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Incompressibility and Existence of Navier- Stokes Equations

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ABSTRACT

In this paper, we represents incompressible Navier-Stokes equations, i.e. fluid is incompressible in the domain $\mathbb{R}^2$. Navier-Stokes equations have wide applications in fluid mechanics, air and sea navigation. Mathematicians have not proven yet these equations. In this paper we describe incompressibility and existence of these equations in the domain $\mathbb{R}^n$ (n=2), Navier-Stokes problem is now included in the Millenium problems of Clay mathematics Institute.

Keywords; Newton's second law of motion; incompressibility; viscosity; molecular dynamics.

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INTRODUCTION

In this paper, we study the motion of fluid membrane and Incompressibilty of fluid, and existence of Navier-Stokes's three impotant equations. Navier-Stokes equations are the most important and famous equations for turbulent flow of fluid or wind from a century. The Proof of Navier-Stokes equations is most challenging for all Mathematician from all around the world.

Navier-Stokes equations were originally derived in the 1840s on the basis of conservation laws and first- order approximations. But if one assumes sufficient randomness in microscopic molecular in processes in fluid they can also be derived from molecular dynamics, as done in the early 1900s.

The major difficulty in obtaining a time-accurate solution for an incompressible flow arises from the fact that the continuity equation does not contain the time-derivative explicitly. The constraint of mass conservation is achieved by an implicit coupling between the continuity equation and the pressure in the momentum equations. One can use an explicit time advancement scheme which obtains the pressure at the current time-step such that the continuity equation at the next step is satisfied. However, for fully implicit or semi implicit schemes, the aforementioned difficulty prevents the use of the conventional Alternating Direction Implicit (ADI) scheme to advance in time as is the case for compressible flows.

For very low Reynolds numbers and simple geometries, it is often possible to find explicit formulas for solutions to the Navier-Stokes equations. But even in the regime of flow where regular arrays of eddies are produced, analytical methods have never yielded complete explicit solutions. In this regime, however, numerical approximations are fairly easy to find.

Since about the 1960s computers have been powerful enough to allow computations at least nominally to be extended to considerably higher Reynolds numbers. And it has become increasingly common to see numerical results given far into the turbulent regime - leading sometimes to the assumption that turbulence has somehow been derived from the Navier-Stokes equations. But just what such numerical results actually have to do with detailed solutions to the Navier-Stokes equations is not clear. Our understanding of them remians minimal.The challenge is to make substantial progress towards a mathematical theory which will unlock hidden secrets in Navier-Stokes equations.

Mathematical Approach of Navier-Stokes Equations

The Euler and Navier-Stokes equations describe the motion of a fluid in $\mathbb{R}^n$ (n=2,3). These equations can be solved for an unknown velocity vector $u(x,t)=u_i(x,t)\in\mathbb{R}$, $1\le i\le n$ and pressure $P(x,t)\in\mathbb{R}$ defined for position $x\in\mathbb{R}$ and time $t\ge 0$

$$\frac{\partial u_i}{\partial t}+\sum_{k=1}u_k\frac{\partial u_i}{\partial x_k}=\nu\Delta u_i-\frac{\partial P}{\partial x_i}+f_i(x,t) \qquad \text{where } \mathbb{R}^n \qquad ....(1)$$

$$\text{div u} =\sum_{i=1}^{n}\frac{\partial u_i}{\partial x_i}=0 \qquad x\in\mathbb{R}^n, t\ge 0 \qquad .....(2)$$

with intial conditions $u(x,0)=u^0(x),\ \ x\in R^n$ .....(3)

Equation (1) is just Newton's second law of motion f = ma for a fluid element subject to the external force $f=f_i(x,t)$, $1\le x\le n$ and to the forces arising from pressure and friction. Equation (2) just says the incompressibility of fluid . Where $f_i(x,t)$ are the components of a given externally applied force (for e.g. gravity) , ν is +ve coefficient of viscosity and $\Delta=\sum_{i=1}^{n}\frac{\partial^2}{\partial x_i^2}$ is a Laplacian operator in the space variables. $u^0(x)$ is a given, C^∞ divergence-free vector field on $\mathbb{R}^n$. Equation (1), (2), (3) are **Euler equations** with ν equal to zero.

Incompressibility and Existence

Cosider a fluid membrane. Let P'Q'R'S' shape of an element an element of the membrane at any time t. Let PQRS be projections of P'Q'R'S' on xy-plane. We wish to obtain position change $u(x,y,t)$ at any point (x,y) at any time t>0. First we cosider the following assumptions:

(a) The mass of the fluid membrane per unitvarea is constant ρ (say)

(b) The whole motion takes place in a direction perpendicular to xy- plane.

(c) The surface tension T per unit length developed by fluid membrane molecules is same at all points. We consider T does not change during the motion.

(d) The position change of $u(x, y, t)$ is small as compared to the size of the membrane. All angles of inclination are small.

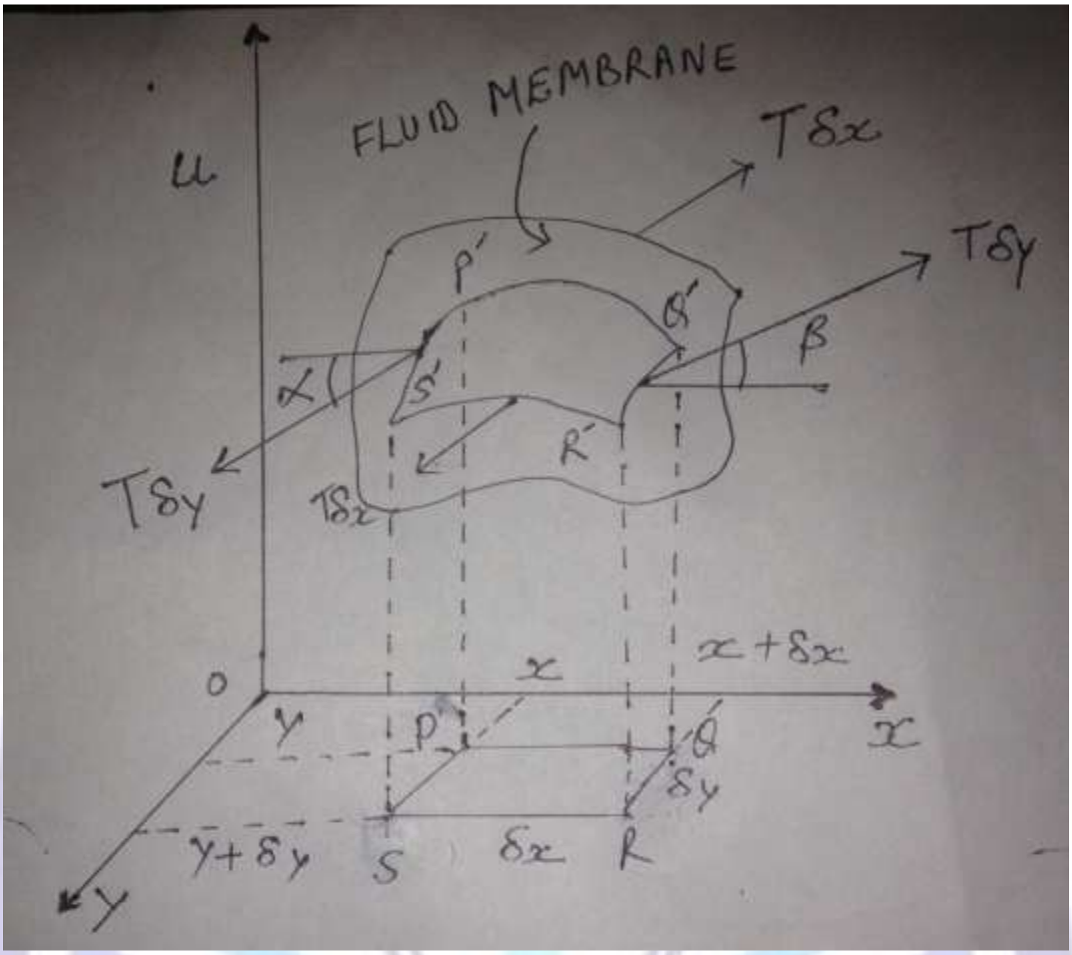

Fig 1.1

If we consider motion of the element P'Q'R'S' , The forces $T\delta y$ on its opposite edges Q'R' of length δy act at angles α and β to the horizontal.

These have vertical components $= T(\delta y)\sin\beta - T(\delta y)\sin\alpha$

$$= T\delta y(\sin\beta - \sin\alpha)$$

Since α and β are so small so that $\sin\alpha = \alpha = \tan\alpha$, $\sin\beta = \beta = \tan\beta$

$$= T\delta y(\tan\beta - \tan\alpha)$$

$$= T\delta y\left[\left(\frac{\partial u}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial u}{\partial x}\right)_{x}\right]$$

Where $\tan\beta = \left(\frac{\partial u}{\partial x}\right)_{x+\delta x}$ is the slope and $\tan\alpha = \left(\frac{\partial u}{\partial x}\right)_{x}$ is also slope.

So similarly the forces $T\delta x$ act on the edges P'Q' and S'R' of length δx have a vertical component

$$= T\delta x\left[\left(\frac{\partial u}{\partial y}\right)_{y+\delta y} - \left(\frac{\partial u}{\partial y}\right)_{y}\right]$$

Area of the portion P'Q'R'S' is $\delta x \delta y$ and mass is $\rho \delta x \delta y$. Again the acceleration of the fluid membrane in vertical direction is $\left(\frac{\partial^2 u}{\partial^2 t}\right)$. Using Newton's laws of motion **P = mf** the equation of the portion P'Q'R'S' is

$$T\delta y\left[\left(\frac{\partial u}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial u}{\partial x}\right)_{x}\right] + T\delta x\left[\left(\frac{\partial u}{\partial y}\right)_{y+\delta y} - \left(\frac{\partial u}{\partial y}\right)_{y}\right] = (\rho \delta x \delta y)\left(\frac{\partial^2 u}{\partial t^2}\right)$$

Dividing both sides by $\delta x \delta y$, we have

$$\frac{T\delta y\left[\left(\frac{\partial u}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial u}{\partial x}\right)_{x}\right]}{\delta x} + \frac{T\delta x\left[\left(\frac{\partial u}{\partial y}\right)_{y+\delta y} - \left(\frac{\partial u}{\partial y}\right)_{y}\right]}{\delta y} = \frac{\rho}{T}\left(\frac{\partial^2 u}{\partial^2 t}\right) \qquad \text{....(4)}$$

As $\delta y \to 0$ and $\delta y \to 0$, then equation (4) gives the result

$$\frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y} = \frac{\rho}{T}\frac{\partial^2 u}{\partial^2 t} \qquad \text{....(5)}$$

From equation (3) the intial condition $u(x,0) = u^0(x)$,

$$\therefore \qquad \frac{\partial u}{\partial t} = 0 \Rightarrow \frac{\partial^2 u}{\partial t^2} = 0$$

Then Eq.(5) becomes $\frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y} = 0$

$$\therefore \qquad \Delta^2 u = 0$$

This result shows that Eq.(2) is incompressible i.e. the incompressibility of fluid in two dimensional ($\mathbb{R}^2$)

There fore **Navier-Stokes equations** exits. Equations (1), (2), (3) are **Euler equations** with ν equal to zero.

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