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A new Characterization of some Distributions based on the r th conditional moment of doubly Truncated Mean function

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Abstract

New characterizations of doubly truncated exponential, Pareto and power function distributions are presented by using the s th conditional expectation in terms of their failure rate and reversed failure rate. These results may serve as generalization of several other results in the literature. This characterization generalizes a result of Nofal(2010).

Key Words: characterization; failure rate; reversed failure rate; conditional expectation; exponential; Pareto; power function.

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1. Introduction

Characterizations of distributions have always played an important role in statistical theory and are widely published in the literature. Several functions are defined related to the residual life. The failure rate function, defined by: $h(x) = \frac{f(x)}{1-F(x)}$ represents the failure rate of X (or F) at age x and $\tau(x) = \frac{f(x)}{F(x)}$ represents the inversed failure rate of X (or F) at age x where $F(x) = P(X \le x)$, and $f(x)$ is the density function when X is continuous, or $f(x) = P(X = x)$ when X is discrete. Another interesting function is the mean residual life function, defined by$E(X - x \mid X \ge x)$, and it represents the expected additional life length for a unit which is alive at age x. This function is equivalent to the left censored mean function, also called vitality function (see Gupta (1975)), defined by$E(X \mid X \ge x)$.Applications of hazard functions are quite well known in the statistical literature. Another interesting function is the mean inactivity time, defined by $E(x - X \mid X \le x)$ and it is equivalent to the left censored lifetimes .it become quite popular among the statisticians, see for example Gupta and Han (2001). Anderson et al. (1993) show that the reversed hazard function plays the same role in the analysis of left-censored data as the hazard function plays in the analysis of right-censored data. Interestingly, The properties of the mean inactivity time have been considered by many authors, see, eg., Kayid and Ahmad (2004), and Ahmad , Kayid and Pellery (2005). Several characterizations of probability models have been obtained in the last 30 years based on the univariate failure rate or mean residual life functions. The problems of characterization of distributions are today a substantial part of probability theory and mathematical statistics. The mean inactivity time and mean residual life are applicable in biostatistics and many other actuarial science, engineering, economics, biometry and applied probability areas. They also are useful in survival analysis studies when we take are faced with left or right censored data.

2. Preliminaries

In this section we shall introduce notation, definitions and basic facts used throughout.

2.1. Definition

Let X be a non-negative random variable with probability density function(or probability mass function) f(x), cdf $F(x)$, survival$\bar{F}(x) = 1 - F(x)$.

2.2. Definition

We define the doubly truncated trimmed conditional r th moment of X, by $E(X^r|x \le X \le y)$, where

$$E(X^r \mid x \le X \le y) = \int_x^y u^r f(u)du \,/(F(y) - F(x))$$

3. Main Results

In this section we introduce some new results about some distributions as exponential, Pareto and power function.

3.1. Characterization of the doubly truncated exponential distribution.

In this section we shall characterize the trimmed exponential distribution in terms of their failure rate and reversed failure rate. This is contained in the following.

Theorem 3.1

Let we have a nonnegative continuous random variable with cdf, $F(x)$, pdf, $f(x)$; then X has Exponential distribution with:

$$f(x) = \lambda e^{-\lambda x} \quad , \quad x > 0 \quad , \quad \lambda > 0$$

If and only if

$$E(X^r \mid x \le X \le y) = \frac{r!}{\lambda^r} + \frac{r!\, g(x)h(y)\tau(x) - r!\,[g(y) - g(x)][\tau(x)\tau(y)] - r!\, g(y)h(x)\tau(y)}{[\tau(x)h(y) - h(x)\tau(y)]} \qquad (3.1)$$

where

$$g(t) = r! \sum_{i=0}^{r-1} \frac{t^{r-i}}{\lambda^i (r-i)!}$$

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We prove Theorem 2.1 through a series of lemmas (2.2, 2.3 and 2.4 of Nofal, 2010)

Proof of Theorem

1. Necessity

Observe that

$$E(X^r \mid x \le X \le y) = \frac{1}{[F(y)-F(x)]}\int_x^y u^r f(u)du$$

$$= \frac{1}{[F(y)-F(x)]}A_1,$$

where

$$A_1 = \int_x^y u^r f(u)du = \int_x^y u^r \lambda e^{-\lambda u} du\,.$$

Now integrating by parts, gives

$$A_1 = -y^r e^{-\lambda y} + x^r e^{-\lambda x} + r\int_x^y u^{r-1} e^{-\lambda u} du.$$

A second integrating by parts leads to

$$A_1 = -y^r e^{-\lambda y} + x^r e^{-\lambda x} - \frac{r}{\lambda} y^{r-1} e^{-\lambda y} + \frac{r}{\lambda} x^{r-1} e^{-\lambda x} + \frac{r(r-1)}{\lambda}\int_x^y u^{r-2} e^{-\lambda u} du$$

Using recursive integration by parts, one gets

$$A_1 = -e^{-\lambda y}\left[y^r + \frac{r}{\lambda}y^{r-1} + \frac{r(r-1)}{\lambda^2}y^{r-2} + \cdots + \frac{r!}{\lambda^r}\right]$$

$$+e^{-\lambda x}\left[x^r + \frac{r}{\lambda}x^{r-1} + \frac{r(r-1)}{\lambda^2}x^{r-2} + \cdots + \frac{r!}{\lambda^r}\right] \qquad (3.2)$$

One Equation (1.2), after doing the necessary manipulations can be rewritten as

$$A_1 = -r!\, e^{-\lambda y}\left[\sum_{i=0}^{r} \frac{y^{r-i}}{\lambda^i (r-i)!}\right] + r!\, e^{-\lambda x}\left[\sum_{i=0}^{r} \frac{x^{r-i}}{\lambda^i (r-i)!}\right]$$

$$= -r!\, \bar{F}(y)\left[\sum_{i=0}^{r} \frac{y^{r-i}}{\lambda^i (r-i)!}\right] + r!\, \bar{F}(x)\left[\sum_{i=0}^{r} \frac{x^{r-i}}{\lambda^i (r-i)!}\right]$$

$$= -r!\, \bar{F}(y)\left[\sum_{i=0}^{r-1} \frac{y^{r-i}}{\lambda^i (r-i)!} + \frac{1}{\lambda^r}\right] + r!\, \bar{F}(x)\left[\sum_{i=0}^{r-1} \frac{x^{r-i}}{\lambda^i (r-i)!} + \frac{1}{\lambda^r}\right]$$

$$= -\frac{r!}{\lambda^r}[\bar{F}(y) - \bar{F}(x)] - r!\, \bar{F}(y)\left[\sum_{i=0}^{r-1} \frac{y^{r-i}}{\lambda^i (r-i)!}\right] + r!\, \bar{F}(x)\left[\sum_{i=0}^{r-1} \frac{x^{r-i}}{\lambda^i (r-i)!}\right]$$

$$= -\frac{r!}{\lambda^r}[\bar{F}(y) - \bar{F}(x)] - r!\, \bar{F}(y)g(y) + r!\, \bar{F}(x)g(x)$$

Summing up, it follows that

$$E(X^r \mid x \le X \le y)\frac{1}{[F(y)-F(x)]}\left[-\frac{r!}{\lambda^r}[\bar{F}(y) - \bar{F}(x)] - r!\, \bar{F}(y)g(y) + r!\, \bar{F}(x)g(x)\right]$$

$$= \frac{r!}{\lambda^r} + \frac{-r!\bar{F}(y)g(y) + r!\bar{F}(x)g(x)}{[F(y)-F(x)]}$$

Using Lemma (2.3) in Nofal (2010), one gets

$$E(X^r \mid x \le X \le y) = \frac{r!}{\lambda^r} + \frac{r!\, g(x)h(y)\tau(x) - r!\,[g(y)-g(x)][\tau(x)\tau(y)] - r!\, g(y)h(x)\tau(y)}{[\tau(x)h(y) - h(x)\tau(y)]} \qquad (3.3)$$

2. Sufficiency

Equation 3.3 can be written as

$$\int_x^y u^r f(u)du = -\frac{r!}{\lambda^r}[\bar{F}(y) - \bar{F}(x)] - r!\,\bar{F}(y)g(y) + r!\,\bar{F}(x)g(x).$$

By differentiating both sides with respect to , one finds

$$\frac{d}{dy}\int_x^y u^r f(u)du = \frac{d}{dy}\left[-\bar{F}(y)\left[\frac{r!}{\lambda^r} + r!\,g(y)\right] + \bar{F}(x)\left[\frac{r!}{\lambda^r} + r!\,g(x)\right]\right]$$

This implies that

$$y^r f(y) = f(y)\left[\frac{r!}{\lambda^r} + r!\,g(y)\right] - r!\,\bar{F}(y)\acute{g}(y)$$

Where,

$$\acute{}g(y) = \sum_{i=0}^{r-1}\frac{y^{r-i-1}}{\lambda^i(r-i-1)!},$$

then

$$y^r f(y) = f(y)\left[\frac{r!}{\lambda^r} + \sum_{i=0}^{r-1}\frac{r!\,y^{r-i}}{\lambda^i(r-i)!}\right] - r!\,\bar{F}(y)\acute{g}(y) \qquad (3.4)$$

We can rewrite Equation (3.4) as

$$r!\,\bar{F}(y)\acute{g}(y) = f(y)\left[\frac{r!}{\lambda^r} + \sum_{i=0}^{r-1}\frac{r!\,y^{r-i}}{\lambda^i(r-i)!} - y^r\right] = \frac{r!}{\lambda}f(y)\acute{g}(y),$$

This implies that

$$\frac{1}{\lambda}f(y) = \bar{F}(y)$$

We can write it as

$$\frac{f(y)}{\bar{F}(y)} = \frac{1}{\lambda}$$

By integrating both sides with respect to y, one finds that

$$\bar{F}(y) = e^{-\lambda x}$$

This is the survival function of exponential distribution

Lemma3.2

In Theorem 3.1 if one puts $r = 1$, then one can get the result of (Nofal (2010)) as

$$E(X^r\,|\,x \le X \le y) = \frac{1}{\lambda} + \frac{xh(y)\tau(x) - [y-x][\tau(x)\tau(y)] - yh(x)\tau(y)}{[\tau(x)h(y) - h(x)\tau(y)]},$$

which g(t) = t .

Lemma 3.3

In Theorem 3.1 if one puts $x = 0$, then one can get the result of (Zakria (2013)) as

$$E(X^r\,|\,x \le X \le y) = \frac{r!}{\lambda^r} - r!\,\tau(y)\sum_{i=0}^{r-1}\frac{t^{r-i}}{\lambda^{i+1}(r-i)!}.$$

Lemma 3.4

In Theorem 3.1 if one puts$x \sim \infty$, then one can get the result of (Zakria (2013)) as

$$E(X^r\,|\,x \le X \le y) = \frac{r!}{\lambda^r} + \frac{r!}{\lambda}h(y)\sum_{i=0}^{r-1}\frac{t^{r-i}}{\lambda^{i+1}(r-i)!}.$$

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3.2 Characterization of the doubly truncated Pareto distribution.

In this section we shall characterize the trimmed Pareto distribution in terms of their failure rate and reversed failure rate .This is contained in the following.

Theorem 3.5

Let we have a nonnegative continuous random variable with cdf$F(x)$, pdf$f(x)$; then Xhas Pareto distribution with:

$$f(x)=\frac{\theta}{\alpha}\left(\frac{x}{\alpha}\right)^{-(\theta+1)},x>\alpha,\alpha,\theta>0.$$

If and only if,

$$E(X^r\,|\,x\le X\le y)=\frac{bx^r h(y)\tau(x)-by^r h(x)\tau(y)-[by^r-bx^r][\tau(x)\tau(y)]}{[\tau(x)h(y)-h(x)\tau(y)]}$$

We prove Theorem 3.5 through a series of Lemmas (2.2, 2.3 and 2.4 of Nofal , 2010).

Proof of Theorem

1. Necessity

Observe that

$$E(X^r\,|\,x\le X\le y)=\frac{1}{[F(y)-F(x)]}\int_x^y u^r f(u)du$$

$$=\frac{1}{[F(y)-F(x)]}A_2, \tag{3.5}$$

where

$$A_2=\int_x^y u^r f(u)du=\int_x^y u^r\left[\frac{\theta}{\alpha}\left(\frac{u}{\alpha}\right)^{-(\theta+1)}\right]du.$$

We rewrite A_2 as

$$A_2=\int_x^y \alpha^\theta\theta u^{-\theta+r-1}du. \tag{3.6}$$

Now integrating Equation (3.6) by parts, one gets

$$A_2=\frac{\alpha^\theta\theta}{-\theta+r}u^{-\theta+r}\Big|_{x}^{y},$$

$$=\frac{\alpha^\theta\theta}{-\theta+r}[y^{-\theta+r}-x^{-\theta+r}],$$

$$=\frac{x^r\theta}{\theta-r}\left(\frac{x}{\alpha}\right)^{-\theta}-\frac{y^r\theta}{\theta-r}\left(\frac{y}{\alpha}\right)^{-\theta}, \tag{3.7}$$

Equation (3.7) can be written as

$$A_2=bx^r\bar{F}(x)-by^r\bar{F}(y),$$

where

$$b=\frac{\theta}{\theta-r}.$$

One can rewrite Equation (3.7) as

$$E(X^r\,|\,x\le X\le y)=\frac{bx^r\bar{F}(x)-by^r\bar{F}(y)}{[F(y)-Fx]},$$

Using Lemmas (2.2, 2.3 and 2.4 of Nofal, 2010), one obtains

$$E(X^r\,|\,x\le X\le y)=\frac{bx^r h(y)\tau(x)-by^r h(x)\tau(y)-[by^r-bx^r][\tau(x)\tau(y)]}{[\tau(x)h(y)-h(x)\tau(y)]}. \tag{3.8}$$

2. Sufficiency

Equation (3.8) can be written as

$$\int_x^y u^r f(u)du = bx^r \bar{F}(x) - by^r \bar{F}(y).$$

By differentiating both sides with respect to y, one gets

$$y^r f(y) = by^r f(x) - bry^{r-1}\bar{F}(y). \tag{3.9}$$

One can rewrite Equation (3.9) as

$$\frac{-f(y)}{\bar{F}(y)} = \frac{-br}{y(1-b)},$$

integrating both sides with respect to y, one gets

$$ln\bar{F}(y) = lny^{-\theta},$$

then

$$\bar{F}(y) = \left(\frac{y}{\alpha}\right)^{-\theta}.$$

This completes the proof.

3.3 Characterization of the doubly truncated power function distribution.

In this section we shall characterize the trimmed power function distribution in terms of their failure rate and reversed failure rate .This is contained in The following.

Theorem 3.6

Let we have a nonnegative continuous random variable with cdf F(x), pdf f(x); then X has

Power function distribution with:

$$\mathrm{f(x)} = \alpha(1-\mathrm{x})^{\alpha-1}, 0 < x < 1, \alpha > 0$$

If and only if,

$$E(X^r \mid x \le X \le y) = \frac{r!\,\alpha!\,W(x)h(y)\tau(x) - r!\,\alpha!\,W(y)h(x)\tau(y) - r!\,\alpha!\,[W(y) - W(x)][\tau(x)\tau(y)]}{[\tau(x)h(y) - h(x)\tau(y)]} \tag{3.10}$$

Proof of Theorem

1. Necessity

One can notice that

$$E(X^r \mid x \le X \le y) = \frac{1}{[F(y) - F(x)]}\int_x^y u^r f(u)du$$

$$= \frac{1}{[F(y) - F(x)]}A_3,$$

where

$$A_3 = \int_x^y u^r f(u)du = \int_x^y u^r[\alpha(1-u)^{\alpha-1}]du.$$

Now integrating by parts, gives

$$A_3 = x^r(1-x)^\alpha - y^r(1-y)^\alpha + r\int_x^y u^{r-1}(1-u)^\alpha du.$$

A second integrating by parts, one gets

$$A_3 = x^r(1-x)^\alpha - y^r(1-y)^\alpha + \frac{rx^{r-1}(1-x)^{\alpha+1}}{\alpha+1} - \frac{ry^{r-1}(1-y)^{\alpha+1}}{\alpha+1}$$

$$+\frac{r(r-1)}{\alpha+1}\int_x^y u^{r-2}\,(1-u)^{\alpha}du.$$

Using recursive integration by parts, one gets

$$A_3 = x^r(1-x)^{\alpha} - y^r(1-y)^{\alpha} + \frac{rx^{r-1}(1-x)^{\alpha+1}}{\alpha+1} - \frac{ry^{r-1}(1-y)^{\alpha+1}}{\alpha+1} + \frac{r(r-1)x^{r-2}(1-x)^{\alpha+2}}{(\alpha+1)(\alpha+2)} - \frac{r(r-1)y^{r-2}(1-y)^{\alpha+2}}{(\alpha+1)(\alpha+2)} + \cdots + \frac{r(r-1)(r-2)\ldots(r-(r-1))}{(\alpha+1)(\alpha+2)\ldots(\alpha+(r-1))}\int_x^y (1-u)^{\alpha+r-1}du. \quad (3.11)$$

Equation (3.11) can be written as

$$A_3 = x^r(1-x)^{\alpha} + \frac{rx^{r-1}(1-x)^{\alpha+1}}{\alpha+1} + \frac{r(r-1)x^{r-2}(1-x)^{\alpha+2}}{(\alpha+1)(\alpha+2)} + \cdots + \frac{r!\,\alpha!\,(1-x)^{\alpha+r}}{(\alpha+r)!} - y^r(1-y)^{\alpha} + \frac{ry^{r-1}(1-y)^{\alpha+1}}{\alpha+1} + \frac{r(r-1)y^{r-2}(1-y)^{\alpha+2}}{(\alpha+1)(\alpha+2)} + \cdots + \frac{r!\,\alpha!\,(1-y)^{\alpha+r}}{(\alpha+r)!}, \quad (3.12)$$

one can rewrite Equation (3.12) as

$$A_3 = \sum_{i=0}^{r}\frac{r!\,\alpha!\,x^{r-i}(1-x)^{\alpha+i}}{(r-i)!\,(\alpha+i)!} - \sum_{i=0}^{r}\frac{r!\,\alpha!\,y^{r-i}(1-y)^{\alpha+i}}{(r-i)!\,(\alpha+i)!},$$

or

$$A_3 = r!\,\alpha!\,\bar{F}(x)\sum_{i=0}^{r}\frac{x^{r-i}(1-x)^{i}}{(r-i)!\,(\alpha+i)!} - r!\,\alpha!\,\bar{F}(y)\sum_{i=0}^{r}\frac{r!\,\alpha!\,y^{r-i}(1-y)^{i}}{(r-i)!\,(\alpha+i)!}, \quad (3.13)$$

one can write Equation (3.13) as

$$A_3 = r!\,\alpha!\,\bar{F}(x)W(x) - r!\,\alpha!\,\bar{F}(y)W(y), \quad (3.14)$$

where

$$W(t) = \sum_{i=0}^{r}\frac{r!\,\alpha!\,t^{r-i}(1-t)^{i}}{(r-i)!\,(\alpha+i)!}.$$

One can rewrite Equation (3.13) as

$$E(X^r\,|\,x \le X \le y) = \frac{r!\,\alpha!\,\bar{F}(x)W(x) - r!\,\alpha!\,\bar{F}(y)W(y)}{[F(y)-F(x)]},$$

using Lemmas (2.2, 2.3 and 2.4 of Nofal , 2010), one obtains

$$E(X^r\,|\,x \le X \le y) = \frac{r!\,\alpha!\,W(x)h(y)\tau(x) - r!\,\alpha!\,W(y)h(x)\tau(y) - r!\,\alpha!\,[W(y)-W(x)][\tau(x)\tau(y)]}{[\tau(x)h(y)-h(x)\tau(y)]}. \quad (3.15)$$

2. Sufficiency

Equation (3.15) can be written as

$$\int_x^y u^r f(u)du = r!\,\alpha!\,\bar{F}(x)W(x) - r!\,\alpha!\,\bar{F}(y)W(y).$$

By differentiating both sides with respect to y, one gets

$$y^r f(y)du = r!\,\alpha!\,f(y)W(y) - r!\,\alpha!\,\bar{F}(y)W'(y), \quad (3.16)$$

where

$$W'(y) = \sum_{i=0}^{r} \frac{(r-i)y^{r-i-1}(1-y)^i - iy^{r-i}(1-y)^i}{(r-i)!\,(\alpha+i)!},$$

$$= \frac{ry^{r-1}}{r!\,\alpha!} + \frac{(r-1)y^{r-2}(1-y) - y^{r-1}}{(r-1)!\,(\alpha+1)!}$$

$$+ \frac{(r-2)y^{r-2}(1-y)^2 - 2y^{r-2}(1-y)}{(r-1)!\,(\alpha+1)!} + \cdots, \tag{3.17}$$

we can rewrite (3.17) as,

$$W'(y) = \left[\frac{ry^{r-1}}{r!\,\alpha!} - \frac{y^{r-1}}{(r-1)!\,(\alpha+1)!}\right] + \left[\frac{(r-1)y^{r-2}(1-y)}{(r-1)!\,(\alpha+1)!} - \frac{2y^{r-2}(1-y)}{(r-2)!\,(\alpha+2)!}\right]$$

$$+ \left[\frac{(r-2)y^{r-3}(1-y)^2}{(r-2)!\,(\alpha+2)!} - \frac{3y^{r-3}(1-y)^2}{(r-3)!\,(\alpha+3)!}\right] + \cdots,$$

$$= \frac{(\alpha+1)ry^{r-1} - ry^{r-1}}{r!\,(\alpha+1)!} + \frac{(\alpha+2)(r-1)y^{r-2}(1-y) - 2(r-1)y^{r-2}(1-y)}{(r-1)!\,(\alpha+2)!}$$

$$+ \frac{(\alpha+3)(r-2)y^{r-3}(1-y)^2 - 3(r-2)y^{r-3}(1-y)^2}{(r-2)!\,(\alpha+3)!} + \cdots$$

This implies that

$$W'(y) = \frac{\alpha r y^{r-1}}{r!\,(\alpha+1)!} + \frac{\alpha(r-1)y^{r-2}(1-y)}{(r-1)!\,(\alpha+2)!} + \frac{\alpha(r-2)y^{r-3}(1-y)^2}{(r-2)!\,(\alpha+3)!} + \cdots,$$

$$= \alpha\left[\frac{y^{r-1}}{(r-1)!(\alpha+1)!} + \frac{y^{r-2}(1-y)}{(r-2)!(\alpha+2)!} + \frac{y^{r-3}(1-y)^2}{(r-3)!(\alpha+3)!} + \cdots\right]$$

$$= \alpha\left[\sum_{i=1}^{r} \frac{y^{r-i}(1-y)^{i-1}}{(r-i)!\,(\alpha+i)!}\right].$$

One can rewrite Equation (3.16) as

$$r!\,\alpha!\,\bar{F}(y)\alpha\left[\sum_{i=1}^{r} \frac{y^{r-i}(1-y)^{i-1}}{(r-i)!\,(\alpha+i)!}\right] = f(y)[r!\,\alpha!\,W(y) - y^r]. \tag{3.18}$$

But

$$[r!\,\alpha!\,W(y) - y^r] = r!\,\alpha!\,(1-y)\left[\sum_{i=1}^{r} \frac{y^{r-i}(1-y)^{i-1}}{(r-i)!\,(\alpha+i)!}\right].$$

Then Equation (3.18) can be written as

$$r!\,\alpha!\,\bar{F}(y)\alpha\left[\sum_{i=1}^{r} \frac{y^{r-i}(1-y)^{i-1}}{(r-i)!\,(\alpha+i)!}\right] = r!\,\alpha!\,(1-y)f(y)\left[\sum_{i=1}^{r} \frac{y^{r-i}(1-y)^{i-1}}{(r-i)!\,(\alpha+i)!}\right].$$

This implies that

$$\alpha\bar{F}(y) = (1-y)f(y). \tag{3.19}$$

One can rewrite Equation (3.19) as

$$\frac{-f(y)}{\bar{F}(y)} = \alpha\left[\frac{-1}{1-y}\right].$$

Integrating both sides with respect to y, one gets

$$\bar{F}(y) = (1-y)^{\alpha}.$$

This completes the proof.

References

[1] Ahmad, I.A., Kayid, M., &Pellerey, F.(2005). Further results involving the MIT order and the IMIT class, Probability in the Engineering and Informational Sciences 19(3), 377.395.

[2] Ahsanullah, M. (2009). On some characterizations of univariate distributions based on truncated moments of order statistics. Pak. J. statist. Vol. 25(2), 83-91.

[3] Ahsanullah, M. and Shakil, M. (2012). A note on the characterizations of Pareto distributions by upper record values.Commun. Korean Math. Soc. 27, No. 4, pp. 835.842

[4] Ahsanullah, M. and Hamedani, G.G. (2007). Certain characterizations of power function and Beta distributions based on order statistics. J. Statist. Theor. Appl., 6, 220-226.

[5] Ahsanullah, M. and Raqab, M (2004).Characterizations of distributions by conditional expectations of generalized order statistics. J. Appl. Statist. Sc., 13, No.l, 41-48.

[6] Anderson, P.K., Borgan, O., Gill, R.D. and Kieding, N. (1993). Statistical Methods Based on Counting Processes. Springer Verlag. New York.

[7] Galambos, J. and KOtz, S.(1978). Characterization of Probability Distributions; Springer-Verlage.

[8] Gupta, R.C. (1975).On characterization of distributions by conditional expectations. Comm. Statist. 4 (1), 99-103.

[9] Gupta, R.C. and Kirmani, S.N.U.A (2004). Some characterization of distributions by function of failure rate and mean residual life, Comm. Statist. Theory Methods, 33, 3115- 3131.

[10] Hamedani, G.G., Ahsanullah, M. and Sheng, R. (2008). Characterizations of certain continuous univariate distributions based on truncated moment of the .rst order statistics. To appear in Aligarh J. Statist.

[11] Kayid, M. and Ahmad, I. (2004). On the mean inactivity time ordering with reliability applications. Probability in the Engineering and Informational Science. 18,395- 409.

[12] Kotz, S. and Shanbhag, D.N. (1980). Some new approaches to probability distributions. Adv. in Appl. Probab., 12, 903-912.

[13] Nofal, Z. M. (2010). Characterization of exponential and geometric distributions based on the doubly truncated mean function. The 45 th Annual Conference on Stat. and Computer Science and o. r.

[14] Nofal, Z. M. (2011). Characterization of Beta and gamma distribution based on the doubly truncated mean function. Journal of Applied Statistical Science. Vol. 19, No. 2, pp. 159-168.

[15] Zakria, M. (2013). On characterizations of probability distributions.MSc.D. thesis, BenhaUniversity.