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Odd and Even Ratio Edge Antimagic Labeling

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ABSTRACT

In this paper we introduce a new labeling namely odd and even ratio edge antimagic labeling and study the existence of this labeling for basic graph structures.

Keywords

Graph; labeling; antimagic; odd antimagic; even antimagic.

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INTRODUCTION

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs are becoming an increasingly useful family of Mathematical Models from a broad range of applications. The graph labeling problem that appears in graph theory has a fast development recently. This problem was first introduced by Alex Rosa in 1967. Since Rosa's article, many different types of graph labeling problems have been defined around this. This is not only due to its mathematical importance but also because of the wide range of the applications arising from this area, for instance, x-rays, crystallography, coding theory, radar, astronomy, circuit design, and design of good Radar Type Codes, Missile Guidance Codes and Convolution Codes with optimal autocorrelation properties and communication design. An enormous body of literature has grown around the subject in about 1500 papers. They gave birth to families of graphs with attractive names such as graceful, Harmonious, felicitous, elegant, cordial, magic antimagic, bimagic and prime labeling etc. A useful survey to know about the numerous graph labeling methods is the one by J.A. Gallian recently [2012][2]. All graphs considered here are finite simple and undirected.

The vertex-weight of a vertex v in G under an edge labeling is the sum of edge labels corresponding to all edges incident with v. Under a total labeling, vertex-weight of v is defined as the sum of the label of v and the edge labels corresponding to the entire edges incident with v. If all vertices in G have the same weight k, we call the labeling vertex-magic edge labeling or Vertex-magic total labeling respectively and we call k a magic constant. If all vertices in G have different weights, then the labeling is respectively called vertex-antimagic edge labeling or vertex-antimagic total labeling. The edge-weight of an edge e under a vertex labeling is defined as the sum of the labels of the end vertices of e under a total labeling, we also add the label of e. Using edge-weight, and we derive edge-magic vertex or edge-magic total labeling and edge-antimagic vertex or edge-antimagic total labeling.

In [5] J.Jayapriya and K.Thirusangu introduced a new labeling called Max-min labeling. As this labeling is the ratio between the maximum and minimum of the labels of the end vertices of the edges, here after we call the max-min labeling as Ratio labeling.

In this paper we examine the existence of odd and even ratio edge antimagic labeling, for some class of graph.

ODD AND EVEN RATIO EDGE ANTIMAGIC LABELING

Definition 2.1: Let G (V, E) be a simple graph with p vertices and q edges. A bijective function f : V(G) $\rightarrow$ {1,3,5,….,2p-1} is said to be odd ratio edge antimagic labeling if for every edge uv in E, the edge weights λ(uv) = $\frac{\max\{f(u), f(v)\}}{\min\{f(u), f(v)\}}$ are distinct.

Definition 2.2: A bijective function f : V(G) $\rightarrow$ {2,4,6,…., 2p} is said to be even ratio edge antimagic labeling if for every edge uv in E, λ(uv) = $\frac{\max\{f(u), f(v)\}}{\min\{f(u), f(v)\}}$ are distinct.

Theorem 2.3: P_n admits both odd and even ratio edge antimagic labeling.

Proof: The graph P_n has n vertices and n-1 edges. Let the vertices be V = {v_1,v_2,…,v_n} and the edge set E = {v_i v_{i+1}: 1 ≤ i ≤ n-1}.

First we will prove P_n admits odd ratio edge antimagic labeling. For this, let us define a map f : V→ {1,3,5,…..2n-1} such that f (v_i)= 2i-1;1 ≤ i ≤ n. The edge weights are calculated as follows.

For 1≤ i ≤ n-1, λ (v_i v_{i+1}) = $\frac{\max\{f(v_i), f(v_{i+1})\}}{\min\{f(v_i), f(v_{i+1})\}}$

$$= \frac{(2i+1)}{2i-1}.$$

When i≠j, for 1≤ i, j≤n-1 , λ (v_i v_{i+1}) ≠ λ (v_j v_{j+1}).

If λ (v_i v_{i+1}) = λ (v_j v_{j+1}), then $\frac{(2i+1)}{2i-1} = \frac{(2j+1)}{2j-1}$.

This implies i = j,which is a contradiction and therefore all the edge labels are distinct. Hence P_n admits odd ratio edge antimagic labeling.

To prove the existence of even ratio edge antimagic labeling let us define a map f: V→{2,4,6,…,2n} such that f(v_i)=2i;1≤i≤n.

For1≤i≤n-1, λ (v_i v_{i+1}) = $\frac{\max\{f(v_i), f(v_{i+1})\}}{\min\{f(v_i), f(v_{i+1})\}}$

$$= \frac{(i+1)}{i}.$$

For $1 \le i, j \le n-1$, clearly $\lambda(v_i v_{i+1}) \neq \lambda(v_j v_{j+1})$ whenever $i \neq j$.

If $\lambda(v_i v_{i+1}) = \lambda(v_j v_{j+1})$ then we have $i=j$, which is a contradiction. Hence all edge labels are distinct, proving the existence of even ratio edge antimagic labeling.

Theorem 2.4: C_n admits both odd and even ratio edge antimagic labeling.

Proof: The graph C_n has n vertices and n edges. Let the vertices be $V= \{v_1, v_2,\dots,v_n\}$ and the edge set $E = \{v_i v_{i+1}: 1 \le i \le n-1\} \cup \{v_1v_n\}$.First we will prove C_n admits odd ratio edge antimagic labeling.

For this, let us define a map

$f: V \rightarrow \{1,3,5,\dots,2n-1\}$such that $f(v_i)= 2i-1$; $1 \le i \le n$.

The edge weights are calculated as follows.

For $1 \le i \le n-1$, $\lambda(v_iv_{i+1}) = \dfrac{\max\{f(v_i), f(v_{i+1})\}}{\min\{f(v_i), f(v_{i+1})\}}$

$$= \frac{2i+1}{2i-1}.$$

When $i \neq j$, for $1 \le i, j \le n-1$, clearly $\lambda(v_iv_{i+1}) \neq \lambda(v_jv_{j+1})$.

If $\lambda(v_iv_{i+1}) = \lambda(v_jv_{j+1})$ then we have,

$$\frac{(2i+1)}{2i-1} = \frac{(2j+1)}{2j-1}.$$

This implies $i=j$, which is a contradiction and therefore all edge labels are distinct.

Also $\lambda(v_1v_n) = \dfrac{\max\{f(v_1), f(v_n)\}}{\min\{f(v_1), f(v_n)\}} = 2n-1$.

If $\lambda(v_i v_{i+1}) = \lambda(v_1v_n)$ then, $2i+1 = (2n-1)(2i-1)$, which is a contradiction and therefore all edge labels are distinct. Hence C_n admits odd ratio edge antimagic labeling.

To prove the existence of even ratio edge antimagic let us define a map

$f: V \rightarrow \{2, 4, 6,\dots, 2n\}$ such that $f(v_i)= 2i$;$1 \le i \le n$.

For $1 \le i \le n-1$, $\lambda(v_iv_{i+1}) = \dfrac{\max\{f(v_i), f(v_{i+1})\}}{\min\{f(v_i), f(v_{i+1})\}} = \dfrac{(i+1)}{i}$.

When $i \neq j$, for $1 \le i, j \le n-1$, clearly $\lambda(v_iv_{i+1}) \neq \lambda(v_jv_{j+1})$.

If $\lambda(v_iv_{i+1}) = \lambda(v_jv_{j+1})$, then $\dfrac{(i+1)}{i} = \dfrac{(j+1)}{j}$.

This implies $i=j$, which is a contradiction and therefore all edge labels are distinct.

Also $\lambda(v_1v_n) = \dfrac{\max\{f(v_1), f(v_n)\}}{\min\{f(v_1), f(v_n)\}} = n$.

For $1 \le i \le n-1$, clearly, $\lambda(v_i v_{i+1}) \neq \lambda(v_1v_n)$.

If $\lambda(v_i v_{i+1}) = \lambda(v_1v_n)$, then $(i+1) = ni$ .This implies $i = j$, which is a contradiction and therefore all edge labels are distinct.

Hence C_n admits even ratio edge antimagic labeling.

Theorem 2.5: Star- **$K_{1,n}$ admits both odd and even ratio edge antimagic labeling.**

Proof: The graph $K_{1,n}$ has $n + 1$ vertices and n edges. Let the vertices be $V=\{v_1,v_2,\dots,v_{n+1}\}$, and the edge set be $E = \{v_1 v_i: 2 \le i \le n+1\}$. First we will prove $K_{1,n}$ admits odd ratio edge antimagic labeling. For this, let us define a map Let $f: V \rightarrow \{1,3,5,\dots..2n-1\}$ such that $f(v_1)= 1$, $f(v_i)= 2i-1$; $2 \le i \le n+1$. The edge weights are calculated as follows.

For $2 \le i \le n+1$, $\lambda(v_1v_i) = \dfrac{\max\{f(v_1), f(v_i)\}}{\min\{f(v_1), f(v_i)\}} = 2i-1$.

Thus, $\lambda(v_1v_i) \neq \lambda(v_1v_j)$.

When i≠j, if λ (v_1v_i) = λ (v_1v_j) then 2i-1 = 2j-1.

This implies i=j, which is a contradiction and therefore all edge labels are distinct.

Hence $K_{1,n}$ admits odd ratio edge antimagic labeling.

To prove the existence of even ratio edge antimagic let us define a map *f*: V→ {2, 4, 6,…, 2n} such that *f* (v_1)= 2 and *f* (v_i)= 2i ;2≤i≤n+1.

For 2≤ i ≤n+1, λ (v_1v_i) = $\frac{\max\{f(v_1), f(v_i)\}}{\min\{f(v_1), f(v_i)\}} = \frac{2i}{2}$ =i.

When i≠j, for 2≤ i, j ≤n+1, clearly, λ (v_1v_i) ≠ λ (v_1v_j) .

If λ (v_1v_i) = λ (v_1v_j) and i ≠ j then 2i= 2j.

This implies i=j, which is a contradiction and therefore all edge labels are distinct.

Hence $K_{1,n}$ admits even ratio edge antimagic labeling.

Theorem 2.6: The Fan graph admits both odd and even ratio edge antimagic labeling.

Proof: The fan f_n (n ≥ 2) is obtained by joining all vertices of P_n(Path of n vertices) to a further vertex called the center and contains n+ 1 vertex and 2n-1 edges. Let the vertices be V={v_1,v_2,…,v_{n+1}} and the edge set E ={ v_1v_i :2 ≤ i ≤n+1}∪{ v_iv_{i+1}: 2 ≤ i ≤n}. First we will prove fan graph admits odd ratio edge antimagic labeling. For this, let us define a map *f*:V→{1,3,5,…,2n-1}such that *f* (v_1) = 1, *f* (v_i)= 2i-1;2≤i≤n+1. The edge weights are calculated as follows.

For 2 ≤ i ≤n+1, λ (v_1 v_i) = $\frac{\max\{f(v_1), f(v_i)\}}{\min\{f(v_1), f(v_i)\}} = \frac{(2i-1)}{1}$ = 2i-1.

When i≠j, for 2 ≤ i, j≤n+1, clearly λ (v_1 v_i) ≠ λ (v_1 v_j).

If λ (v_1 v_i) = λ (v_1 v_j), then i=j, which is a contradiction and therefore all edge labels are distinct.

For 2≤i≤n , λ (v_iv_{i+1}) = $\frac{\max\{f(v_i), f(v_{i+1})\}}{\min\{f(v_i), f(v_{i+1})\}} = \frac{2i+1}{2i-1}$.

When i≠j, for 2≤ i,j≤n, clearly λ (v_iv_{i+1}) ≠ λ (v_jv_{j+1}).

If λ (v_iv_{i+1}) = λ (v_jv_{j+1}) then i=j, which is a contradiction and therefore all edge labels are distinct.

This implies i=j, which is a contradiction and therefore all edge labels are distinct.

Hence fan graph admits odd ratio edge antimagic labeling.

To prove the existence of even ratio edge antimagic let us define a map f: V→ {2, 4, 6,…, 2n} such that f (v_1) = 2 and f (v_i) = 2i; 2≤i≤n+1.

For 2 ≤ i ≤n+1, λ (v_1 v_i) = $\frac{\max\{f(v_1), f(v_i)\}}{\min\{f(v_1), f(v_i)\}} = \frac{2i}{2}$ = i.

When i≠j, for 2 ≤ i,j≤n+1, Clearly λ (v_1 v_i) ≠ λ (v_1 v_j) .

If λ (v_1 v_i) = λ (v_1 v_j), then i=j, which is a contradiction and therefore all edge labels are distinct.

For 2≤i≤n , λ (v_iv_{i+1}) = $\frac{\max\{f(v_i), f(v_{i+1})\}}{\min\{f(v_i), f(v_{i+1})\}} = \frac{(i+1)}{i}$.

When i≠j, for 1≤ i, j≤n-1, clearly λ (v_iv_{i+1}) ≠ λ (v_jv_{j+1}).

If λ (v_iv_{i+1}) = λ (v_jv_{j+1}), then i=j, which is a contradiction and therefore all edge labels are distinct. Hence fan graph admits even ratio edge antimagic labeling.

Theorem 2.7: The Friendship graph admits both odd and even ratio edge antimagic labeling.

Proof: The friendship graph has 2n+1 vertices and 3n edges. Let the vertices be V= {v_1}∪{ u_i: 1≤i≤n}∪{w_i:1≤i≤n}and the edge set E = { v_1u_i; 1≤i≤n}∪{ w_i u_i; 1≤i≤n}∪{ v_1w_i; 1≤i≤n}. First we will prove fan graph admits odd ratio edge antimagic labeling. For this, let us define a map *f*:V→{1,3,5,…,2n-1} such that *f* (v_1) = 1, *f* (u_i) = {4i-1:1≤i≤n} and *f* (w_i) = {4i+1:1≤i≤n }.

The edge weights are calculated as follows.

For 1≤i≤n, λ (v_1 u_i) = $\frac{\max\{f(v_1), f(u_i)\}}{\min\{f(v_1), f(u_i)\}}$ = 4i-1.

For 1≤i,j≤n, clearly λ (v_1 u_i) ≠ λ (v_1 u_j) and i≠j.

If λ (v_1 u_i) = λ (v_1 u_j), then i =j, which is a contradiction and therefore all edge labels are distinct.

For 1≤i≤n, λ (v_1 w_i) = $\frac{\max\{f(v_1), f(w_i)\}}{\min\{f(v_1), f(w_i)\}}$ =4i+1.

For 1≤i≤n, λ (w_i u_i) = $\frac{\max\{f(w_i), f(u_i)\}}{\min\{f(w_i), f(u_i)\}} = \frac{4i+1}{4i-1}$.

For 1≤i, j≤n, clearly λ (w_i u_i) ≠ λ (w_j u_j) and i≠j.

If λ (w_i u_i) ≠ λ (w_j u_j), then i =j, which is a contradiction and therefore all edge labels are distinct.

Hence the friendship graph admits odd ratio edge antimagic labeling.

To prove the existence of even ratio edge antimagic let us define a map *f*: V→ {2, 4, 6,…, 2n} such that *f* (v_1) = 2 , *f* (u_i) = {4i;1≤i≤n} and *f* (w_i) = {4i+2:1≤i≤n }.

The edge weights are calculated as follows.

For 1≤i≤n, λ (v_1 u_i) = $\frac{\max\{f(v_1), f(u_i)\}}{\min\{f(v_1), f(u_i)\}}$ = 2i.

For 1≤i,j≤n, clearly λ (v_1 u_i) ≠ λ (v_1 u_j) and i≠j.

If λ (v_1 u_i) = λ (v_1 u_j), then i =j, which is a contradiction and therefore all edge labels are distinct.

For 1≤i≤n, λ (v_1 w_i) = $\frac{\max\{f(v_1), f(w_i)\}}{\min\{f(v_1), f(w_i)\}}$ =2i+1.

For 1≤i≤n, λ (w_i u_i) = $\frac{\max\{f(w_i), f(u_i)\}}{\min\{f(w_i), f(u_i)\}} = \frac{4i+2}{4i}$.

For 1≤i, j≤ n, clearly λ (w_i u_i) ≠ λ (w_j u_j) and i≠j

If λ (w_i u_i) ≠ λ (w_j u_j), then i =j, which is a contradiction and therefore all edge labels are distinct.

For 1≤i≤n, λ (v_1 u_i) ≠ λ (w_i u_i), if λ (v_1 u_i) =λ (w_i u_i) then $4i^2$=2i+1, which is a contradiction and therefore all edge labels are distinct. For 1≤i≤n, λ (v_1 w_i) ≠ λ (w_i u_i), if λ (v_1 w_i) =λ (w_i u_i) then $4i^2$=1, which is a contradiction and therefore all edge labels are distinct.Hence the friendship graph admits even ratio edge antimagic labeling.

CONCLUSION

In our work we determined odd and even ratio edge antimagic for certain classes of graph .Complete graph K_n does not admit even ratio edge antimagic ,it admits odd ratio edge antimagic only if n=4. Also 1< λ (uv) ≤ p always.

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