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Ephemerides Of Visual Binaries Of Highly Eccentric Orbits

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ABSTRACT

Ephemerides of binary stars are important information for astronomical observation and research. In this paper, ephemerides of the visual binaries ADS784, ADS13665, A1529, ADS836, ADS 3434, ADS1105 and ADS 3315 have been calculated using a computational algorithm to the successive approximations method. Ephemerides prediction to the visual binary systems of highly eccentric orbits is evaluated up to the year 2021. Comparisons with the observations are in good agreement.

Keywords

Ephemerides; visual binaries; orbits; successive approximations method; highly eccentric orbits.

Academic Discipline And Sub-Disciplines

Dynamical Astronomy - Celestial Mechanics - Ephemerides of celestial objects - orbit determination.

Mathematics Subject Classification

Mathematical Astrophysics.

Type (Method/Approach)

Successive approximations method.

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INTRODUCTION

A binary star is a star system consisting of two stars orbiting around their common center of mass. One of them was called primary (brighter one) while the other was called the companion or secondary.

Studies of binary stars is very important and useful in astrophysics for many reasons, like the studies of the orbits allow the masses of their component stars to be directly determined, then many parameters could be calculated. Also, to test evolutionary models and star formation theories.

One of the types of the binary system may be called the visual binaries, into which the angular separation between two components is great enough to observe them as a double star in a telescope. The stars in this type are gravitationally bound to each other but not interact like in other close binaries.

By carefully measuring position angle and separation over a period of many years, we can determine the apparent orbit of the secondary star relative to the primary of a binary star system. It is the projection of the true orbit onto a plane perpendicular to the observer's line of sight. The terms used to describe the true orbit are called the elements of the orbit. They are defined as follows, see [1].

a: semi-major axis

e: eccentricity

i: inclination

P: period (in years)

τ : epoch of periastron

Ω : position angle of the node

ω : argument of periastron

The relationship between the apparent and true orbits and the elements Ω, ω and i are illustrated in Fig. 1

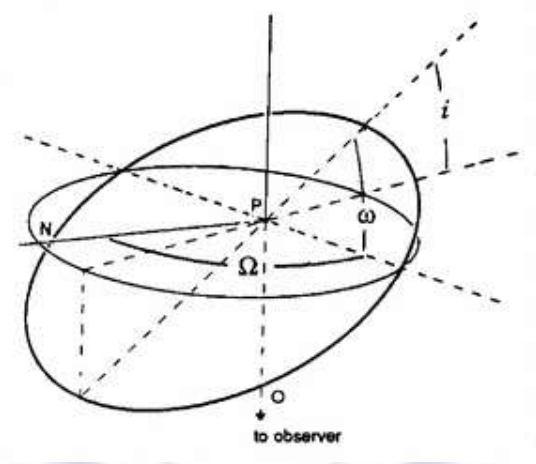

Figure 1: The projection of the true orbit of a binary star onto the plane of the apparent orbit

In this paper, a computational algorithm based on the successive approximations method is used to determine the ephemerides of some visual binaries [2].The ephemerides of binary systems with highly eccentric orbits are predicted up to the year 2021. The ephemerides (θ,ρ) could be determined from a set of elements of the orbit; where θ is the apparent position angle in degrees and ρ is the angular separation in seconds of arc. When a set of elements is known, (θ,ρ) at the observing times t are calculated by ephemerides formulae.

BASIC FORMULATIONS

Successive Approximations Method

The method of successive approximations depends on the expressing Kepler's equation in terms of parameter λ , i.e.

$$\lambda = \frac{1-e}{1+e}, \tag{1}$$

which also appears in the fundamental relations

$$\tan\frac{1}{2}E=\sqrt{\lambda}\tan\frac{1}{2}f \text{ , for elliptic orbit} \tag{2}$$

and

$$\tan\frac{1}{2}H=\sqrt{\lambda}\tan\frac{1}{2}f \text{ , for hyperbolic orbit} \tag{3}$$

where, E, H and f are, in respectively, elliptic eccentric anomaly, hyperbolic eccentricity and the true anomaly.The relation between the radial distance r and the orbital parameter p is given for both types as

$$r=\frac{p}{1+e\cos f}. \tag{4}$$

By writing

$$1+e\cos f=(1-e)\cos^2\frac{1}{2}f\left\{\frac{1}{\lambda}+\tan^2\frac{1}{2}f\right\}=(1+e)\left\{\frac{1+\lambda W^2}{1+W^2}\right\}, \tag{5}$$

Equation (4) becomes

$$r=q\frac{1+W^2}{1+\lambda W}, \tag{6}$$

where

$$W=\tan\frac{1}{2}f, \tag{7}$$

and

$$q=a(1-e), \text{ for elliptic orbits} \tag{8}$$

and

$$q=-a(1-e), \text{ for hyperbolic orbits} \tag{9}$$

Where, q is the pericenter distance. One seeks solution of Kepler's equation as a power series in λ

$$W=\sum_{j=1}^{\infty}a_j\lambda^j. \tag{10}$$

Now, according to the law of areas, we have

$$\frac{\sqrt{\mu p}}{2q^2}dt=\frac{1+W^2}{\left(1+\lambda W^2\right)^2}dW, \tag{11}$$

where, μ is the gravitational constant. Then, expanding the right-hand side by polynomial division to produce a power series in λ and integrating term by term, yields

$$\frac{\sqrt{\mu p}}{2q^2}(t-\tau)=\sum_{j=0}^{\infty}(-1)^j(j+1)\left\{\frac{W^{2j+1}}{2j+1}+\frac{W^{2j+3}}{2j+3}\right\}\lambda^j, \tag{12}$$

where, τ is the time of pericenter passage. Finally, we substitute for W from Equation (10) and equate coefficients of corresponding powers of λ. The zeroth – order term a_0 is the one and real root of

$$a_0^3+3a_0=\frac{3\sqrt{\mu p}}{2q^2}(t-\tau). \tag{13}$$

The solution of the cubic Equation (13) can be written as [3],

$$a_0=\frac{2y}{3}G, \tag{14}$$

where

$$G = \frac{F\left(\frac{2}{3},\frac{4}{3},\frac{3}{2};-y^2\right)}{F\left(\frac{2}{3},\frac{1}{3},\frac{1}{2};-y^2\right)}, \tag{15}$$

$$y = \frac{3\sqrt{\mu p}}{4q^2}(t-\tau) > 0, \tag{16}$$

and $F(\alpha,\beta,\gamma;z)$ is the hypergeometric function.

When a set of elements known, (θ,ρ) at the observing times *t* are calculated by ephemerides formulae, and the residuals $(O-C)$ can be found. They should be sufficiently small and mostly randomly distributed for an acceptable orbit. The ephemeris formulae for these cases are exactly the same formulae which relate position and time in the corresponding conic section of the two-body motion of the celestial mechanics [4] together with the common formulae

$$\tan(\theta-\Omega) = \tan(f+\omega)\cos i \text{ and } \rho = r\cos(f+\omega)\sec(\theta-\Omega) \tag{17}$$

where Ω, *ω*, *i*, *f* and *r* have their usual meaning for orbits. Equations (17) convertthe *f* and *r* of the companion in the true orbit into θ and ρ.

Computional Algorithm

- *Purpose:* To compute (θ,ρ) for a visual binary system.
- *Input data:* $q,\ i,\ \omega,\ \Omega,\ \mu,\ t,\ \tau$ and e.
- *Computational sequence*

  1. Compute λ from Equation (1).
  2. Compute $p = q(1+e)$.
  3. y from Equation (16).
  4. Solve the cubic Equation (13) for a_0 by the continued fraction.
  5. Compute $a_j;\ j = 1,\ 2,\ ....,\ 10.$
  6. $W = \sum_{j=1}^{10} a_j \lambda^j.$
  7. $f = 2\tan^{-1}(W).$
  8. For elliptic orbits $E = 2\tan^{-1}\left(W\sqrt{\lambda}\right).$
  9. For hyperbolic orbits $H = \ln\frac{1+W\sqrt{-\lambda}}{1-W\sqrt{-\lambda}}.$
  10. *r* from Equation (6).
  11. Calculate Q from

  $$Q = \theta - \Omega = \tan^{-1}\left\{\frac{\sin(f+\omega)\cos i}{\cos(f+\omega)}\right\}. \tag{18}$$

  12. Calculate θ from

  $$\theta = Q + \Omega. \tag{19}$$

  13. Calculate ρ from

  $$\rho = \frac{r\cos(f+\omega)}{\cos Q}. \tag{20}$$

  14. The algorithm is completed.

Numerical Results

A computational algorithm to the successive approximations method is used for calculating ephemerides of the visual binaries ADS784, ADS13665, A1529, ADS836, ADS 3434, ADS1105 and ADS 3315. The adopted constants are taken as tolerance $tol = 10^{-4}$ and $\mu = 4\pi^2 (m_1 + m_2)\pi''^3$, where π'' is the dynamical parallax and $m_{1,2}$ are the masses of respective system. The input data for the binary systems taken from their references [5], [6], [7], [8] and [9] are given in Table 1.

[7] used orbital elements were previously announced by [10]. On the other hand, [6] used Washington Double Star Catalog *WDS*[11] and [12] to calculate the orbital elements, into which these elements were determined using the Kovalski-Olevic [13] method; the dynamical parallaxes π'' were calculated for stars on the main sequence using [14] method, also he used total masses of the systems with trigonometric parallaxes published in the Hipparcos and Tycho Catalogues [15].

Tables 2, 3 and 4 present the ephemerides of the binary systems ADS 784, ADS 13665, A1529, ADS 836 and ADS3434 and comparison with the observations $(O-C)$. For future observations we have predicted ephemerides of the highly eccentric orbits of binary systems A1529, ADS 1105, and ADS 3315 that are shown in Tables 5 and 6. Ten epochs between the years 2009.0 to 2018.0 are considered to the system ADS 1105, while the epochs of the system ADS 3315 were between the years 2012 and 2021. The accuracy of orbit determination has been checked using the two-body conditions

$$r - \frac{q(1+e)}{1+e\cos f} = 0; \; r - \frac{q(1-e\cos E)}{1-e} = 0; \; E = 2Arc\tan(W\sqrt{\lambda}). \qquad (21)$$

that were of order 10^{-8}.

Table 1: Visual binaries orbital elements

Object	τ^y	q''	e	i°	ω°	Ω°	π''	$m_{1,2}$	*Ref.*
ADS 784	1952.9	0.1953	0.225	53.5	342	170.1	0.007	6*M*	[5]
ADS 13665	1969.8	0.1748	0.87	99	261	106.8	0.028	1.3 *M*	[5]
A 1529	1966.0	0.042771	0.731	98	208.1	165.4	0.00256	21.5 *M*	[7]
ADS 836	1950.56	0.37521	0.621	69.3	330.2	48.4	0.0028	19.2 *M*	[6]
ADS 3434	1903.6	0.289152	0.424	40.3	200.9	3.8	0.00814	2.4 *M*	[6]
ADS 1105	1984.6	0.059547	0.931	98.3	128.5	138.7	0.01686	2.8 *M*	[8]
ADS 3315	1966.0	0.072186	0.894	105.9	137.9	34.9	0.00789	2.7 *M*	[9]

Table 2: Ephemerides of ADS 784 and ADS 13665 and comparison to [5]

Epoch	ADS 784				ADS 13665			
t	$\theta^{(\circ)}$	$\rho^{('')}$	$(O-C)\theta^{(\circ)}$	$(O-C)\rho^{('')}$	$\theta^{(\circ)}$	$\rho^{('')}$	$(O-C)\theta^{(\circ)}$	$(O-C)\rho^{('')}$
1976.0	290.1	0.18	0.1	0.00	104.9	0 .43	0.1	0.00
1978.0	297.7	0.19	0.0	0.00	103.2	0.50	0.0	0.00
1980.0	303.8	0.21	0.0	0.00	102.1	0.55	0.1	0.00
1982.0	308.5	0.22	0.0	0.00	101.1	0.60	0.1	0.00
1984.0	312.3	0.23	0.0	0.00	100.3	0.64	0.2	0.00
1986.0	315.5	0.24	0.0	0.00	99.5	0.67	0.1	0.00
1988.0	318.2	0.24	0.0	0.00	98.8	0.69	0.2	0.00
1990.0	320.3	0.25	0.0	0.00	98.1	0.72	0.1	0.00
1992.0	322.2	0.25	0.0	0.00	97.4	0.74	0.0	0.00

Table 3: Ephemerides of A 1529 and comparison to [7]

Epoch	A 1529			
t	$\theta^{(\circ)}$	$\rho^{('')}$	$(O-C)\theta^{(\circ)}$	$(O-C)\rho^{('')}$
2006.0	163.6	0.244	- 0.8	0.007
2010.0	163.6	0.247	- 1.3	0.005
2015.0	163.1	0.250	- 1.4	- 0.001

Table 4: Ephemerides of ADS 836, ADS 3434 and ADS 6381and comparison to [6]

Epoch	ADS 836				ADS3434			
t	$\theta^{(\circ)}$	$\rho^{('')}$	$(O-C)\theta^{(\circ)}$	$(O-C)\rho^{('')}$	$\theta^{(\circ)}$	$\rho^{('')}$	$(O-C)\theta^{(\circ)}$	$(O-C)\rho^{('')}$
2007.0	61.3	0.40	- 0.3	0	356.9	0 .647	1.2	0.008
2008.0	61.7	0.407	- 0.3	0	357.2	0.649	1.4	0.008
2009.0	61.1	0.406	- 0.3	0	357.5	0.651	1.7	0.009
2010.0	62.5	0.406	- 0.3	0	357.7	0.652	1.9	0.01

Table 5: Ephemerides prediction of ADS 1105 and comparison to [8]

Epoch	ADS 1105			
t	$\theta^{(\circ)}$	$\rho^{('')}$	$(O-C)\theta^{(\circ)}$	$(O-C)\rho^{('')}$
2009.0	162.4	0.308	- 0.3	0.007
2010.0	161.8	0.322	- 0.3	0.008
2011.0	161.2	0.340	- 0.3	0.005
2012.0	160.7	0.350	- 0.3	0.009
2013.0	160.3	0.363	- 0.4	0.01
2014.0	159.8	0.376	- 0.3	0.011
2015.0	159.4	0.389	- 0.3	0.012
2016.0	159.1	0.401	- 0.4	0.014
2017.0	158.7	0.413	- 0.4	0.016
2018.0	158.4	0.425	- 0.4	0.017

Table 6: Ephemerides prediction of ADS 3315and comparison to [9]

Epoch	ADS 3315			
t	$\theta^{(\circ)}$	$\rho^{('')}$	$(O-C)\theta^{(\circ)}$	$(O-C)\rho^{('')}$
2012.0	73.4	0.262	0.1	- 0.001
2013.0	73.0	0.268	- 0.1	- 0.001
2014.0	72.2	0.274	0.1	- 0.001
2015.0	72.0	0.281	- 0.2	- 0.001
2016.0	71.2	0.287	- 0.1	- 0.001
2017.0	71.0	0.293	- 0.2	- 0.001
2018.0	70.2	0.299	0.1	0.00
2019.0	70.0	0.305	- 0.2	0.00
2020.0	69.4	0.311	0.00	0.00
2021.0	69.0	0.317	0.00	0.00

Conclusion

In concluding the present paper, ephemerides of the visual binaries ADS784, ADS13665, A1529, ADS836, ADS 3434, ADS1105 and ADS 3315 have been calculated using a computational algorithm to the successive approximations method. Ephemerides prediction to the visual binary systems of highly eccentric orbits is also evaluated up to the year 2021. Comparisons with observations and different authors are in good agreement that shows the efficiency of the used computational algorithm.

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REFERENCES

[1] Michael, K., Gainer 2007, Real astronomy with small telescopes, Springer, 103-113

[2] Saad, A. S. 2004, Conrib. Astron. Obs. SkalnatePleso, 34, 119-127

[3] Wall, H.S. 1948, Analytic theory of continued fraction, D. van Co., New York

[4] Denby, J.M.A. 1988, Fundamentals of Celestial Mechanics, 2nd Edition William – bell, Inc. Richmond

[5] Heintz, W.D.1978, AJ. Suppl. Ser., 37, 71

[6] Novakovic, B. 2007, Chin. J. Astron. Astrophys., 7, 415

[7] Vakhtang, S.T. and Jose, A. D. 2006, AJ, 131, 2681

[8] Docobo, J.A. and Ling, J.F., 2009, AJ, 138, 1159-1170

[9] Ling, J.F. 2012, AJ, 143, 1

[10] Docobo, J.A. and Tamazain, V. S. 2005, IAU Commission 26, Circ. 157

[11] Mason, B.D., Wycoff, G.L., Hartkopf, W.I., Douglass, G.G. and Worley, C.E. 2001, AJ, 122, 3466

[12] Mason, B.D., Wycoff, G.L., Hartkopf, W.I. 2006, The Washington Double Star Catalog, US Naval Observatory, Washington electronic version http://ad.usno.navy.mil/wds/wds.html

[13] Olevic, D. and Cvetkoic, Z. 2004, A&A, 415, 259

[14] Abgelov, T. 1993, Bull. Obs. Astron. Belgrade, 148, 1

[15] ESA, 1997, TheHipparcos and Tycho Catalogues (Noordwijk: ESA Pub. Div. ESTEC)