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Linear and Weakly Non-Linear Analyses of Gravity Modulation and Electric Field on the Onset of Rayleigh-Bénard Convection in a Micropolar Fluid

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ABSTRACT

The effect of time periodic body force (or g-jitter or gravity modulation) on the onset of Rayleigh-Bénard electro-convection in a micropolar fluid layer is investigated by making linear and non-linear stability analysis. The stability of the horizontal fluid layer heated from below is examined by assuming time periodic body acceleration. This normally occurs in satellites and in vehicles connected with micro gravity simulation studies. A linear and non-linear analysis is performed to show that gravity modulation can significantly affect the stability limits of the system. The linear theory is based on normal mode analysis and perturbation method. Small amplitude of modulation is used to compute the critical Rayleigh number and wave number. The shift in the critical Rayleigh number is calculated as a function of frequency of modulation. The non-linear analysis is based on the truncated Fourier series representation. The resulting non-autonomous Lorenz model is solved numerically to quantify the heat transport. It is observed that the gravity modulation leads to delayed convection and reduced heat transport.

Keywords: Gravity modulation; Electric field; Micropolar fluid; Rayleigh-Bénard convection; Lorenz model.

Subject classification: Fluid Mechanics.

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1. Introduction

A significant class of natural convection problem is anxious with the effort in evading the convection in the earth's gravitational field even when the basic temperature gradient is identical and interfacial instabilities can be overlooked.Owing to numerous inevitable sources of residual acceleration experienced by a spacecraft, the gravity field in an orbiting laboratory is not constant in a microgravity environment, but itis randomly fluctuating. This fluctuating gravity is referred to as g-jitter.

The effect of gravity modulation on a convection stable configuration can significantly influence the stability of a system by increasing or decreasing its susceptibility to convection. In general, a distribution of stratifying agency that is convectively stable under constant gravity conditions can be destabilized when a time-dependent component of the gravity field is introduced certain combinations of thermal gradients, physical properties and modulation parameters may lead to parametric resonance and hence, to the stability of the system. Gresho and Sani [1],Wheeler et al. [2], Siddheshwar andPranesh [3,4], Malashetty and Basavaraja [5], Siddheshwar and Abraham [6], Swamy et al. [7] and more recently by Bhadauria and Kiran [8] have studied the effects of gravity modulation on the onset of convection in Newtonian and non-Newtonian fluids.

Micropolar fluids are the fluids with microstructure. Physically, they represent fluids consisting of randomly oriented particles suspended in a medium, where the deformation of the fluid particles is ignored. This constitutes a substantial generalization of the Navier–Stokes model and opens a new field of potential applications including a large number of complex fluids. The theory of micropolar fluid introduced byEringen[9] have become an important field of research especially in many industrially important fluids like paints, polymeric suspensions, colloidal fluids, and also in physiological fluids such as normal human blood and synovial fluids.A detailed survey of the theory of micropolarfluid and its applications are considered in the books of Erigen[10,11], Lukasazewicz[12] and power[13]. The theory of thermomicropolar convection was studied by many authors Datta and Sastry [14], Ahmadi [15], Rama Rao [16], Bhattacharya and Jena [17], Siddheshwar andPranesh [18,19], Pranesh and Kiran [20], Pranesh and Riya [21], Joseph et al. [22] and Pranesh [23].

In most part of the last century the engineering applications of fluid mechanics were restricted to systems in which electric and magnetic fields played no role. In recent years, the study of the interaction of electromagnetic fields with fluids started gaining attention with the promise of applications in areas like nuclear fusion, chemical engineering, medicine and high speed noiseless printing. The investigation of convective heat transfer together with the electrical and magnetic forces in non-Newtonian fluids is of practical importance. A systematic study through a proper theory is essential to understand the physics of the complex flow behaviour of these fluids and also to obtain invaluable scaled up information for industrial applications.

In dielectric fluids with low values of conductivity, the electric effects will essentially govern the motion. The forces that are exerted by an electric field on free charges present in a liquid are transmitted by collision to the neutral molecules. The fluid will be set in motion, thus changing the distribution of charges that in turn modifies the electric field. There is an analogy between Rayleigh-Bénard instability and pure electroconvection. In the latter case, the destabilizing force is proportional to the mean charge gradient. If alternating electric fields of sufficiently high frequency are employed, then Kelvin or polarization body force becomes the driving force for convection.

Onset of natural convection inthe presence of an external electric field has been studied by Takashima and Gosh [24], Siddheshwar and Abraham [25] and Rudraiah et al. [26]

Main object of this paper is to study the effects of gravity modulation and electric field on the stability of convective flow in a micropolar fluid by considering free-free, isothermal and no spin boundaries.

2. MATHEMATICAL FORMULATION

Consider a layer of electrically conducting micropolar fluid confined between two infinite horizontal walls distant 'd' apart (fig. 1). The uniform magnetic field is directed along the z-axis. A Cartesian co-ordinate system is taken with origin in the lower boundary and z-axis vertically upwards.

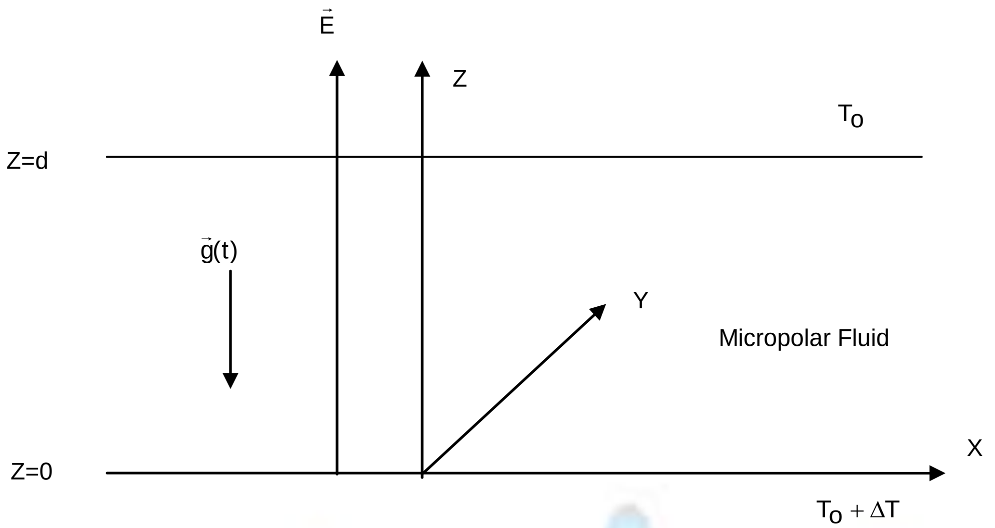

Figure 1: Schematic diagram for the problem.

The governing equations are:

Continuity equation:

$$\nabla.\vec q = 0 \, , \tag{1}$$

Conservation of Linear Momentum:

$$\rho_0\left[\frac{\partial \vec q}{\partial t} + (\vec q.\nabla)\vec q\right] = -\nabla p + \rho \vec g(t)\hat k + (2\zeta + \eta)\nabla^2 \vec q + \zeta \nabla \times \vec \omega + (\vec P.\nabla)\vec E \, , \tag{2}$$

$$\vec g(t) = -g_0[1 + \delta \cos(\gamma t)]\hat k \, , \tag{3}$$

Conservation of Angular Momentum:

$$\rho_0 I\left[\frac{\partial \vec \omega}{\partial t} + (\vec q.\nabla)\vec \omega\right] = (\lambda' + \eta')\nabla(\nabla.\vec \omega) + \eta' \nabla^2 \vec \omega + \zeta(\nabla \times \vec q - 2\vec \omega) \, , \tag{4}$$

Conservation of energy:

$$\frac{\partial T}{\partial t} + (\vec q.\nabla)T = \frac{\beta}{\rho C_v}(\nabla \times \vec \omega).\nabla T + \chi \nabla^2 T \, , \tag{5}$$

Equation of state:

$$\rho = \rho_0[1 - \alpha(T - T_0)] \, , \tag{6}$$

Equation of state for dielectric constant:

$$\epsilon_r = (1 + \chi_e) - e(T - T_0) \, , \tag{7}$$

Faraday's law:

$$\left.\begin{aligned} \nabla \times \vec E &= 0, \\ \vec E &= -\nabla \phi, \end{aligned}\right\} , \tag{8}$$

Equation of polarisation field:

$$\left.\begin{aligned} \nabla.(\varepsilon_0 \vec E + \vec P) &= 0, \\ \vec P &= \varepsilon_0(\varepsilon_r - 1)\vec E, \end{aligned}\right\} , \tag{9}$$

where, $\vec q$ is the velocity , ρ_0 is density of the fluid at temperature T = T_0, p is the pressure, ρ is the density, $\vec g$ is acceleration due to gravity, g_0 is the mean gravity, δ is the small amplitude of gravity modulation, γ is the frequency. ζ is coupling viscosity coefficient or vortex viscosity, $\vec{P}$ is dielectric polarization, $\vec{E}$ is the electric field, λ and η are the bulk and shear spin-viscosity coefficients, $\vec{\omega}$ is the angular velocity, I is moment of inertia, λ' and η' are bulk and shear spin-viscosity coefficients, T is the temperature, χ is the thermal conductivity, β is micropolar heat conduction coefficient, α is coefficient of thermal expansion, σ is electrical conductivity, ε_r is the dielectric constant, $e = -\left(\frac{\partial \varepsilon_r}{\partial T}\right)_{T=T_o}$, χ_e is electric susceptibility, ε_o is the electric permittivity of free space and ϕ is the electric scalar potential.

3. BASIC STATE

The basic state of the fluid is quiescent and is described by:

$$\vec{q}_b = (0,0,0), \vec{\omega} = \vec{\omega}_b(0,0,0), p = p_b(z), \rho = \rho_b(z), \vec{E} = \vec{E}_b(z), \vec{P} = \vec{P}_b(z), T = T_b(z), \varepsilon_r = \varepsilon_{rb}(z).\}. \quad (10)$$

Substituting equation (10) into basic governing equations (1)-(9), we obtain the quiescent state solutions as:

$$\frac{dp_b}{dz} = -\rho_b g_0[1+\delta\cos(\gamma t)] + P_b\frac{dE_b}{dz}, \quad (11)$$

$$\frac{\partial T_b}{\partial z} = \chi\frac{\partial^2 T_b}{\partial z^2}, \quad (12)$$

$$\left.\begin{aligned} &\rho_b = \rho_o[1-\alpha(T_b - T_o)],\\ &\varepsilon_r = (1+\chi_e) - e(T_b - T_0),\\ &\vec{E}_b = \left[\frac{(1+\chi_e)E_0}{(1+\chi_e)+\frac{e\Delta T}{h}z}\right]\hat{k},\\ &\vec{P}_b = \varepsilon_0 E_0(1+\chi_e)\left[1-\frac{1}{(1+\chi_e)+\frac{e\Delta T_z}{h}}\right]\hat{k}. \end{aligned}\right\}. \quad (13)$$

4. STABILITY ANALYSIS

Let the basis state be disturbed by an infinitesimal thermal perturbation. We now have:

$$\vec{q} = \vec{q}_b + \vec{q}', \vec{\omega} = \vec{\omega}_b + \vec{\omega}', p = p_b + p', \rho = \rho_b + \rho', T = T_b + T', \vec{E} = \vec{E}_b + \vec{E}', \vec{P} = \vec{P}_b + \vec{P}', \varepsilon_r = \varepsilon_{rb} + \varepsilon_r'.\}. \quad (14)$$

The prime indicates that the quantities are infinitesimal perturbations.

From equation (9), we get,

$$\left.\begin{aligned} &P_1' = \varepsilon_0\chi_e E_i' \quad \text{for } i = 1,2\\ &P_3' = \varepsilon_0\chi_e E_3' - e\varepsilon_0 T' E_0 \end{aligned}\right\}, \quad (15)$$

Substituting equation (14) into equations (1) to (9) and using the basic state solution, we get,

$$\nabla.\vec{q}' = 0, \quad (16)$$

$$\rho_o\left[\frac{\partial \vec{q}'}{\partial t} + (\vec{q}'.\nabla)\vec{q}'\right] = -\nabla p' - \rho'[1+\delta\cos(\gamma t)]g_0\hat{k} + (2\zeta+\eta)\nabla^2\vec{q}' + (\zeta\nabla\times\vec{\omega}') + (\vec{P}_b.\nabla)\vec{E}' + (\vec{P}'.\nabla)\vec{E}_b + (\vec{P}'.\nabla)\vec{E}', \quad (17)$$

$$\rho_o I\left[\frac{\partial \vec{\omega}'}{\partial t} + (\vec{q}'.\nabla)\vec{\omega}'\right] = (\lambda'+\eta')\nabla(\nabla\vec{\omega}') + (\eta'\nabla^2\vec{\omega}') + \zeta(\nabla\times\vec{q}' - 2\vec{\omega}'), \quad (18)$$

$$\frac{\partial T'}{\partial t} + (\vec{q}'.\nabla)T' - w\frac{\Delta T}{d} = \chi\nabla^2 T' + \frac{\beta}{\rho_o C_r}\left[\nabla\times\vec{\omega}'.\left(-\frac{\Delta T}{d}\right)\hat{k}\right] + \frac{\beta}{\rho_o C_r}[\nabla\times\vec{\omega}'.\nabla T'], \quad (19)$$

$$\rho' = -\alpha\rho_0 T', \quad (20)$$

$$\varepsilon_r' = -\varepsilon_0 e T', \quad (21)$$

$$\nabla.\left(\varepsilon_0 E' + P'\right) = 0\,. \tag{22}$$

We consider only two dimensional disturbances and thus restrict ourselves to the xz-plane; we now introduce the stream functions in the form:

$$u' = \frac{\partial \psi'}{\partial z}, \quad w' = \frac{\partial \psi'}{\partial x}\,. \tag{23}$$

which satisfies the continuity equation (16).

Introducing the electric potential ϕ' through the relation $\vec{E}' = -\nabla\phi'$, substituting equation (20) in equation (17), eliminating p by differentiating X-component of the resulting equation with respect to z, differentiating Z-component of the equation with respect to x and subtracting the two resulting equation from one another and non-dimensionalizing using the following definitation

$$(x^*, y^*, z^*) = \left(\frac{x'}{d}, \frac{y'}{d}, \frac{z'}{d}\right), t^* = \frac{t'}{d^2/\chi}, \psi^* = \frac{\psi'}{\chi/d}, T^* = \frac{T'}{\Delta T}, \phi^* = \frac{\phi'}{eE_0\Delta Td/1+\chi_e}, \omega_z = \frac{(\nabla\times\vec{\omega})}{\chi/d^3}.\Bigg\}, \tag{24}$$

we get the following dimensionless equations:

$$\frac{1}{Pr}\frac{\partial}{\partial t}\left(\nabla^2\psi\right) = -R[1+\delta\cos(\Omega t)]\frac{\partial T}{\partial t} + (1+N_1)\nabla^4\psi - N_1\nabla^2\omega_y + \frac{1}{Pr}J\left(\psi,\nabla^2\psi\right) + L\frac{\partial^2\phi}{\partial x\partial y} - L\frac{\partial T}{\partial t} + LJ\left(T,\frac{\partial\phi}{\partial z}\right), \tag{25}$$

$$\frac{N_2}{Pr}\frac{\partial\omega_y}{\partial t} = N_3\nabla^2\omega_y + N_1\nabla^2\psi - 2N_1\omega_y + \frac{N_2}{Pr}J\left(\psi,\omega_y\right), \tag{26}$$

$$\frac{\partial T}{\partial t} = -\frac{\partial\psi}{\partial x} + \nabla^2 T - N_5\frac{\partial\omega_y}{\partial x} + N_5 J\left(\omega_y, T\right) + J(\psi, T)\,, \tag{27}$$

$$\nabla^2\phi = \frac{\partial T}{\partial z}, \tag{28}$$

where the asterisks have been dropped for simplicity and the non-dimensional parametersPr, R,N_1, N_2, N_3, N_5 and L are given as

$Pr = \dfrac{\mu}{\rho_o\chi}$ (Prandtl number),

$R = \dfrac{\rho_o\alpha g\Delta Td^3}{\chi(\zeta+\eta)}$ (Rayleigh number),

$N_1 = \dfrac{\zeta}{\zeta+\eta}$ (Coupling parameter),

$N_2 = \dfrac{I}{d^2}$ (Inertia Parameter),

$N_3 = \dfrac{\eta'}{(\zeta+\eta)d^2}$ (Couple Stress Parameter),

$N_5 = \dfrac{\beta}{\rho_o C_v d^2}$ (Micropolar Heat Conduction Parameter),

$L = \dfrac{\varepsilon_0 e^2 E_0^2\Delta T^2 d^2}{(1+\chi_e)(\zeta+\eta)\chi}$ (Electric number) and

$\Omega = \dfrac{\gamma d^2}{\chi}$ (Non-dimensional modulation frequency).

Equations (31) to (34) are solved subject to the conditions

$$\psi = \frac{\partial^2\psi}{\partial z^2} = T = \omega_y = \frac{\partial\phi}{\partial z} = 0 \text{ at } z = 0,1. \tag{29}$$

5. LINEAR STABILITY ANALYSIS

In this section, we discuss the linear stability analysis considering marginal and over-stable states. To make this study we neglect the Jacobians in equations (25) to (28). The linearized version of equations (25) to (28) are:

$$\left[\frac{1}{Pr}\frac{\partial}{\partial t}-(1+N_1)\nabla^2\right]\nabla^2\psi=-R[1+\delta\cos(\Omega t)]\frac{\partial T}{\partial t}-L\frac{\partial T}{\partial t}-N_1\nabla^2\omega_y+L\frac{\partial^2\phi}{\partial x\partial y}, \tag{30}$$

$$\left[\frac{N_2}{Pr}\frac{\partial}{\partial T}+2N_1-N_3\nabla^2\right]\omega_y=N_1\nabla^2\psi, \tag{31}$$

$$\left[\frac{\partial}{\partial t}-\nabla^2\right]T=-\frac{\partial\psi}{\partial x}-N_5\frac{\partial\omega_y}{\partial x}, \tag{32}$$

$$\nabla^2\phi=\frac{\partial T}{\partial z}. \tag{33}$$

Eliminating T, ω_y and ϕ from equations (30) to (33), we get an equation for ψ in the form:

$$\left.\begin{aligned}&\left\{\left(\frac{N_2}{Pr}\frac{\partial}{\partial t}-N_3\nabla^2+2N_1\right)\left[\left(\frac{\partial}{\partial t}-\nabla^2\right)\nabla^4\left(\frac{1}{Pr}\frac{\partial}{\partial t}-(1+N_1)\nabla^2\right)-L\nabla_1^4\right]+N_1^2\nabla^6\left(\frac{\partial}{\partial t}-\nabla^2\right)-LN_1N_5\nabla^2\nabla_1^4\right\}\psi\\&=\nabla^2\nabla_1^2\left\{\frac{N_2}{Pr}\delta f'+(-N_3\nabla^2+2N_1+N_1N_5\nabla^2)(1+\delta f)\right\}\psi\end{aligned}\right\}, \tag{34}$$

where

$$f=\text{Real part of}(e^{-i\Omega t})\ \text{and}\ f'=-i\Omega\,\text{Real part of}(e^{-i\Omega t}).$$

In dimensionless form, the velocity boundary conditions for solving equation (34) are obtained from equations (30) to (33) and (29) in the form

$$\psi=\frac{\partial^2\psi}{\partial z^2}=\frac{\partial^4\psi}{\partial z^4}=\frac{\partial^6\psi}{\partial z^6}=\frac{\partial^8\psi}{\partial z^8}=0\,. \tag{35}$$

6. Perturbation procedure

We now seek the eigen-function ψ and eigen-values R of the equation (34) in the form

$$(R,\psi)=(R_0,\psi_0)+\delta(R_1,\psi_1)+\delta^2(R_2,\psi_2)+\ldots\ldots\ldots\,. \tag{36}$$

Substituting equation (36) into equation (34) and equating like powers of δ on both sides, we get:

$$L_1\psi_0=0\,, \tag{37}$$

$$L_1\psi_1=\frac{N_2}{Pr}\nabla^2\nabla_1^2 f' R_0\psi_0+\nabla^2\nabla_1^2(-N_3\nabla^2+2N_1+N_1N_5\nabla^2)(fR_0+R_1)\psi_0, \tag{38}$$

$$L_1\psi_2=\nabla^2\nabla_1^2\frac{N_2}{Pr}f'(R_1\psi_0+R_0\psi_1)+\nabla^2\nabla_1^2(-N_3\nabla^2+2N_1+N_1N_5\nabla^2)[(R_1+fR_0)\psi_1+(R_2+fR_1)\psi_0], \tag{39}$$

where

$$L_1=\left\{\begin{aligned}&\left(\frac{N_2}{Pr}\frac{\partial}{\partial t}-N_3\nabla^2+2N_1\right)\left[\left(\frac{\partial}{\partial t}-\nabla^2\right)\nabla^4\left(\frac{1}{Pr}\frac{\partial}{\partial t}-(1+N_1)\nabla^2\right)-L\nabla_1^4\right]\\&+N_1^2\nabla^6\left(\frac{\partial}{\partial t}-\nabla^2\right)-LN_1N_5\nabla^2\nabla_1^4-\nabla^2\nabla_1^2(-N_3\nabla^2+2N_1+N_1N_5\nabla^2)R_0\end{aligned}\right\}. \tag{40}$$

Each of ψ_n is required to satisfy the boundary condition (35).

6.1. Solution to the zeroth order problem

The zeroth order problem is equivalent to the Rayleigh-Bénard problem of micropolar fluid with electric field in the absence of gravity modulation.The marginally stable solution of the problem is the general solution of the equation (37), i.e.

$$\psi_0 = \sin(\pi\alpha x)\sin(\pi z), \tag{41}$$

corresponding to the lowest mode of convection with the corresponding eigen value

$$R_0 = \frac{N_3(1+N_1)k^{10} + N_1(2+N_1)k^8 + L\pi^2\alpha^2(\pi^2 - k^2)[(N_3 - N_1N_5)k^2 + 2N_1]}{[(N_3 - N_1N_5)k^2 + 2N_1]\pi^2k^2\alpha^2}, \tag{42}$$

where

$$k^2 = \pi^2(\alpha^2 + 1).$$

6.2. Solution to the first order problem

Equation (44) on using equation (47) becomes

$$L_1\psi_1 = \frac{N_2}{Pr}k^2\pi^2\alpha^2 f' R_0\psi_0 + k^2\pi^2\alpha^2[(N_3 - N_1N_5)k^2 + 2N_1](fR_0 + R_1)\psi_0, \tag{43}$$

If the above equation is to have a solution, the right hand side must be orthogonal to the null-space of the operator L. This implies that the time independent part of the RHS of the equation (43) must be orthogonal to $\sin(\pi z)$. Since f varies sinusoidal with time, the only steady term on the RHS of equation (43) is $[(N_3 - N_1N_5)k^2 + 2N_1]k^2\pi^2\alpha^2R_1$ so that $R_1 = 0$. It follows that all the odd coefficients i.e. $R_1 = R_3 = \dots\dots = 0$ in equation (36).

Using equation (36), we find that

$$L_1[\sin(\pi z)e^{i(lx+my-\Omega t)}] = L_1(\Omega)\sin(\pi z)e^{i(lx+my-\Omega t)}, \tag{44}$$

where

$$L_1(\Omega) = Y_1 + iY_2,$$

$$Y_1 = -\frac{\Omega^2}{Pr}\left(\frac{1}{Pr} + (1+N_1)\right)N_2k^2 + (N_3k^2 + 2N_1)\left(-\frac{\Omega^2}{Pr}k^4 + (1+N_1)k^8\right) - N_1^2k^8 - (\alpha^4\pi^4L + R_0\alpha^2\pi^2k^2)[(N_3 - N_1N_5)k^2 + 2N_1],$$

$$Y_2 = \Omega\left\{\frac{N_2}{Pr}\left(-\frac{\Omega^2}{Pr}k^4 - (1+N_1)k^8 + \alpha^4\pi^4L\right) - (N_3k^2 + 2N_1)\left(\frac{1}{Pr} + (1+N_1)\right)k^6 + N_1k^6\right\}.$$

The particular solution of equation (43) is

$$\psi_1 = \frac{R_0k^2a^2}{|L(\Omega)|^2}\left[\frac{N_2}{Pr}(-\Omega Y_1\sin(\Omega t) - \Omega Y_2\cos(\Omega t)) + A_1(Y_1\cos(\Omega t) - Y_2\sin(\Omega t))\right]\sin(\pi z), \tag{45}$$

The equation of ψ_2 is

$$L_1\psi_2 = k^2\alpha^2\pi^2\frac{N_2}{Pr}f' R_0\psi_1 + k^2\alpha^2\pi^2A_1fR_0\psi_1 + k^2\alpha^2\pi^2A_1R_2\psi_0, \tag{46}$$

where

$$A_1 = [(N_3 - N_1N_5)k^2 + 2N_1].$$

We shall not solve equation (46), but will use this to determine R_2. For the existence of a solution of equation (46), it is necessary that the steady part of its right hand side is orthogonal to $\sin(\pi z)$. This gives,

$$\int_0^1\left[k^2\alpha^2\pi^2\frac{N_2}{Pr}f' R_0\psi_1 + k^2\alpha^2\pi^2A_1fR_0\psi_1 + k^2\alpha^2\pi^2A_1R_2\psi_0\right]\sin(\pi z)dz = 0,$$

Taking time average, we get,

$$R_2 = -\frac{2N_2R_0}{A_1Pr}\int_0^1\overline{f'\psi_1}\sin(\pi z)dz - 2R_0\int_0^1\overline{f\psi_1}\sin(\pi z)dz, \tag{47}$$

and finally

$$R_2 = -\frac{R_0k^2\alpha^2\pi^2}{2|L_1(\Omega)|^2}\left[\left(\frac{N_2}{Pr}\right)^2\frac{\Omega^2Y_1}{A_1} + A_1Y_1\right]. \tag{48}$$

7. NON-LINEAR THEORY

The finite amplitude analysis is carried out here via Fourier series representation of stream function ψ, the spin ω_y and the temperature distribution T. Although the linear stability analysis is sufficient for obtaining the stability condition of the motionless solution and the corresponding eigen functions describing qualitatively the convective flow, it cannot provide information about the values of the convection amplitudes, nor regarding the rate of heat transfer. To obtain this additional information, we perform nonlinear analysis, which is useful to understand the physical mechanism with minimum amount of mathematical analysis and is a step forward towards understanding full nonlinear problem.

The first effect of non-linearity is to distort the temperature field through the interaction of ψ and T. The distortion of the temperature field will correspond to a change in the horizontal mean, i.e., a component of the form $\sin(2\pi z)$ will be generated. Thus, truncated system which describes the finite-amplitude free convection is given by (see Veronis [27]):

$$\psi(x,y,t) = A(t)\sin(\pi\alpha x)\sin(\pi z)\,, \tag{49}$$

$$\omega_y(x,y,t) = B(t)\sin(\pi\alpha x)\sin(\pi z)\,, \tag{50}$$

$$T(x,y,t) = E(t)\cos(\pi\alpha x)\sin(\pi z) + F(t)\sin(2\pi z)\,, \tag{51}$$

$$\phi(x,z,t) = \frac{1}{\pi}M(t)\cos(\pi\alpha x)\cos(\pi z)\,, \tag{52}$$

where the time dependent amplitudes A, B, E, F and M are to be determined from the dynamics of the system. The functions ψ and ω_y do not contain an x- independent term because the spontaneous generation of large scale flow has been discussed.

Substituting equations (49)-(52) into coupled non-linear partial differential equations (25)-(28) and equating the coefficient of like terms we obtain the following non-linear autonomous Lorenz system (fourth order) of differential equations:

$$\dot{A} = \frac{-R[1+\delta\cos(\Omega t)]\Pr\pi\alpha}{k^2}E - \Pr(1+N_1)k^2A - N_1\Pr B - \frac{L\Pr\pi\alpha}{k^2}E + \frac{L\pi^3\alpha\Pr}{k^4}E + \frac{4L\pi^6\alpha^3\Pr}{k^4}EF, \tag{53}$$

$$\dot{B} = \frac{-N_3\Pr k^2}{N_2}B - \frac{N_1\Pr k^2}{N_2}A - \frac{2N_1\Pr}{N_2}B, \tag{54}$$

$$\dot{E} = -\pi\alpha A - k^2E - N_5\pi\alpha B - \pi^2\alpha AF - N_5\pi^2\alpha BF, \tag{55}$$

$$\dot{F} = -4\pi^2F + \frac{1}{2}N_5\pi^2\alpha BE + \frac{1}{2}\pi^2\alpha AE, \tag{56}$$

where over dot denotes time derivative. M in equation (52) is eliminated by substituting equations (50) and (52) in equation (28).

The generalized Lorentz model (53)-(56) is uniformly bounded in time and possesses many properties of the full problem.

The phase-space volume contracts at a uniform rate given by:

$$\frac{\partial}{\partial A}\left(\frac{\partial A}{\partial t}\right) + \frac{\partial}{\partial B}\left(\frac{\partial B}{\partial t}\right) + \frac{\partial}{\partial E}\left(\frac{\partial E}{\partial t}\right) + \frac{\partial}{\partial F}\left(\frac{\partial F}{\partial t}\right) = -\left[\Pr(1+N_1)k^2 + \frac{N_3\Pr k^2}{N_2} + \frac{2N_1\Pr}{N_2} + k^2 + 4\pi^2\right]. \tag{57}$$

which is always negative and therefore the system is bounded and dissipative. As a result, the trajectories are attracted to a set of measure zero in the phase space; in particular they may be attracted to a defined point, a limit cycle or perhaps, a strange attractor. From equation (63), we can conclude that the set of initial points in the phase space occupies a region V (0) at time t = 0. Then after some time t, the end points of the corresponding trajectories will fill a volume

$$V(t) = V(0)\exp\left\{-\left[\Pr(1+N_1)k^2 + \frac{N_3\Pr k^2}{N_2} + \frac{2N_1\Pr}{N_2} + k^2 + 4\pi^2\right]\right\}.$$

This expression indicates that volume decreases exponentially.

The set of non-linear ordinary differential equations possesses an important symmetry for it is invariant under the transformation,

$$(A,B,E,F) \to (-A,-B,-E,-F). \tag{58}$$

The non-linear system of autonomous differential equations (52)-(56) is not amenable to analytical treatment for the general time-dependent variable and we have to solve it using a numerical method. In the case of steady motions, however, these equations can be solved analytically. Such solutions are very useful because they show that a finite amplitude steady solution to the system is possible for sub-critical values of the Rayleigh number.

8. Heat transport

In this section we mainly focus on the influence of gravity modulation on heat transport which is quantified in terms of the Nusselt number (Nu) defined as follows:

$$Nu = \frac{\text{Heat transport by (conduction + convection)}}{\text{Heat transport by (conduction)}},$$

$$Nu = \frac{\left[\frac{k}{2\pi}\int_0^{2\pi/k}(1-z+T)_z\,dx\right]_{z=0}}{\left[\frac{k}{2\pi}\int_0^{2\pi/k}(1-z)_z\,dx\right]_{z=0}}. \tag{59}$$

where subscript in the integrand denotes the derivative with respect to z.

Substituting equation (56) into equation (59) and completing the differentiation and integration, we get the following expression for Nusselt number:

$$Nu = 1 - 2\pi F(t). \tag{60}$$

9. RESULTS AND DISCUSSIONS

We now comprehend the effect of small amplitude gravity modulation and electric field on the onset of Rayleigh – Bénard convection in a horizontal layer of a micropolar fluid for a wide range of frequencies of modulation and the relevant parameters. The linear stability problem is solved based on the method proposed by Venezian [38]. Attention is focused on the determination of the linear stability criterion. The effect of g-jitter on heat transport is also discussed using the non-linear theory based on the truncated representation of Fourier series. The non – autonomous Lorenz model obtained is solved numerically.

The solutions obtained are based on the assumption that the amplitude of the gravity modulation is small. The validity of the results depends on the value of the modulating frequency Ω. When $\Omega < 1$ (i.e. the period of modulation is large) the gravity modulation affects the entire volume of the fluid resulting in the growth of the disturbance. On the otherhand, the effect of modulation disappears for large frequencies. This is due to the fact the buoyancy force takes a mean value leading to the equilibrium state of the unmodulated case. In view of this, we choose only moderate value of Ω in our resent study. It must be noted here that because of the presence of suspended particles in the fluid and according to Einstein's relation for viscosity, the value of Prandtl number is taken higher than those of clean fluid.

We first discuss the result obtained by linear theory followed by a discussion of the results obtained by a weakly non-linear theory. It should be noted that the gravity modulation affects the entire bulk of the fluid between the bounding plates. The figures (2) – (7) shows that the effect of the parameters N_1, N_2, N_3, N_5, L and Pr on R_{2c}.

Figure (2) is the plot of correction Rayleigh number R_{2c}versus frequency of modulation Ω for different values of coupling parameter N_1. We observe that as N_1 increases, R_{2c}also increases. The increase in N_1 implies increase in the concentration of suspended particles. These suspended particles consume the greater part of the energy in forming the gyrational velocity and as a result R_{2c} increases. Thus, increase in N_1 stabilizes the system.

Figure (3) is the plot of R_{2c} versus Ω for different values of inertia parameter N_2. Increase in N_2 is representative of the increase in inertia of the fluid due to the suspended particles. Thus, as is to be expected, we find that as N_2 increases R_{2c}increases and thereby stabilizing the system.

Figure (4) is the plot of R_{2c} versus Ω for different values of couple stress parameter N_3. Increase in N_3 signifies increase in couple stress of the fluid and decrease in gyrational velocities. Hence, as N_3 increases, we observe that R_{2c}decreases and destabilize the system.

Figure (5) is the plot of R_{2c} versus Ω for different values of micropolar heat conduction parameter N_5. When N_5 increases, the heat induced into the fluid due to this, microelements also increases, thus reducing the heat transfer from bottom to top.We find from the figure that as N_5 increases R_{2c} increases and thus stabilizes the system.

Figure (6) is the plot of R_{2c} versus Ω for different values of electric Rayleigh number L. From the figures we observed that, increase in L decreases R_c. The electric Rayleigh number L is the ratio of electric force to the dissipative force, higher the value of L dissipative force becomes negligible and hence it destabilizes the system.

Figure (7) is a plot of R_{2c} versus Ω for different values of Prandtl number Pr. It is observed that as Pr increases R_{2c}also increases. It can be inferred from this that the effect of increasing the concentration of the suspended particle is to stabilize the system. This means that the fluids with suspended particles are more susceptible to stabilization by modulation than clean fluids.

From the figures it is observed that since R_{2c} remains always positive for all values of Ω, gravity modulation leads to delay in onset of convection. The results of this study are helpful in the areas of crystal growth under microgravity conditions.

We now discuss the result pertaining to the non-linear stability analysis of gravity modulated Rayleigh – Bénard convection in a horizontal layer of a micropolar fluid. The influence of N_1, N_2, N_3, N_5, L and Pr on Nusselt number is examined in figures (8) – (16). It is clear from these figures that the effect of having suspended particles is to reduce the amount of heat transfer. Thus the effect of increase in N_1, N_2, N_5 and Pr is to stabilize the system. The increase in N_3 and L increases the amount of heat transfer and thus destabilize the system which reconfirms the results obtained in the linear case.

The figures (14) – (16) are the plot of frequency of modulation Ω, amplitude of modulation δ and for different values of Rayleigh number R, respectively. It is observed that increase in Ω decreases the heat transfer and increase in δ and R increases the heat transfer. Thus, the frequency of modulation stabilizes the system while amplitude of modulation and increase Rayleigh number destabilizes the system.

From the computation we observed that for a slight change in the initial condition, Nu is fluctuating more often throughout the system when Rayleigh number is increased to ten times as well fifteen times. This shows that chaos sets for higher value of Rayleigh number and chaos are very sensitive to the initial condition, hence difficult to measure the rate of heat transfer in the system. It is also clear from the figures (8) – (16) that the phase space of the system is not uniform.

10. CONCLUSIONS

The effect of coupling parameter N_1, inertia parameter N_2, micropolar heat conduction parameter N_5 and Prandtl number Pris to reduce the amount of heat transfer whereas the opposite effect is observed in the case of couple stress parameter N_3 and electric Rayleigh number L. It is observed that gravity modulation or g-jitter leads to delay in convection and frequency of gravity modulation also plays an important role in controlling heat transfer in the system.

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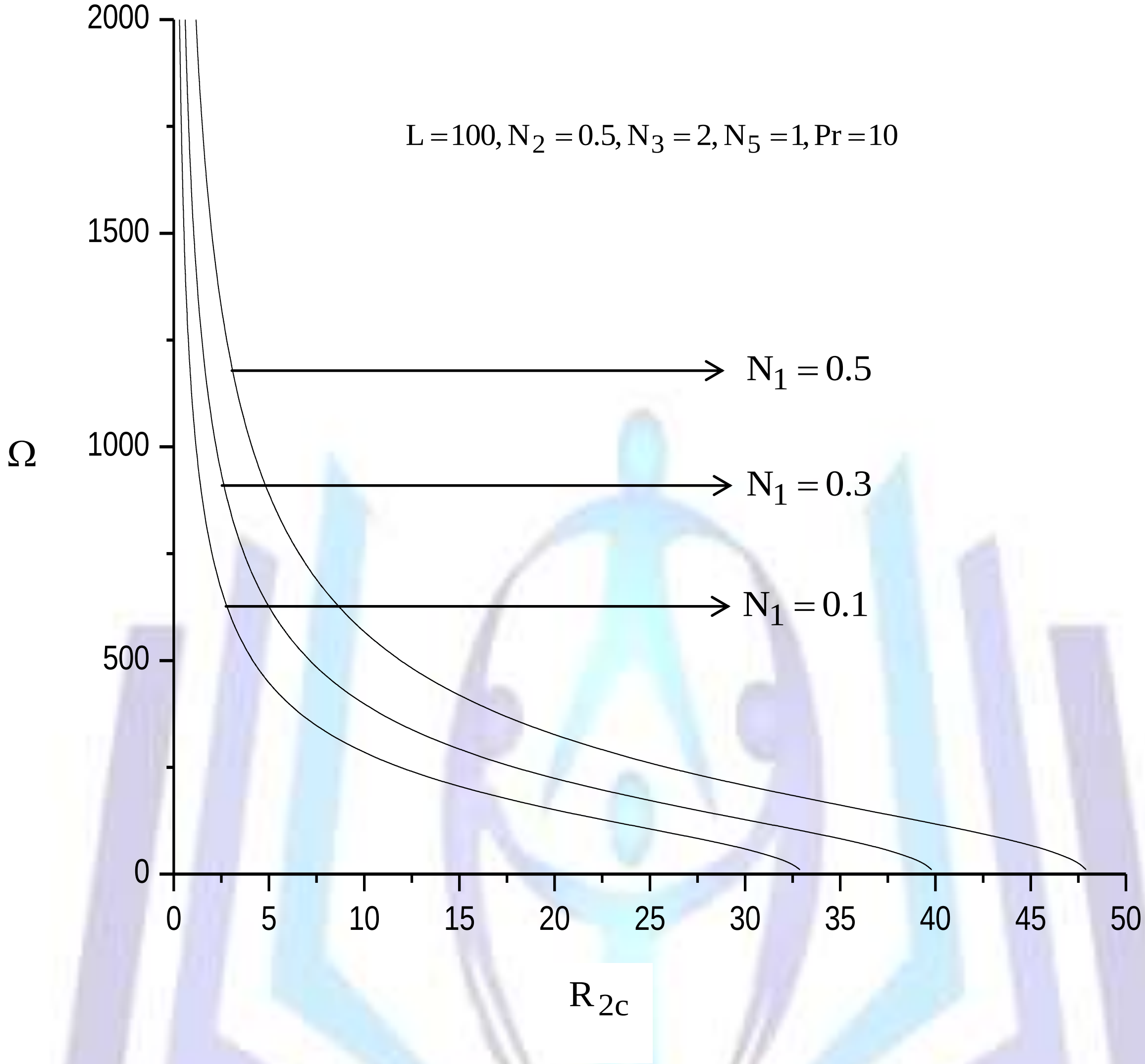

Figure 2: Plot of correction Rayleigh number R_{2c} versus frequency of modulation Ω for different values of coupling parameter N_1.

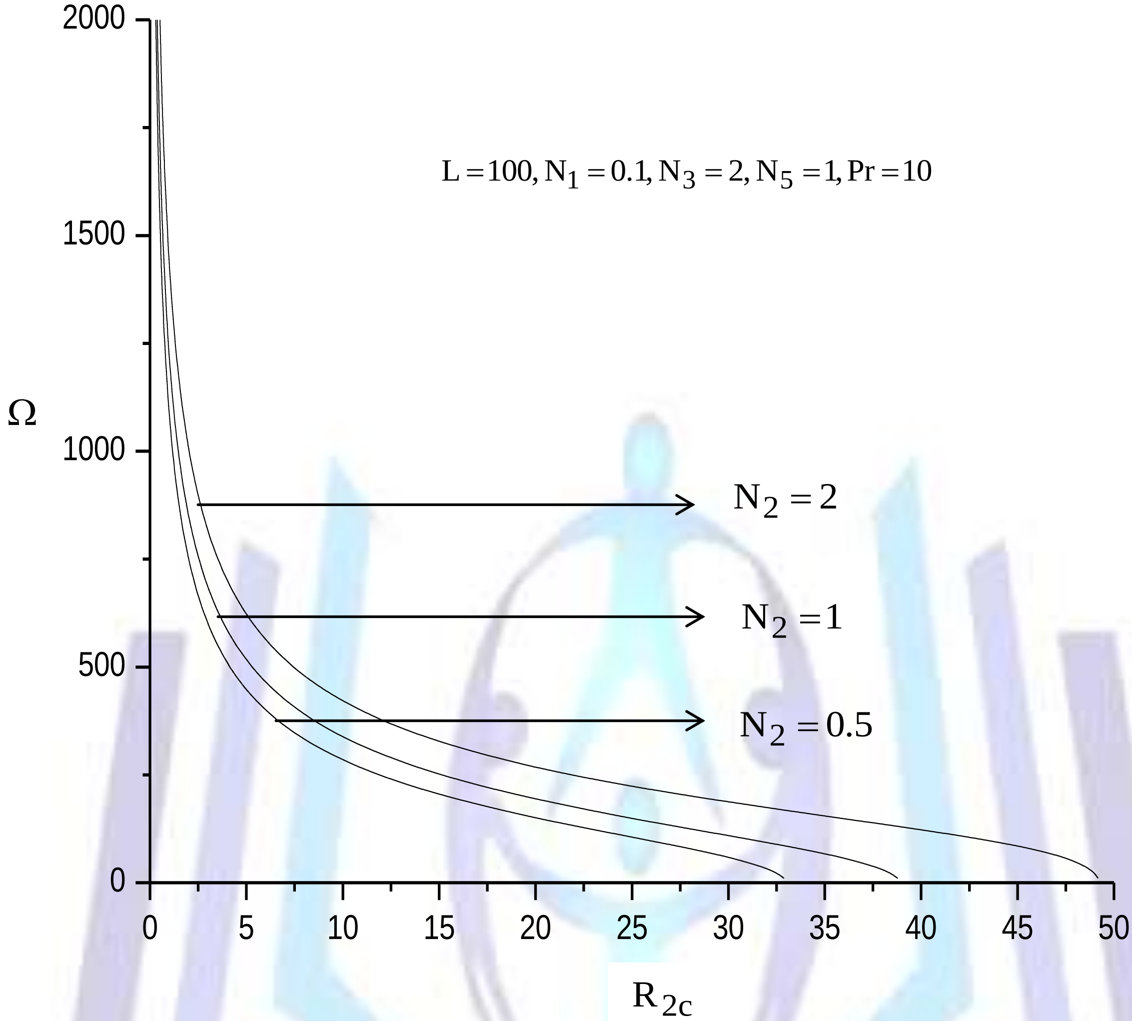

Figure 3: Plot of correction Rayleigh number R_{2c} versus frequency of modulation Ω for different values of inertia parameter N_2.

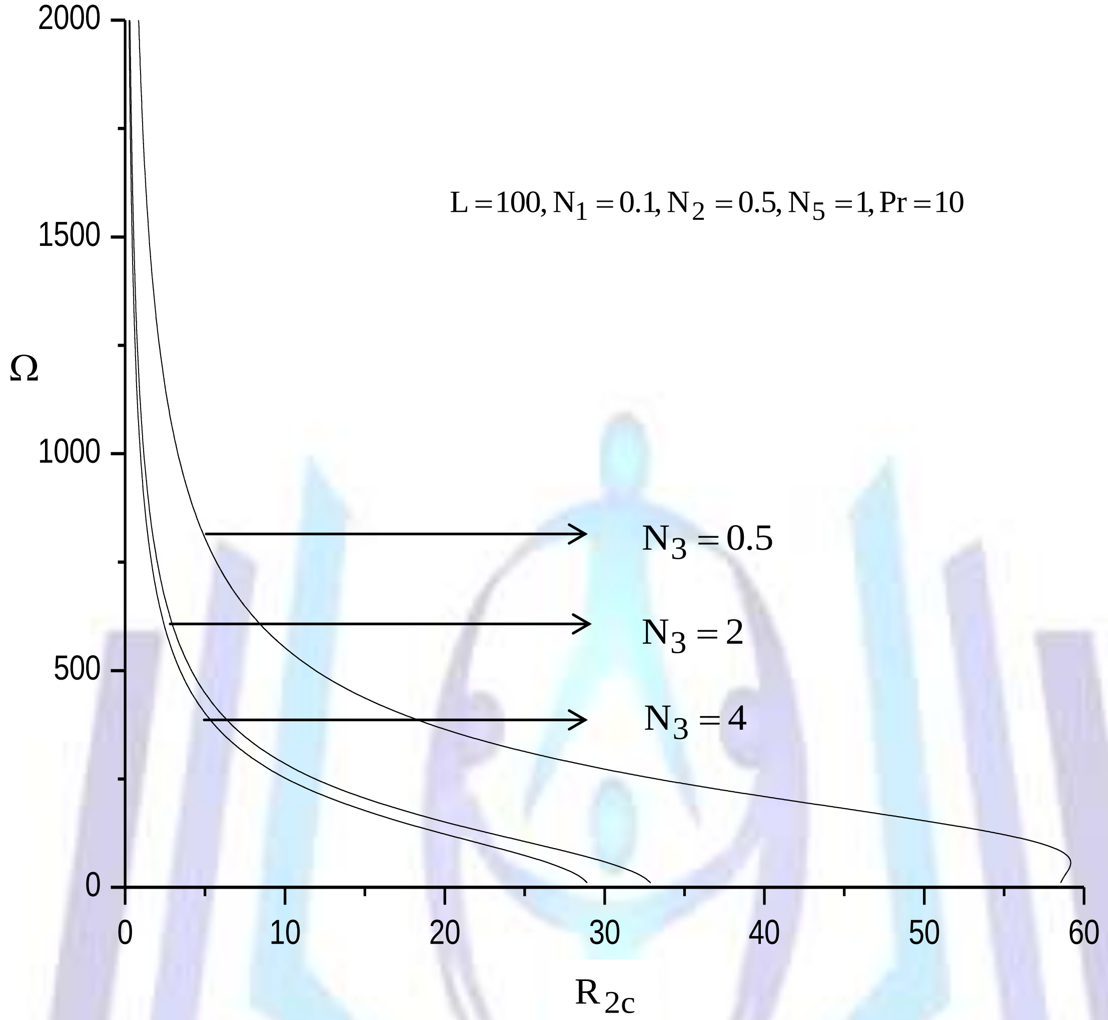

Figure 4: Plot of correction Rayleigh number R_{2c} versus frequency of modulation Ω for different values of couple stress parameter N_3.

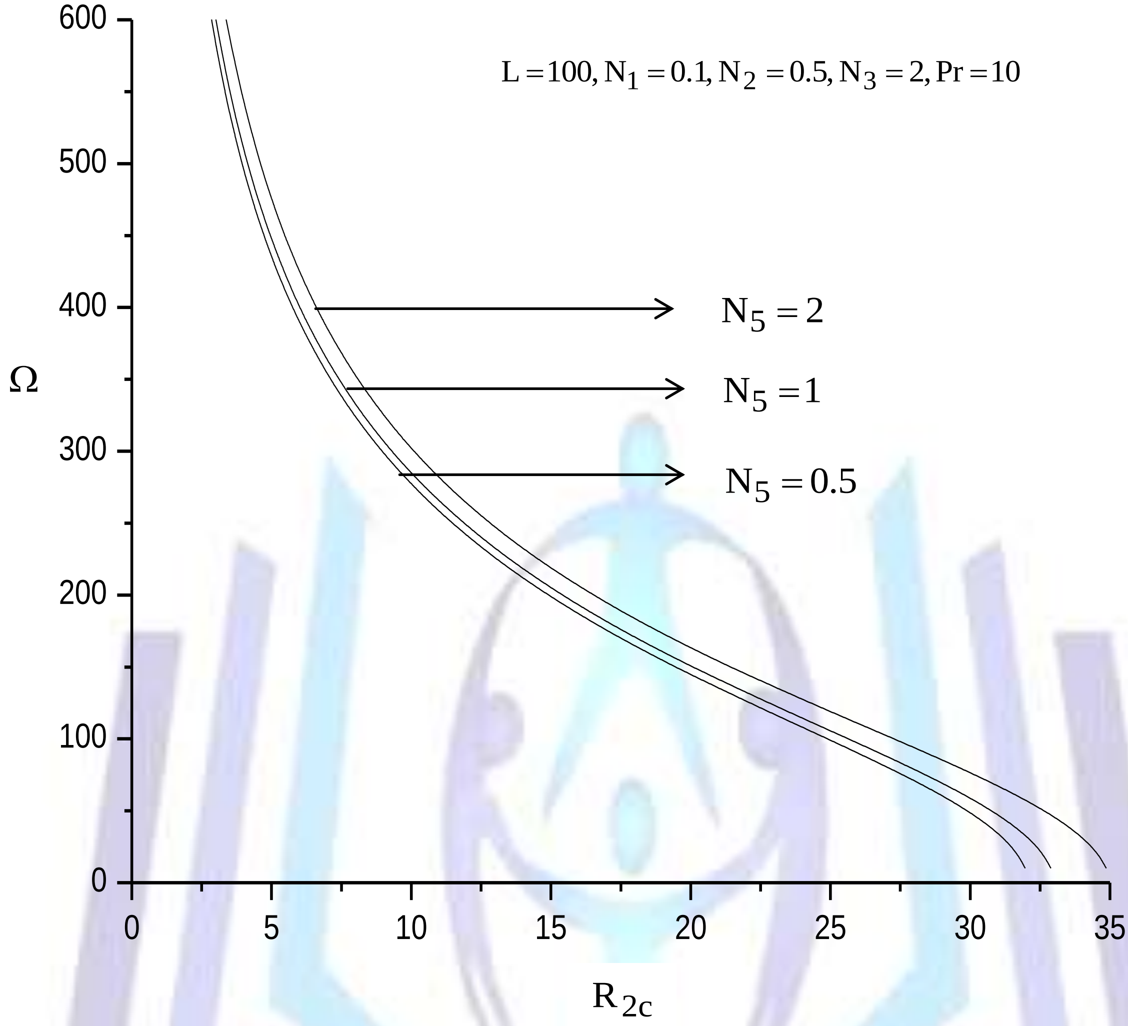

Figure 5: Plot of correction Rayleigh number R_{2c} versus frequency of modulation Ω for different values of micropolar heat conduction parameter N_5.

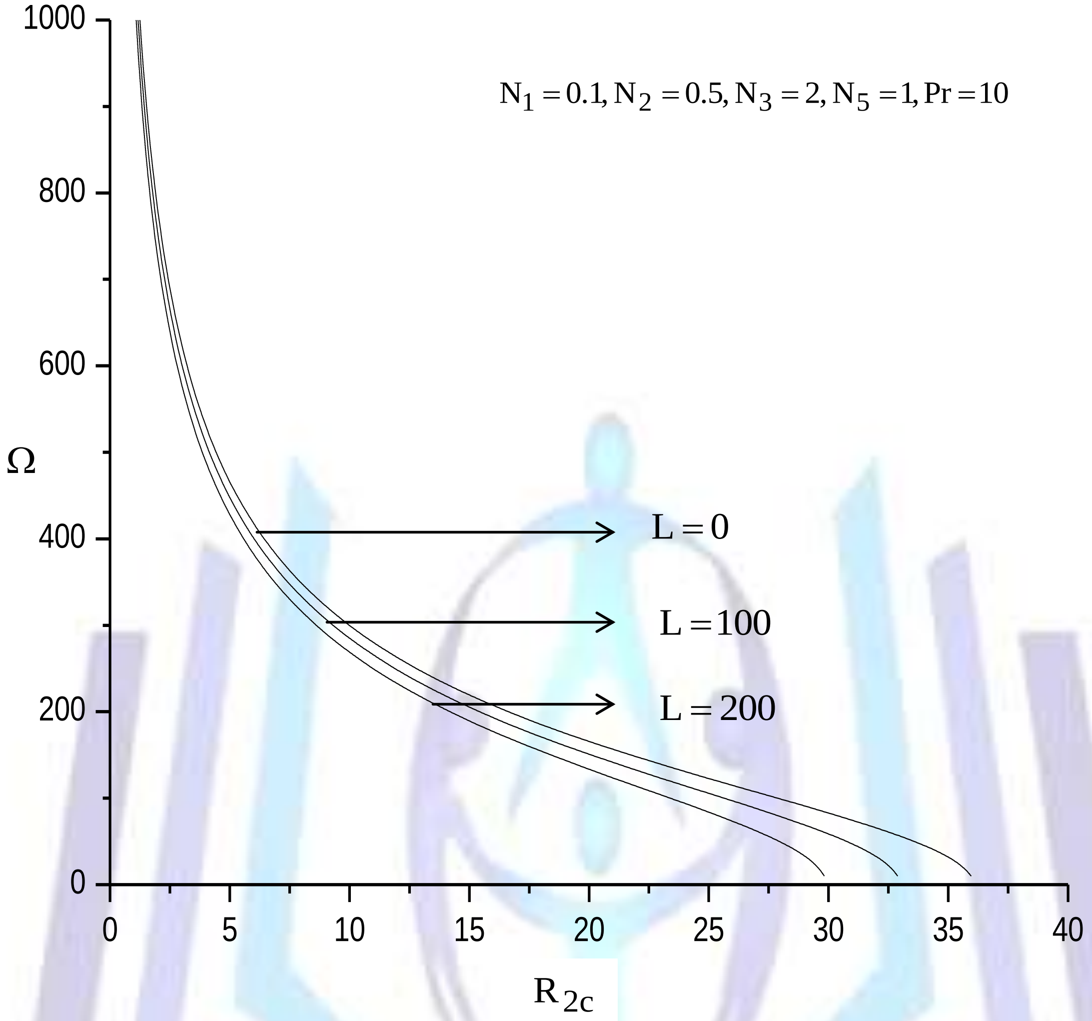

Figure 6: Plot of correction Rayleigh number R_{2c} versus frequency of modulation Ω for different values of electric Rayleigh number L.

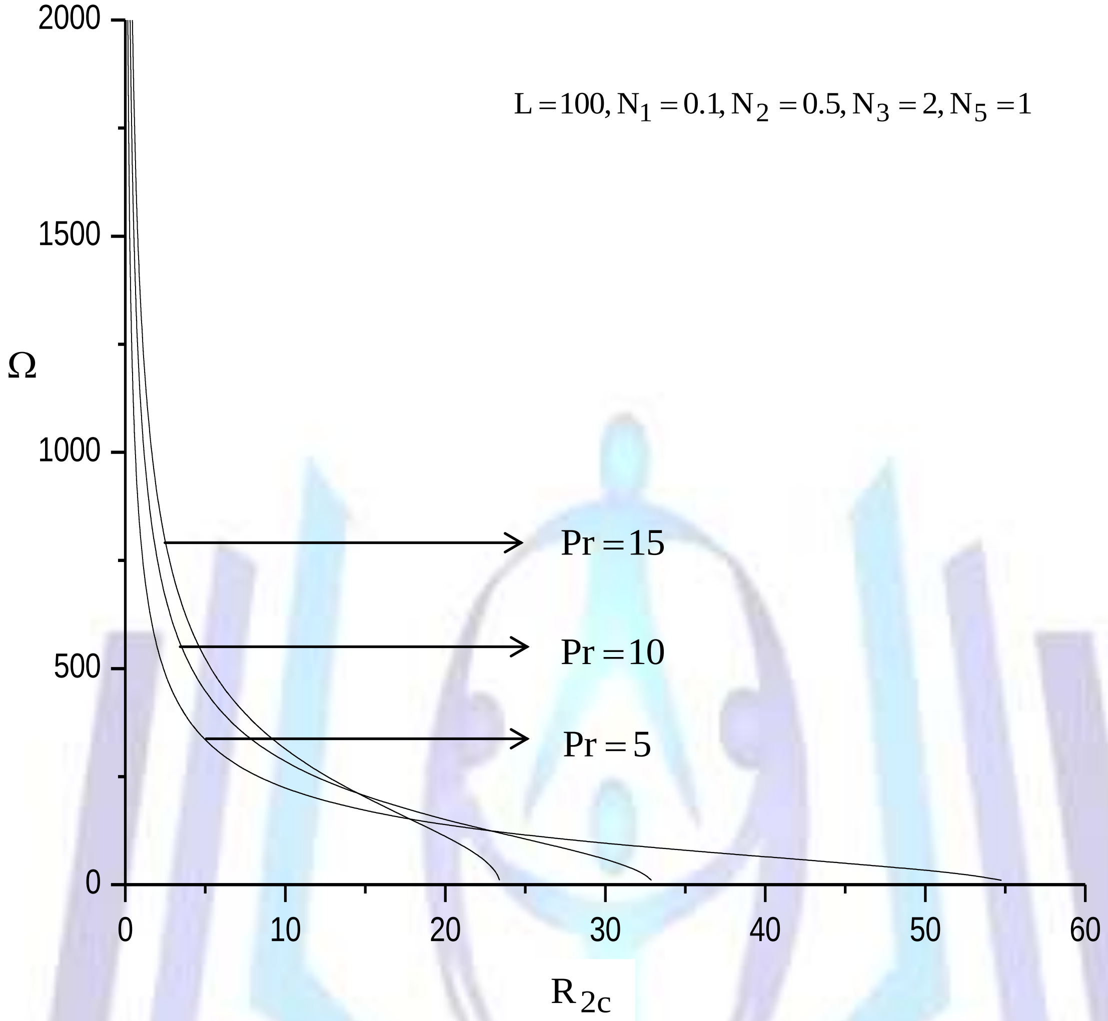

Figure 7: Plot of correction Rayleigh number R_{2c} versus frequency of modulation Ω for different values of Prandtl number Pr.

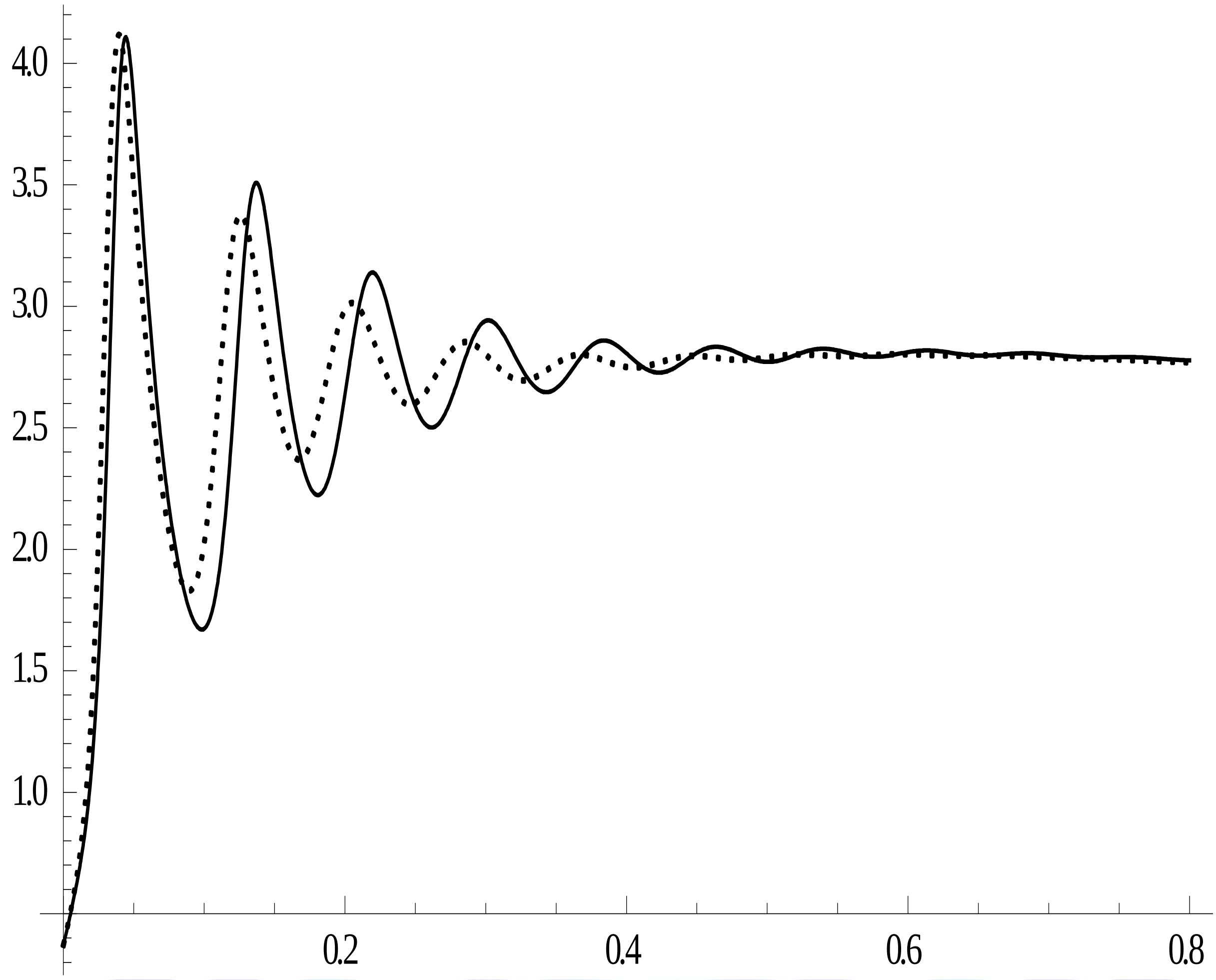

Figure 8: Plot of Nusselt number Nu versus time t for different values of coupling parameter N_1.

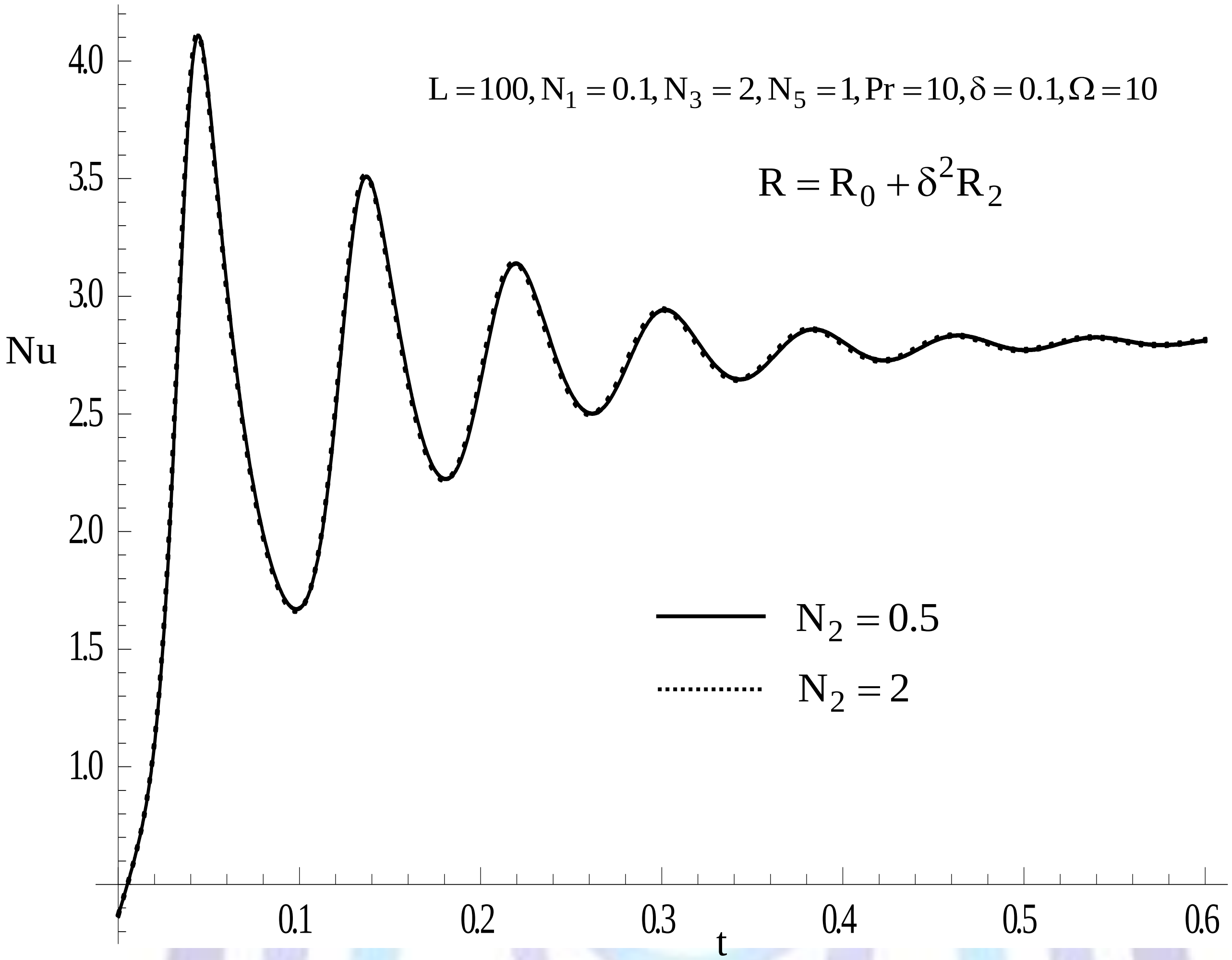

Figure 9: Plot of Nusselt number Nu versus time t for different values of inertia parameter N_2.

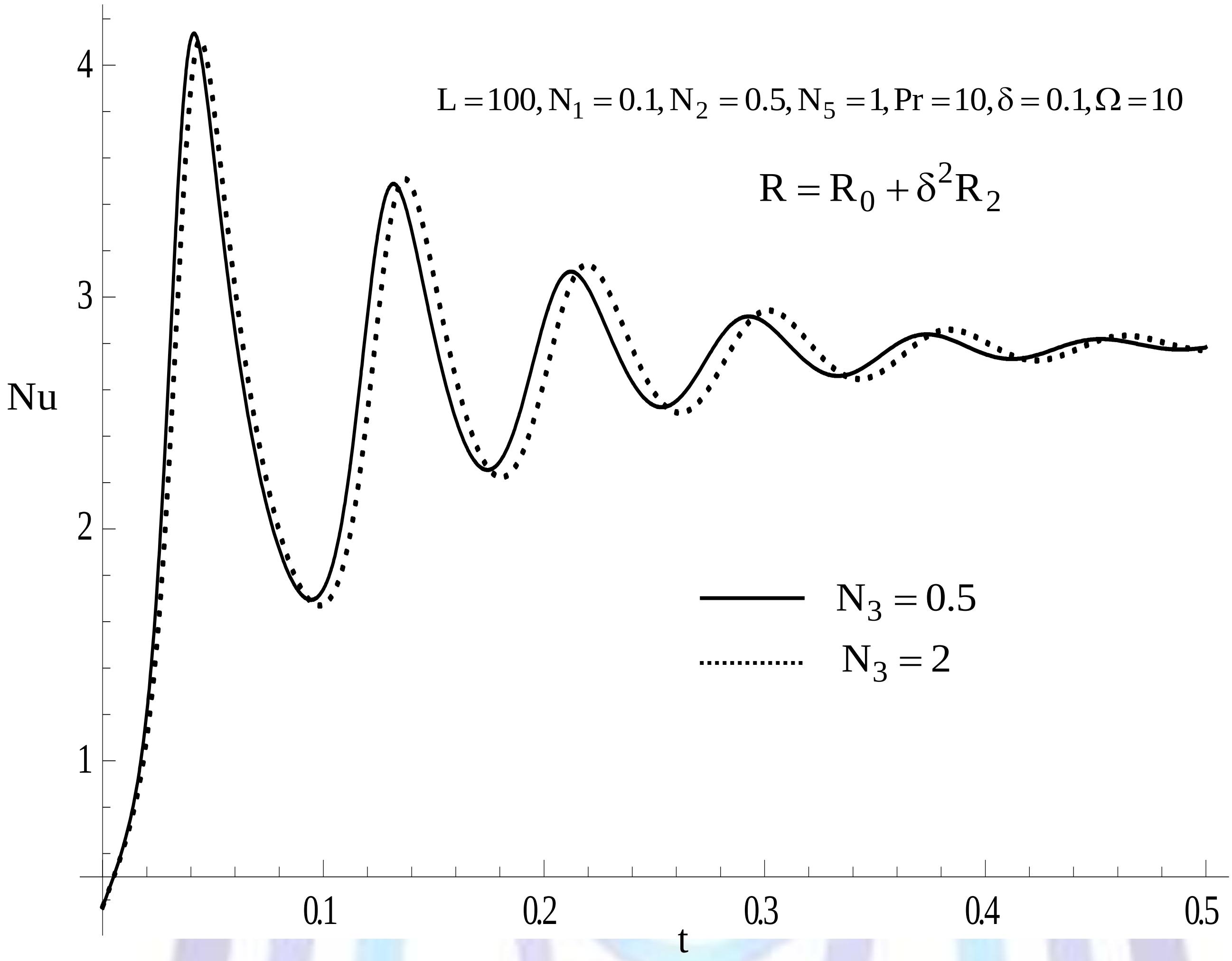

Figure 10: Plot of Nusselt number Nu versus time t for different values of couple stress parameter N_3.

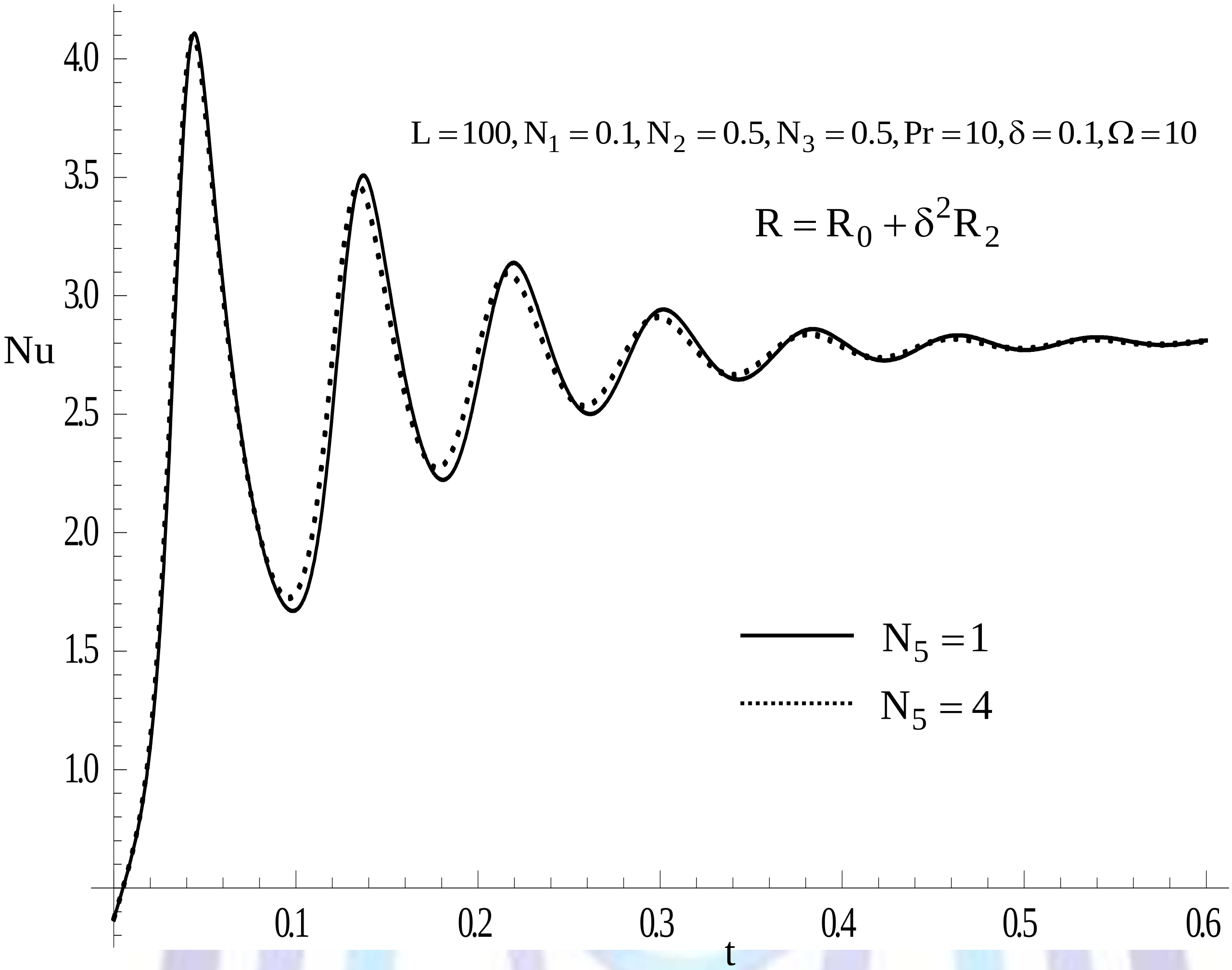

Figure 11: Plot of Nusselt number Nu versus time t for different values of micropolar heat conduction parameter N_5.

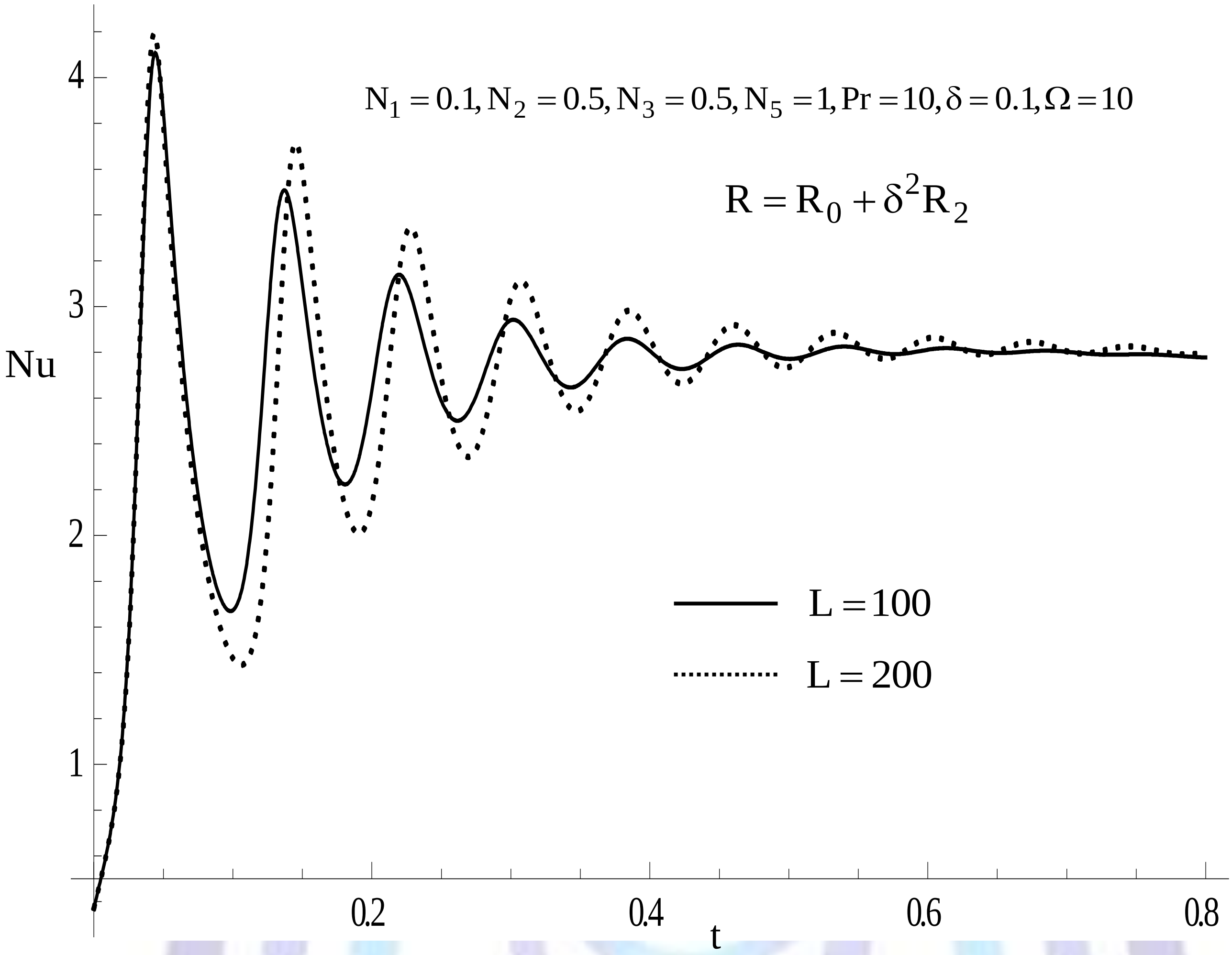

Figure 12: Plot of Nusselt number Nu versus time t for different values of electric Rayleigh number L.

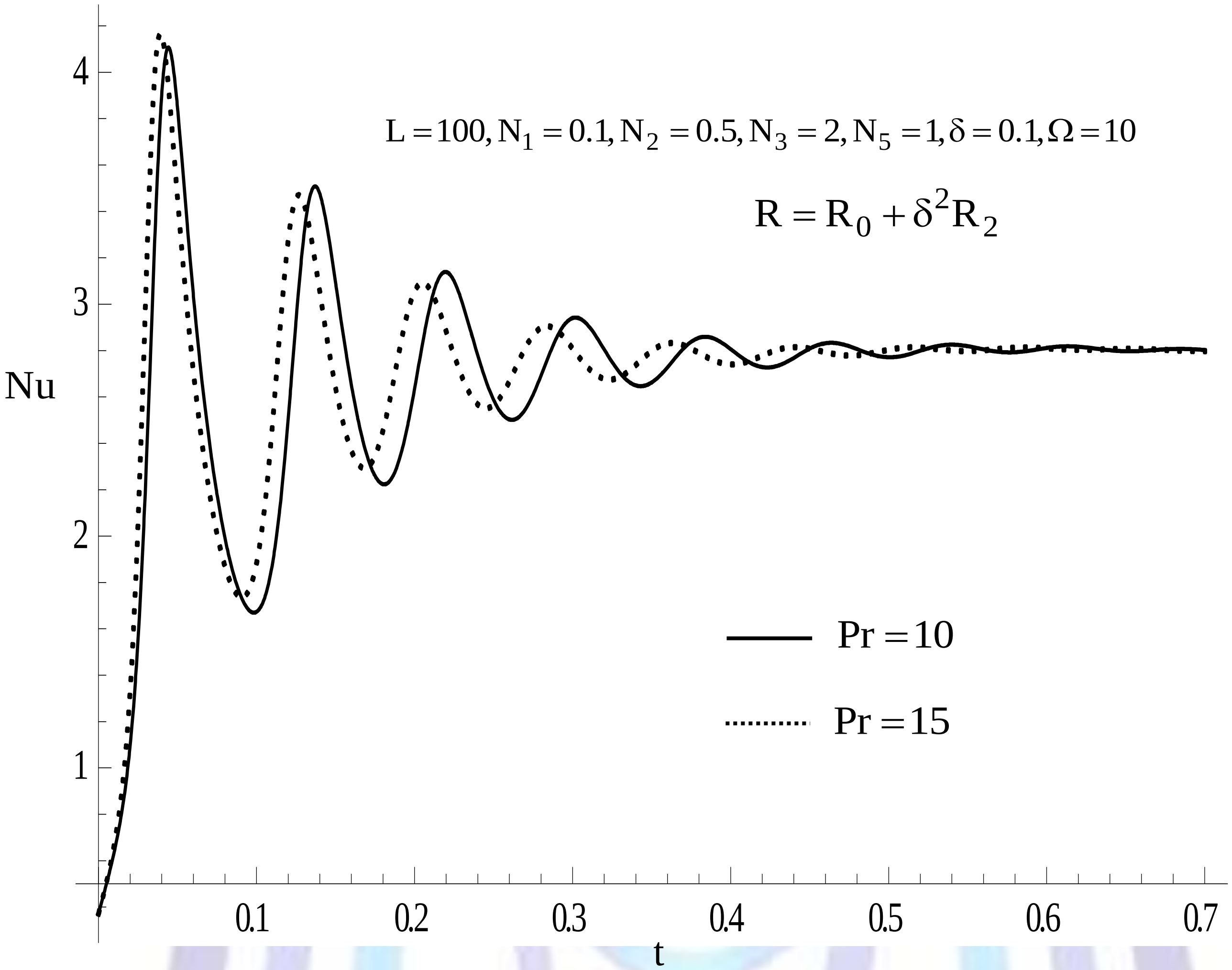

Figure 13: Plot of Nusselt number Nu versus time t for different values of Prandtl number Pr.

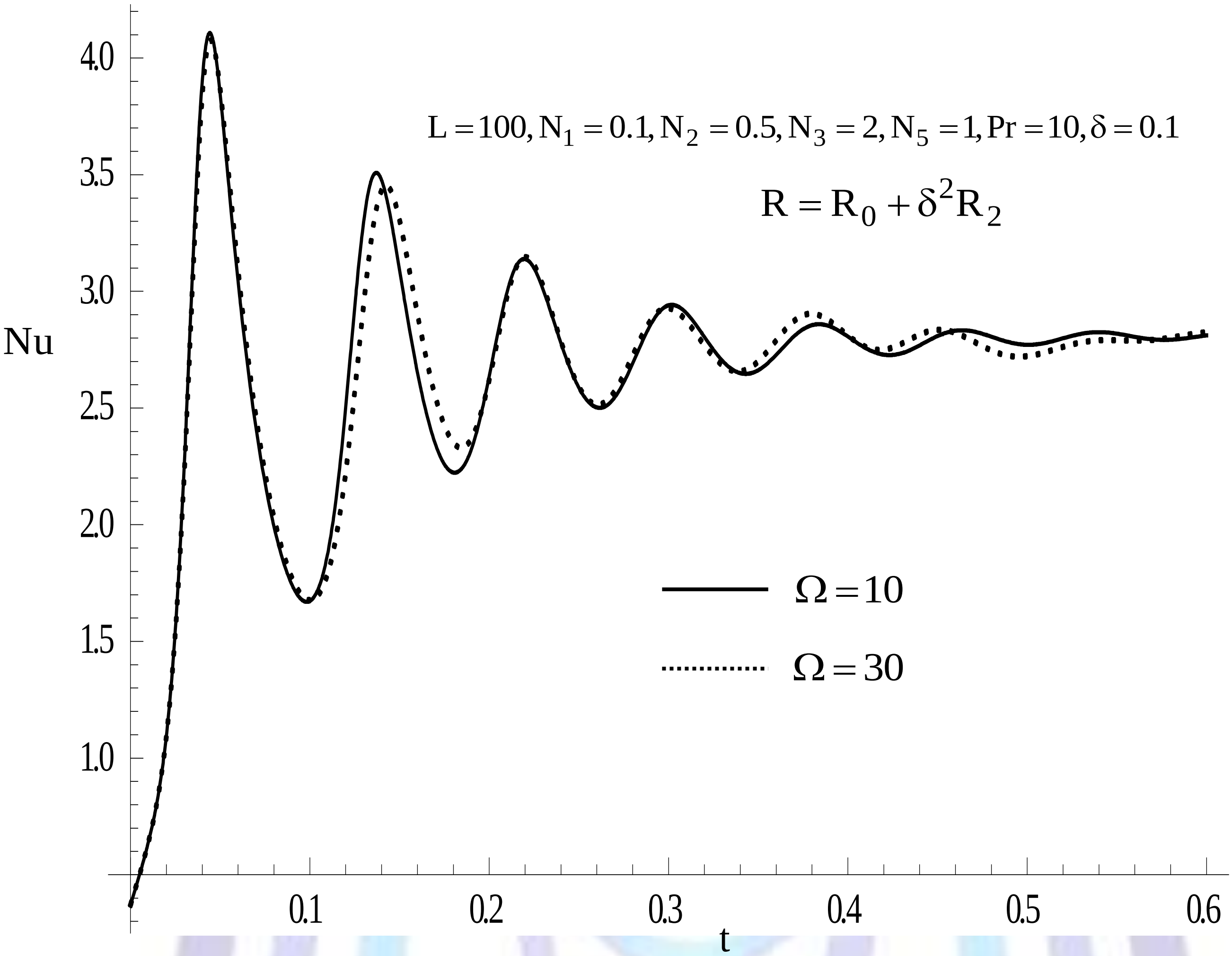

Figure 14: Plot of Nusselt number Nu versus time t for different values of frequency of modulation Ω .

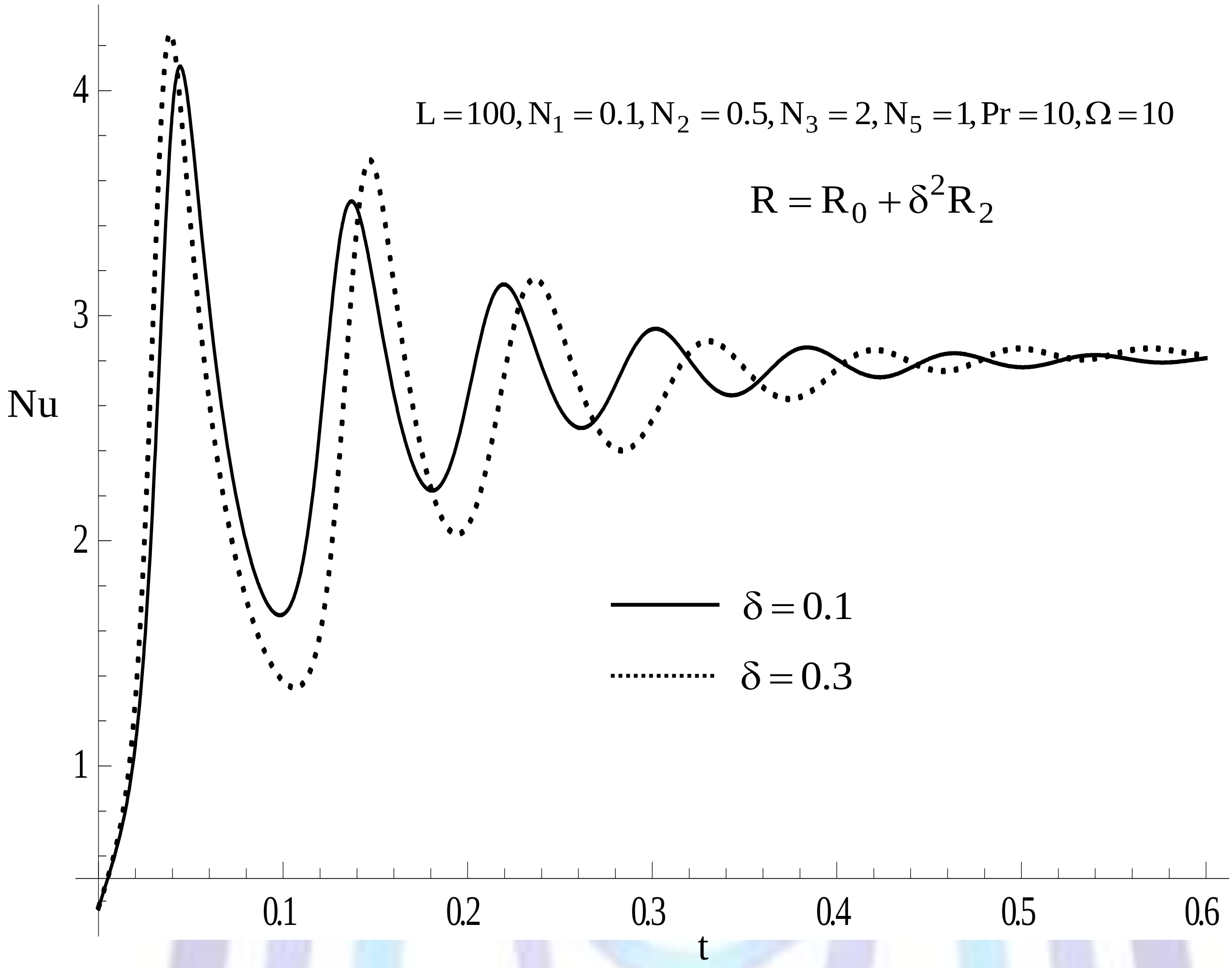

Figure 15: Plot of Nusselt number Nu versus time t for different values of frequency δ.

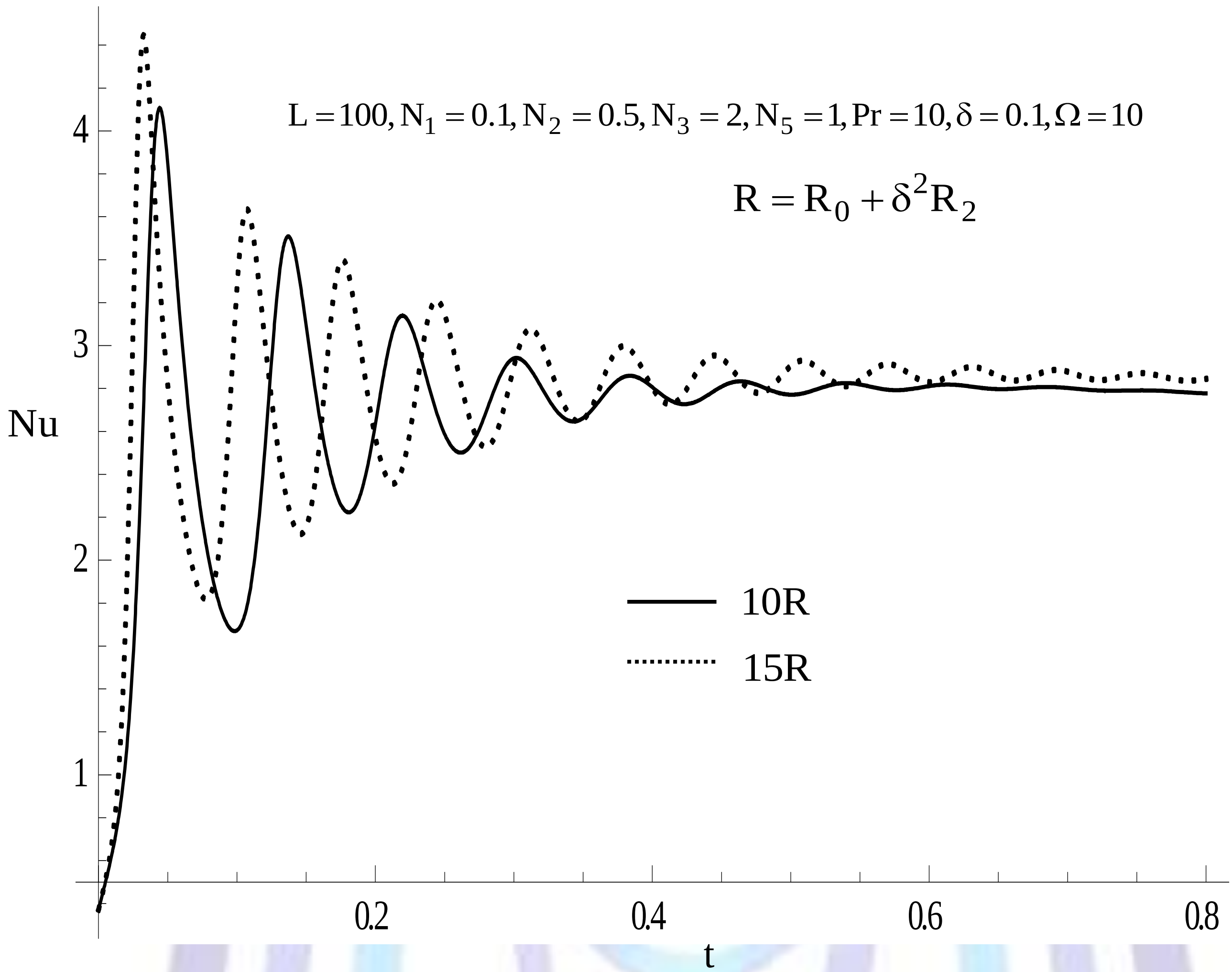

Figure 16: Plot of Nusselt number Nu versus time t for different values of critical Rayleigh number R.