coefficients between different attributes are also given. Finally, new empirical relation was developed between MF and $\Delta\lambda$, with complete error estimates. These error estimates are ,the variance of the fit ,the variance of the least squares solutions vector, the average square distance between the exact and the least-squares solutions. All the precision criteria of this relation are very satisfactory.

2. Linear Least Squares Fit

Let y be represented by the general linear expression of the form:

$$\sum_{i=1}^{n} c_i\varphi_i(x),$$

where φ's are linear independent functions of x. Let $\mathbf{C}$ be the vector of the exact values of the C;s coefficients and $\hat{\mathbf{C}}$ the least squares estimators of $\mathbf{C}$ obtained from the solution of the normal equations of the form $\mathbf{G}\hat{\mathbf{C}}=\mathbf{b}$.The coefficients matrix $\mathbf{G}(n\times n)$ is symmetric positive definite, that is ,all its eigen values $\mu_i; i=1,2,\cdots,n$ are positive. Let $E(z)$ denotes the expectation of z and σ^2 the variance of the fit, defined as:

$$\sigma^2=\frac{q_n}{(N-n)},$$

where

$$q_n=(\mathbf{y}-\mathbf{\Phi}^T\hat{\mathbf{C}})^T(\mathbf{y}-\mathbf{\Phi}^T\hat{\mathbf{C}}),$$

N is the number of observations, **y** is the vector with elements y_k and $\mathbf{\Phi}(n\times N)$ has elements $\Phi_{ik}=\Phi_i(x_k)$. The transpose of a vector or a matrix is indicated by the superscript "T".

According to the least squares criterion, it could be shown that (Kopal and Sharaf,1980):

1-The estimators $\hat{\mathbf{c}}$ obtained by the least squares method gives the minimum of q_n.

2-The estimators $\hat{\mathbf{c}}$ of the coefficients $\mathbf{c}$, obtained by the least squares method, are unbiased; i.e. $E(\hat{\mathbf{c}})=\mathbf{c}$.

3-The variance-covariance matrix $\mathrm{Var}(\hat{\mathbf{C}})$ of the unbiased estimators $\hat{\mathbf{C}}$ is given by:

$$\mathrm{Var}(\hat{\mathbf{C}})=\sigma^2\mathbf{G}^{-1},$$

where $\mathbf{G}^{-1}$ is the inverse of the matrix $\mathbf{G}$.

4-The average squared distance between $\hat{\mathbf{c}}$ and $\mathbf{c}$ is:

$$E(L^2) = \sigma^2 \sum_{i=1}^{n} \frac{1}{\mu_i}.$$

Also it should be noted that, if the precision is measured by probable error e, then:

$$e = 0.6745\,\sigma.$$

3. Analytical Developments

Taking a star's radiation to be that of a black body, and the Earth's atmosphere to be perfectly transparent between two wave lengths λ_1 and λ_2, and completely opaque elsewhere. With these assumptions, the percentage A of the total emitted flux from the star of temperature T and radiates as black body which arrives the Earth's surface can be computed from

$$A = \left(\frac{B_{0-\lambda_2} - B_{0-\lambda_1}}{B_{0-\infty}}\right) \times 100,$$

which is written as:

$$A = (Q_2 - Q_1) \times 100, \tag{1.1}$$

$$Q_i = \left(\frac{B_{0-\lambda_i}}{B_{0-\infty}}\right)_{\lambda_i T} \quad i = 1,2, \tag{1.2}$$

Where the Planck's functions $B_{0-\lambda}$ and $B_{0-\infty_1}$ (Carroll. and Ostlie, 1996)are defined as:

$$B_{0-\lambda} = \int_0^{\lambda} B_\lambda \, d\lambda, \tag{2.1}$$

$$B_{0-\infty} = \int_0^{\infty} B_\lambda \, d\lambda \tag{2.2}$$

and

$$B_\lambda = \frac{2\,h\,c^2}{\lambda^5} \frac{1}{e^{h\nu/k\lambda T} - 1}, \tag{3}$$

is the intensity of radiation at the wave length λ , T is the absolute temperature (in Kelvin) and

$c =$ velocity of light $= 3\times10^{10}\,\text{cm s}^{-1}$,

$h =$ Planck's constant $= 6.57\times10^{-27}$ erg s,

$k =$ Boltzmann constant $= 1.38\times10^{-16}$ erg degree $^{-1}$.

B_λ is the amount of energy measured per wavelength interval If the energy per unit frequency interval is measured, we have ,

$$B_\nu = \frac{2\,h\,\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}. \tag{4}$$

B_λ and B_ν are common abbreviations for the black body energy distribution.

From Equations (2), (3) into Equation (1.2) we deduce that

$$\frac{B_{0-\lambda_i}}{B_{0-\infty}} = \frac{I_x}{I_0}, \tag{5}$$

where

$$I_x = \int_x^\infty \frac{y^3}{e^y - 1} dy = \sum_{n=1}^{\infty} \int_x^\infty y^3 e^{-ny} dy \qquad (6.1)$$

and

$$x = \frac{hc}{\lambda kT} = \frac{1.439773005}{\lambda T} \text{ cm K.} \qquad (6.2)$$

Since

$$\sum_{n=1}^{\infty} \frac{e^{-nx}}{n} = -\int \left(\sum_{n=1}^{\infty} e^{-nx} \right) dx = -\int e^{-x} (1 - e^{-x})^{-1} dx = -\ln(1 - e^{-x}) = -\ln(1 - \cosh x + \sinh x). \qquad (7.1)$$

and the polylogarithm function is defined as (Stegun 1972)

$$Li_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s}, \qquad (7.2)$$

then we get for $B_{0-\lambda_i} / B_{0-\infty}$ the literal analytical expression :

$$\frac{B_{0-\lambda_i}}{B_{0-\infty}} = \frac{15}{\pi^4} \left(-x^3 \ln(1 - \cosh x + \sinh x) + 3x^2 Li_2(e^{-x}) + 6x Li_3(e^{-x}) + 6 Li_4(e^{-x}) \right) \qquad (8)$$

The percentage A of the total emitted flux of Equation (1.1) could be given in terms of polylogarithm functions as::

$$A = \begin{pmatrix} -\frac{15}{\pi^4} \left(T \left(-T^2 \ln(1 - e^{-\lambda_1 T}) \lambda_1^3 + 3T\lambda_1^2 Li_2(e^{-\lambda_1 T}) + 6\lambda_1 Li_3(e^{-\lambda_1 T}) + \lambda_2 \left(T \lambda_2 \left(T \ln(1 - e^{-\lambda_2 T}) \lambda_2 - \right. \right. \right. \right. \\ \left. \left. \left. \left. -3 Li_2(e^{-\lambda_2 T}) \right) - 6 Li_3(e^{-\lambda_2 T}) \right) \right) + 6 \left(Li_4(e^{-\lambda_1 T}) - Li_4(e^{-\lambda_2 T}) \right) \right) \end{pmatrix} \times 100 \qquad (9)$$

Finally the rate of change of A with respect to T for a given values of λ_1 and λ_2 is

$$\frac{dA}{dT} = \frac{15T^3}{\pi^4} \left(\frac{\lambda_1^4}{e^{\lambda_1 T}} - \frac{\lambda_2^4}{e^{\lambda_2 T} - 1} \right). \qquad (10)$$

4. Computational Developments

4. 1The variations of the percentage of the total flux A

For the numerical applications of the above formulations, let us consider the following physical variables :

$T = 500\,(500)\,25500\text{ K}$, $\lambda_1 = 4000(60)7000 A^o$, $\lambda_2 = \lambda_1 + 200 \times k\ A^o; k = 1(1)31$.

The applications of Equation (9) using the above data are shown in Table 1 of Appendix I, in which: the first column represents $\Delta\lambda = (\lambda_2 - \lambda)_1 A^o$.The second column represents the maximum value of the percentage of the total flux in the region from λ_1 to λ_2 which arrives the Earth's surface from a star of temperature T radiates as black body.

The third column gives the temperature in K at the maximum percentage flux.

The variations of the percentage of the total flux A temperature are displayed in Figs. A of Appendix I for some values of $\Delta\lambda$.

4.2 Linear correlation coefficients

Calculating the linear correlation coefficient (Meeus 2000) between the different attributes, we find

1-The temperature TM at maximum flux is inversely proportion to $\Delta\lambda$ with linear correlation coefficient $r = -0.951269.$

2-The maximum flux MF is direct proportion to $\Delta\lambda$ with linear correlation coefficient $r = 0.989804$

3-The temperature at maximum flux is inversely proportion to maximum flux with linear correlation coefficient $r = -0.967801$

The values of all the correlation coefficients ≈ 1, which proves that the attributes are completely related linearly

4.3 Empirical Relation between $\Delta\lambda$ & MF

The formulations of Section 2 were applied to the data of the first two columns of Table 1 of Appendix I and the results are list in what follows:

1. The fitted equation is

$$MF = C_1 + C_2 (\Delta\lambda)^{0.9} + C_3 (\Delta\lambda)^{1.85}$$

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2. The solutions and their probable errors are

$$C_1 = -0.811273 \pm 0.0311391,$$

$$C_2 = 0.0311877 \pm 0.0000498859,$$

$$C_3 = -2.26523 \times 10^{-6} \pm 1.14085 \times 10^{-8}.$$

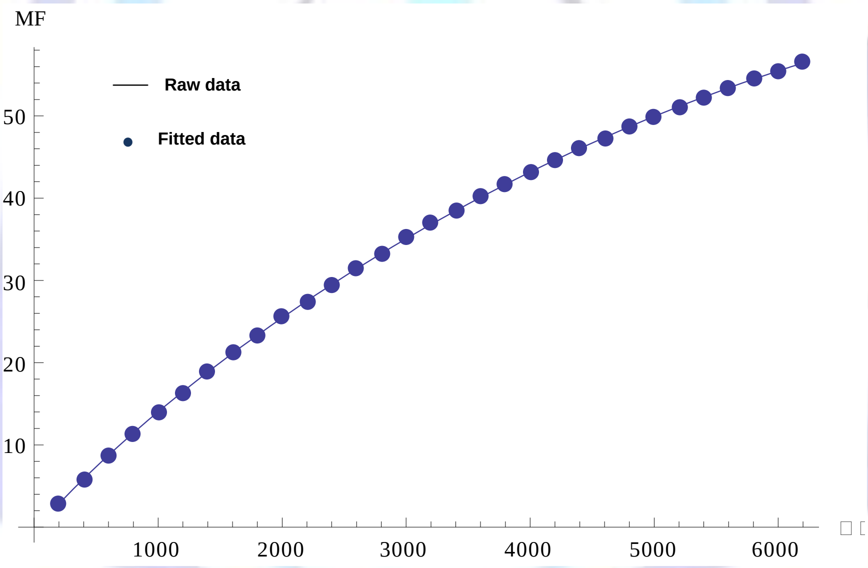
3. The probable error of the fit is

$$e = 0.0493356$$

4. The average squared distance between $\hat{C}$ and C is

$$E(L^2) = 0.00213133$$

5-Graph of the raw and fitted data



This relation is extremely accurate as indicated in the above error analysis report.

In concluding the present paper, new analytical and empirical expressions for the percentage of the star's flux through transparent atmosphere between two wave lengths $\Delta\lambda = \lambda_2 - \lambda_1$ are developed. For the analytical developments, literal expression of the percentage, A, of the total emitted flux from a star of temperature T radiates as black body is established in terms of the polyogarithm functions, together with the rate of change of A, with respect to T.

For the computational developments, the maximum value of the percentage of the total flux MF in the region $\Delta\lambda$ which arrives the Earth's surface from the star is computed together with the corresponding temperature at this maximum. The variations of the percentage of the total flux with temperature are displayed graphically for some values of $\Delta\lambda$. Linear correlation coefficients between different attributes are also given. Finally, new empirical relation was developed between MF and $\Delta\lambda$, with complete error estimates These error estimates are ,the variance of the fit ,the variance of the least squares solutions vector, the average square distance between the exact and the least-squares solutions. All the precision criteria of this relation are very satisfactory.

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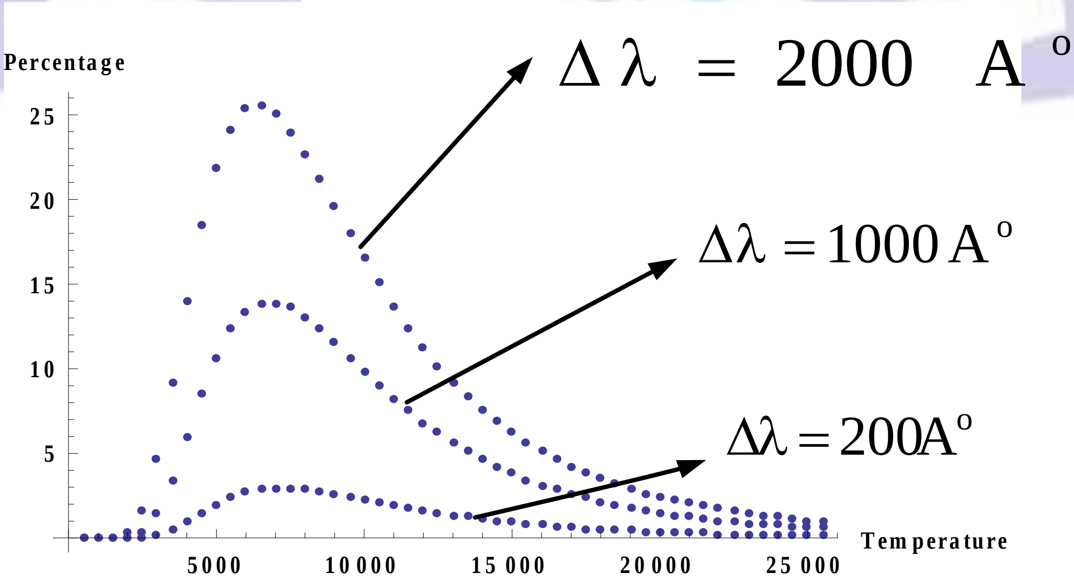
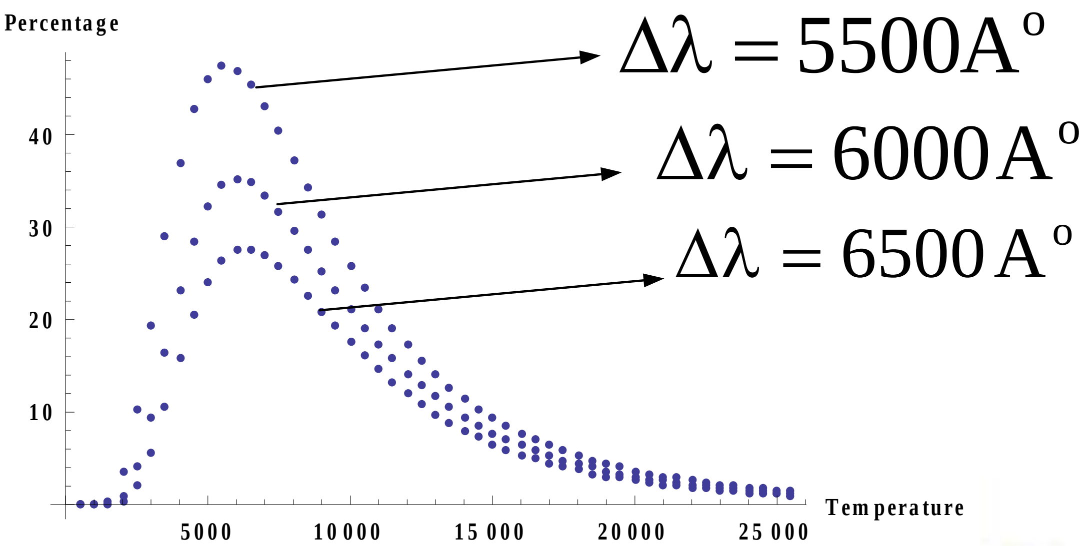
Appendix I

Table 1:Maximum Values of the Percentage of the Total Flux in the Reign $\Delta\lambda$ and the Corresponding Temperature T

□□	Maximum of A	T at A Max
200	2.98679	7000
400	5.87885	7000
600	8.66638	7000
800	11.343	7000
1000	13.9052	7000
1200	16.4	6500
1400	18.8273	6500
1600	21.155	6500
1800	23.3826	6500
2000	25.511	6500
2200	27.5418	6500
2400	29.5085	6000
2600	31.4781	6000
2800	33.3643	6000
3000	35.1689	6000
3200	36.8939	6000
3400	38.5419	6000
3600	40.1153	6000
3800	41.617	6000
4000	43.0498	6000
4200	44.5112	5500
4400	45.954	5500
4600	47.3365	5500
4800	48.6607	5500
5000	49.9288	5500
5200	51.1431	5500
5400	52.3057	5500
5600	53.4187	5500
5800	54.4843	5500
6000	55.5046	5500
6200	56.4814	5500

Figures A: The variations of the percentage of the total flux A with the Temperature for some values of $\Delta\lambda$





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