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Thermal stresses in an infinite body with spherical cavity due to an arbitrary heat flux on its internal boundary surface

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ABSTRACT

In this paper we consider an elastic infinite body $a \le r < \infty$ with a spherical cavity subjected to a arbitrary heat flux on its internal boundary which is assumed to be traction free. The displacement and thermal stresses are obtained and results are compared using constant and time dependent heat flux. Laplace transform technique is used to obtain the temperature distribution. The mathematical model is obtained for copper material. The results are illustrated numerically and graphically.

Keywords

Thermal stresses; infinite body; Laplace transform.

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INTRODUCTION

An understanding of thermally induced stresses in isotropic bodies is essential for a comprehensive study of their response due to an exposure to a temperature field, which may in turn occurs in service or during the manufacturing stages.Thermal stresses may be induced from the heat build-up and cooling processes.

The conduction of heat is a very important phenomenon to the engineering science. Since Fourier's work, *" La Theorie Analytique de la Chaleur"* many mathematics methods have been developed to focus on different types of heat transfer and thermal stress problems. Though the field of Thermoelasticity is developed in many ways, it still needs to work on some challenging fundamental problems. Some examples of these types belonging to geometries with infinite and semi-infinite in spherical coordinates are to be studied, where little literature is available.

The study of problems of determination of temperature and stress distribution in the infinite medium under different thermal and mechanical conditions becomes a subject of extensive research in the field of thermoelasticity. Such an approach with spherical objects is of great importance in engineering practice. The problems of the calculation of field of temperature and its thermal stresses, when massive bodies with spherical cavity are studied under different thermal and mechanical condition are encountered in various physical problems.

Earlier literature on these found to be on the homogeneous and isotropic infinite material with constant and uniform thermo physical properties in the monographs of Carslaw [1], Ozisik [2,12],Noda et al. [3], Yener and Kakac [4],Timoshenko and Goodier [5]. They have discussed variety of problems involving steady and unsteady heat conduction. Noda et al. [3] has discussed the steady and transient problems of thermal stresses in a spherical coordinates in uncoupled thermoelasticiy.

Nowinski J. [6] determined displacement and stresses in infinite medium with spherical cavity assuming thermal and elastic temperature dependent properties by perturbation method. Lahiri et al. [7] discussed the problem of thermoelastic interaction in an unbounded with spherical cavity with eigenvalue approach. Povstenko [8] discussed the non-axisymmertic solutions to time fractional diffusion wave equation with a source term in spherical coordinates are obtained for an infinite medium with a spherical cavity.

Recently Kedar and Deshmukh [9] discussed the determination of quasi-static thermal deflection in a semi infinite solid circular cylinder subjected to arbitrary initial heat supply on the lower surface with curved surface having zero heat flux. Very recently Kedar and Deshmukh [10] studied the determination of thermal deflection in a semi infinite hollow circular cylinder subjected to ramp type heating on the lower face.

In the present paper one dimensional quasi-static problem of a temperature distribution, displacement and thermal stresses in infinite body with spherical cavity is considered. It is heated with arbitrary time dependent heat flux at inner traction free surface. The aim of the work is to obtain the mathematical model for predicting the temperature and stress field with constant and time dependent heat flux with initial constant temperature. The Laplace transform technique is used to obtain temperature distribution function. This` model is useful in having the knowledge of nature of deformation in the massive bodies in the field of engineering.

1. FORMULATION OF PROBLEM

Temperature distribution problem

Consider an infinite elastic body with spherical cavity $a \le r \le \infty$ at constant initial temperature T_i. The inner boundary surface at $r = a$ is exposed to arbitrary heat flux $Q(t)$. One assumes constant thermophysical properties of material. The mathematical formulation for the one dimensional unsteady state distribution of temperature $T(r,t)$,

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1.1}$$

Subjected to the following boundary condition

$$-k\frac{\partial T}{\partial r} = Q(t) \quad \text{on} \quad r = a \tag{1.2}$$

$$T(r,t) = T_i \quad \text{as} \quad r \to \infty \tag{1.3}$$

Initial condition

$$T(r,t) = T_i \quad \text{at} \quad t = 0 \tag{1.4}$$

where k is thermal conductivity and α is thermal diffusivity of the material.

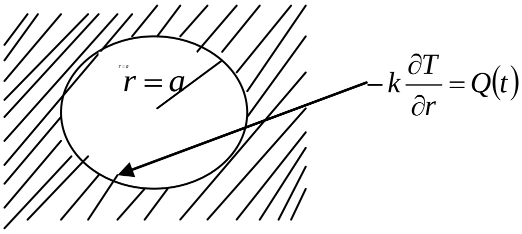

Fig.1: Geometry of infinite body with spherical cavity

Thermoelastic problem

Following Noda et al. [3], for one dimensional problem in spherical coordinate system, which means spherically symmetric problem, the displacement technique is extensively used. The properties in spherical coordinate ϕ and θ direction are identical and u denote the displacement in the radial direction, the strain-displacement relations are,

$$\varepsilon_{rr}=\frac{du}{dr},\ \varepsilon_{\theta\theta}=\frac{u}{r} \tag{1.5}$$

The corresponding thermo elastic stress-strain relation or Hooke's relations are

$$\sigma_{rr}=\lambda e+2\mu\varepsilon_{rr}-(3\lambda+2\mu)a_t\vartheta \tag{1.6}$$

$$\sigma_{\theta\theta}=\sigma_{\phi\phi}=\lambda e+2\mu\varepsilon_{\theta\theta}-(3\lambda+2\mu)a_t\vartheta \tag{1.7}$$

Where σ_{rr}, $\sigma_{\theta\theta}$ and $\sigma_{\phi\phi}$ are the stresses in the radial and tangential direction and ε_{rr}, $\varepsilon_{\theta\theta}$ and $\varepsilon_{\phi\phi}$ are strains in radial and tangential direction. ϑ is the temperature change obtained from the heat conduction equation (1.1), a_t is the coefficient of thermal expansion, e is the strain dilation and λ and μ are the Lame constants related to the modulus of elasticity E and the Poisson's ratio ν as,

$$\lambda=\frac{\nu E}{(1+\nu)(1-2\nu)},\ \mu=\frac{E}{2(1+\nu)} \tag{1.8}$$

The equilibrium equation in the radial direction, excluding the body force and the inertia term is,

$$r\frac{d\sigma_{rr}}{dr}+2(\sigma_{rr}-\sigma_{\theta\theta})=0 \tag{1.9}$$

The radial displacement and thermal stresses for infinite body with a spherical cavity are obtained as [3],

$$u=\frac{1+\nu}{1-\nu}a_t\frac{1}{r^2}\int_a^r\vartheta r^2dr=\frac{1+\nu}{1-\nu}a_t\frac{1}{r^2}F(r,t)$$

$$\sigma_{rr}=-\frac{2a_tE}{1-\nu}\frac{1}{r^3}\int_a^r\vartheta r^2dr=-\frac{2a_tE}{1-\nu}\frac{1}{r^2}F(r,t)$$

$$\sigma_{\theta\theta}=\sigma_{\phi\phi}=\frac{a_tE}{1-\nu}\left(\frac{1}{r^3}\int_a^r\vartheta r^2-\vartheta\right)=\frac{a_t}{1-\nu}\left(\frac{1}{r^3}F(r,t)-\vartheta\right) \tag{1.10}$$

Where $F(r,t)=\int_a^r\vartheta r^2dr$

The sphere is subjected to the traction free boundary condition

$$\sigma_{rr}=0 \quad \text{at} \quad r=a \tag{1.11}$$

Equations (1.1) to (1.11) constitute the mathematical formulation of the problem.

2. SOLUTIONS

Temperature distribution

To find the temperature distribution $T(r,t)$, let $\vartheta = T - T_i$, then the problem (1.1) to (1.4) transformed as,

$$\frac{\partial^2 \vartheta}{\partial r^2} + \frac{2}{r}\frac{\partial \vartheta}{\partial r} = \frac{1}{\alpha}\frac{\partial \vartheta}{\partial t} \tag{2.1}$$

$$-k\frac{\partial \vartheta}{\partial r} = Q(t) \qquad \text{on} \quad r = a \tag{2.2}$$

$$\vartheta(r,t) = 0 \qquad \text{at} \quad t = 0 \tag{2.3}$$

Introducing new variable as [3]

$$U(r,t) = r\vartheta(r,t), \qquad R = r - a \tag{2.4}$$

The problem (2.1) – (2.3) with new variables transformed to,

$$\frac{\partial^2 U}{\partial R^2} = \frac{1}{\alpha}\frac{\partial U}{\partial t} \tag{2.5}$$

$$\frac{\partial U}{\partial R} - \frac{U}{R+a} = -\frac{Q(t)(R+a)}{k} \qquad \text{on} \quad R = 0 \tag{2.6}$$

$$U(R,t) = 0 \qquad \text{at} \quad t = 0 \tag{2.7}$$

On applying Laplace transform to the equations (2.5) - (2.7) one obtains,

$$\frac{d^2\overline{U}}{dR^2} - \frac{p}{\alpha}\overline{U} = 0 \tag{2.8}$$

$$\frac{d^2\overline{U}}{dR^2} - m^2\overline{U} = 0 \tag{2.9}$$

Where $m^2 = \dfrac{p}{\alpha}$

The solution of differential equation (2.9) is obtained as,

$$\overline{U}(R,p) = Ae^{mR} + Br^{-mR}$$

$\overline{U}$ is finite as $R \to \infty$, therefore $A = 0$

$$\overline{U}(R,p) = Be^{-mR} \tag{2.10}$$

Applying the Laplace transform to equation (2.6),

$$\frac{d\overline{U}}{dR} - \frac{\overline{U}}{R+a} = \frac{-\overline{Q(p)}(R+a)}{k}$$ at $R = 0$ it becomes

$$\left(\frac{d\overline{U}}{dR}\right)_{R=0} - \frac{1}{a}\left(\overline{U}\right)_{R=0} = -\frac{a}{k}\overline{Q}(p) \tag{2.11}$$

From (2.10)

$$\left(\frac{d\overline{U}}{dR}\right)_{R=0} = -mB \quad \text{and} \quad \left(\overline{U}\right)_{R=0} = B \tag{2.12}$$

Therefore from (2.11)) one gets,

$$-mB-\frac{B}{a}=-\frac{a}{k}\overline{Q}(p)$$

$$B=\frac{a}{k}\frac{\overline{Q}(p)}{\left(m+\frac{1}{a}\right)} \tag{2.13}$$

Therefore from (2.10) we have,

$$\overline{U}=\frac{a}{k}\left[\overline{Q}(p)\frac{e^{-\sqrt{\frac{p}{\alpha}}R}}{\left(\sqrt{\frac{p}{\alpha}}\right)+\frac{1}{a}}\right] \tag{2.14}$$

The functional form of $\overline{Q}(p)$ is arbitrary and not explicitly specified, one can use the convolution property of the Laplace transform.

We write

$$\frac{k}{a}\overline{U}=\left[\overline{Q}(p)\frac{e^{-\sqrt{\frac{p}{\alpha}}R}}{\left(\sqrt{\frac{p}{\alpha}}\right)+\frac{1}{a}}\right]=L[Q(t)*g(R,t)] \tag{2.15}$$

where, $g(R,t)=L^{-1}\left(\frac{e^{-\sqrt{\frac{p}{\alpha}}R}}{\left(\sqrt{\frac{p}{\alpha}}+\frac{1}{a}\right)}\right)=\frac{1}{\sqrt{\pi\alpha t}}e^{-\frac{R^2}{4\alpha t}}-\frac{1}{a}e^{\frac{R}{a}}e^{\frac{\alpha t}{a^2}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{R}{2\sqrt{\alpha t}}\right)$ (2.16)

The inversion of (2.15) gives, $\frac{k}{a}U(R,t)=Q(t)*g(R,t)$

Using the definition of convolution $Q(t)*g(R,t)$

$$\frac{k}{a}U(R,t)=\int_0^t Q(\tau)g(R,t-\tau)d\tau \tag{2.17}$$

Replacing t by $(t-\tau)$ in (2.16) and introducing it into (2.17),

$$U(R,t)=\frac{a}{k}\int_0^t Q(\tau)\left[\frac{1}{\sqrt{\pi\alpha(t-\tau)}}e^{-\frac{R^2}{4\alpha(t-\tau)}}-\frac{1}{a}e^{\frac{R}{a}}e^{\frac{\alpha(t-\tau)}{a^2}}erfc\left(\frac{\sqrt{\alpha(t-\tau)}}{a}+\frac{R}{2\sqrt{\alpha(t-\tau)}}\right)\right]d\tau \tag{2.18}$$

Using (2.4) one gets,

$$r\vartheta(r,t)=\frac{a}{k}\int_0^t Q(\tau)\left[\frac{1}{\sqrt{\pi\alpha(t-\tau)}}e^{-\frac{(r-a)^2}{4\alpha(t-\tau)}}-\frac{1}{a}e^{\frac{(r-a)}{a}}e^{\frac{\alpha(t-\tau)}{a^2}}erfc\left(\frac{\sqrt{\alpha(t-\tau)}}{a}+\frac{r-a}{2\sqrt{\alpha(t-\tau)}}\right)\right]d\tau$$

$$\vartheta(r,t)=T-T_i=\frac{a}{k}\int_0^t Q(\tau)\left[\frac{1}{\sqrt{\pi\alpha(t-\tau)}}e^{-\frac{(r-a)^2}{4\alpha(t-\tau)}}-\frac{1}{a}e^{\frac{(r-a)}{a}}e^{\frac{\alpha(t-\tau)}{a^2}}erfc\left(\frac{\sqrt{\alpha(t-\tau)}}{a}+\frac{r-a}{2\sqrt{\alpha(t-\tau)}}\right)\right]d\tau$$

(2.19)

The equation (2.19) gives the temperature change ϑ and the temperature distribution function $T(r,t)$ is obtained as,

$$T(r,t)=T_i+\frac{a}{k}\int_0^t Q(\tau)\left[\frac{1}{\sqrt{\pi\alpha(t-\tau)}}e^{-\frac{(r-a)^2}{4\alpha(t-\tau)}}-\frac{1}{a}e^{\frac{(r-a)}{a}}e^{\frac{\alpha(t-\tau)}{a^2}}erfc\left(\frac{\sqrt{\alpha(t-\tau)}}{a}+\frac{r-a}{2\sqrt{\alpha(t-\tau)}}\right)\right]d\tau \qquad (2.20)$$

The equation (2.20) is the general solution for the transient temperature distribution for the infinite spherical body with spherical cavity.

Special cases

As a special case one sets the heat flux on the inner surface of spherical cavity of an infinite body as

1. $Q(t)=Q_0$, Constant heat flux

2. $Q(t)=Q_0 t^{-\frac{1}{2}}$, time dependent heat flux

Due to these heat fluxes, the temperature distribution obtained as follows

Case 1: Constant heat flux Q_0

Using Q_0 in (2.13), the temperature change ϑ is obtained as

$$\vartheta(r,t)=T-T_i=\left(\frac{a^2Q_0}{k\alpha}\right)\frac{1}{r}\left[-e^{\frac{r-a}{a}}e^{\frac{\alpha t}{a^2}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)\right] \qquad (2.21)$$

Hence the tem distributiperatureon $T(r,t)$ is obtained as,

$$T(r,t)=T_i+\left(\frac{a^2Q_0}{k\alpha}\right)\frac{1}{r}\left[-e^{\frac{r-a}{a}}e^{\frac{\alpha t}{a^2}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)\right] \qquad (2.22)$$

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Thermoelastic problem

Using (2.21) in (1.10), one can obtain the displacement and thermal stresses as,

$$u=\frac{1+\nu}{1-\nu}a_t\,\frac{a^2Q_0}{k\alpha}\times$$

$$\frac{1}{r^2}\left[\begin{array}{l}
-\left(a^2-2\alpha t\right)\left\{e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{\alpha t}{a^2}}erfc\left(\frac{\sqrt{\alpha t}}{a}\right)\right\}\\
-2a\sqrt{\alpha t}\left\{\begin{array}{l}
e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}-\frac{a}{2\sqrt{\alpha t}}\right)erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{\alpha t}{a^2}}\\
\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)erfc\left(\frac{\sqrt{\alpha t}}{a}\right)+\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)+\frac{1}{\sqrt{\pi}}\\
\left(1-e^{-\left(\frac{r-a}{2\sqrt{\alpha t}}\right)^2}\right)
\end{array}\right\}+\\
2\sqrt{\frac{\alpha t}{\pi}}\left(1-e^{-\frac{(r-a)^2}{4\alpha t}}\right)+erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)\left\{a(r-a)+\frac{(r-a)^2}{2}\right\}+\alpha t\,erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)\\
-\sqrt{\frac{\alpha t}{\pi}}(r-a)e^{-\frac{(r-a)^2}{4\alpha t}}
\end{array}\right] \tag{2.23}$$

$$\sigma_{rr}=-\frac{2a_tE}{1-\nu}\,\frac{a^2Q_0}{k\alpha}\times$$

$$\frac{1}{r^3}\left[\begin{array}{l}
-\left(a^2-2\alpha t\right)\left\{e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{\alpha t}{a^2}}erfc\left(\frac{\sqrt{\alpha t}}{a}\right)\right\}\\
-2a\sqrt{\alpha t}\left\{\begin{array}{l}
e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}-\frac{a}{2\sqrt{\alpha t}}\right)erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{\alpha t}{a^2}}\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)\\
erfc\left(\frac{\sqrt{\alpha t}}{a}\right)+\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)+\frac{1}{\sqrt{\pi}}\left(1-e^{-\frac{(r-a)^2}{4\alpha t}}\right)
\end{array}\right\}\\
+2\sqrt{\frac{\alpha t}{\pi}}\left(1-e^{-\frac{(r-a)^2}{4\alpha t}}\right)+erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)\left\{a(r-a)+\frac{(r-a)^2}{2}\right\}+\alpha t\,erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)-\\
\sqrt{\frac{\alpha t}{\pi}}(r-a)e^{-\frac{(r-a)^2}{4\alpha t}}
\end{array}\right] \tag{2.24}$$

$$\begin{aligned}
\sigma_{\theta\theta} = \sigma_{\phi\phi} = \frac{a_t E}{1-\nu}\frac{a^2 Q_0}{k\alpha} \times \Bigg[\frac{1}{r^3} \Bigg\{ & -\left(a^2 - 2\alpha t\right)\left[e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}} erfc\left(\frac{\sqrt{\alpha t}}{a} + \frac{r-a}{2\sqrt{\alpha t}}\right) + erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right) - e^{\frac{\alpha t}{a^2}} erfc\left(\frac{\sqrt{\alpha t}}{a}\right)\right] \\
& - 2a\sqrt{\alpha t}\Bigg[e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}\left(\frac{\sqrt{\alpha t}}{a} + \frac{r-a}{2\sqrt{\alpha t}} - \frac{a}{2\sqrt{\alpha t}}\right) erfc\left(\frac{\sqrt{\alpha t}}{a} + \frac{r-a}{2\sqrt{\alpha t}}\right) - \\
& \qquad e^{\frac{\alpha t}{a^2}}\left(\frac{\sqrt{\alpha t}}{a} - \frac{a}{2\sqrt{\alpha t}}\right) erfc\left(\frac{\sqrt{\alpha t}}{a}\right) + \left(\frac{\sqrt{\alpha t}}{a} - \frac{a}{2\sqrt{\alpha t}}\right) erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right) + \\
& \qquad \frac{1}{\sqrt{\pi}}\left(1 - e^{-\frac{(r-a)^2}{4\alpha t}}\right)\Bigg] \\
& + 2\sqrt{\frac{\alpha t}{\pi}}\left(1 - e^{-\frac{(r-a)^2}{4\alpha t}}\right) + \left[a(r-a) + \frac{(r-a)^2}{2}\right] erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right) + \alpha t\, erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right) \\
& - \sqrt{\frac{\alpha t}{\pi}}(r-a)e^{-\frac{(r-a)^2}{4\alpha t}} \Bigg\} \\
& - \frac{1}{r}\left[- e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}} erfc\left(\frac{\sqrt{\alpha t}}{a} + \frac{r-a}{2\sqrt{\alpha t}}\right) + erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)\right] \Bigg]
\end{aligned} \tag{2.25}$$

Case2: Time dependent heat flux $Q_0 t^{-\frac{1}{2}}$

Using this flux in (2.14), the temperature change ϑ is obtained

$$\vartheta(r,t) = T - T_i = \frac{aQ_0}{k}\sqrt{\frac{\pi}{\alpha}}\frac{1}{r}\left[e^{\frac{r-a}{a}} e^{\frac{\alpha t}{a^2}} erfc\left(\frac{\sqrt{\alpha t}}{a} + \frac{r-a}{2\sqrt{\alpha t}}\right)\right] \tag{2.26}$$

Hence the temperature distribution $T(r,t)$ reduced to

$$T(r,t) = T_i + \frac{aQ_0}{k}\sqrt{\frac{\pi}{\alpha}}\frac{1}{r}\left[e^{\frac{r-a}{a}} e^{\frac{\alpha t}{a^2}} erfc\left(\frac{\sqrt{\alpha t}}{a} + \frac{r-a}{2\sqrt{\alpha t}}\right)\right] \tag{2.27}$$

Using (2.26) in (1.10), one can obtain the displacement and thermal stresses as,

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$$u=\frac{1+\nu}{1-\nu}a_t\frac{aQ_0}{k}\sqrt{\frac{\pi}{\alpha}}\times$$

$$\frac{1}{r^2}\left[\begin{aligned}&\left(a^2-2\alpha t\right)\left\{e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{\alpha t}{a^2}}erfc\left(\frac{\sqrt{\alpha t}}{a}\right)\right\}\\&+2a\sqrt{\alpha t}\left\{\begin{aligned}&e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}-\frac{a}{2\sqrt{\alpha t}}\right)erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)\\&-e^{\frac{\alpha t}{a^2}}\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)etfc\left(\frac{\sqrt{\alpha t}}{a}\right)+\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)\\&+\frac{1}{\sqrt{\pi}}\left(1-e^{-\frac{(r-a)^2}{4\alpha t}}\right)\end{aligned}\right\}\end{aligned}\right] \tag{2.28}$$

$$\sigma_{rr}=-\frac{2a_t}{1-\nu}\frac{aQ_0}{k}\sqrt{\frac{\pi}{\alpha}}\times$$

$$\frac{1}{r^3}\left[\begin{aligned}&\left(a^2-2\alpha t\right)\left\{e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{\alpha t}{a^2}}erfc\left(\frac{\sqrt{\alpha t}}{a}\right)\right\}\\&+2a\sqrt{\alpha t}\left\{\begin{aligned}&e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}-\frac{a}{2\sqrt{\alpha t}}\right)erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{\alpha t}{a^2}}\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)\\&erfc\left(\frac{\sqrt{\alpha t}}{a}\right)+\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)+\frac{1}{\sqrt{\pi}}\left(1-e^{-\frac{(r-a)^2}{4\alpha t}}\right)\end{aligned}\right\}\end{aligned}\right] \tag{2.29}$$

$$\sigma_{\theta\theta}=\sigma_{\phi\phi}=\frac{a_tE}{1-\nu}\frac{aQ_0}{k}\sqrt{\frac{\pi}{\alpha}}\times$$

$$\left[\begin{aligned}&\frac{1}{r^3}\left\{\begin{aligned}&\left(a^2-2\alpha t\right)\left[e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{\alpha t}{a^2}}erfc\left(\frac{\sqrt{\alpha t}}{a}\right)\right]\\&+2a\sqrt{\alpha t}\left[\begin{aligned}&e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}-\frac{a}{2\sqrt{\alpha t}}\right)erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{\alpha t}{a^2}}\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)\\&erfc\left(\frac{\sqrt{\alpha t}}{a}\right)+\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)+\frac{1}{\sqrt{\pi}}\left(1-e^{-\frac{(r-a)^2}{4\alpha t}}\right)\end{aligned}\right]\end{aligned}\right\}\\&-\frac{1}{r}\left\{e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)\right\}\end{aligned}\right] \tag{2.30}$$

3. NUMERICAL AND GRAPHCAL ANALYSIS

Radius of spherical cavity $a = 0.2m$

Material properties

The numerical calculations have been obtained for Copper (Pure) with thermo mechanical properties,

Density $\rho = 8954 kg/m^3$

Specific heat $c_p = 383 J/(kgK)$

Thermal diffusivity $\alpha = 112.34 \times 10^{-6} m^2/s$

Thermal Conduction $k = 386 W/mK$

Poisson ratio $\nu = 0.35$

Coefficient of linear expansion $a_t = 16.5 \times 10^{-6}/K$

Constant heat flux $Q_0 = 100 J/sm^2$,

Initial temperature $T_i = 300K$

Modulus of elasticity $E = 117 GPa$

The computational mathematical software MATLAB has been used to carry out the numerical calculations and to obtain the graphs.

Graphical illustration

Case 1: While considering the constant heat flux on boundary surface of infinite body with spherical cavity the observations are made in terms of temperature, displacement and the thermal stresses are shown in following figures and illustrated as follows

In **fig. 2** the temperature distribution is depicted for $t = 5,10,15(s)$ with initial temperature $T_i = 300(K)$. Due to constant heat flux at inner surface of infinite body with cavity the temperature is large in the region near to the surface. The temperature increases with time and the temperature on the inner cavity surface is $355, 385\, and\, 388(K)$ at time $t = 5,10,15(s)$ respectively. It decreases to initial temperature in radial direction. **Fig.3** shows the variation of displacement with radius for constant flux. It is observed that the radial displacement increases along the radial direction and it also shows that there is a small raise with time also. **Fig.4** represents the radial stress distribution with radius. The radial stress is equal to zero at the boundary surface of infinite body, due to the assumed mechanical boundary condition. The stress is tensile in the region near to the surface which is subjected to the flux and it decreases in radial direction and reach to zero as r becomes sufficiently large. **Fig. 5** describes the tangential stress distribution for constant applied heat flux. The region near the surface of spherical cavity is in compression tangentially.

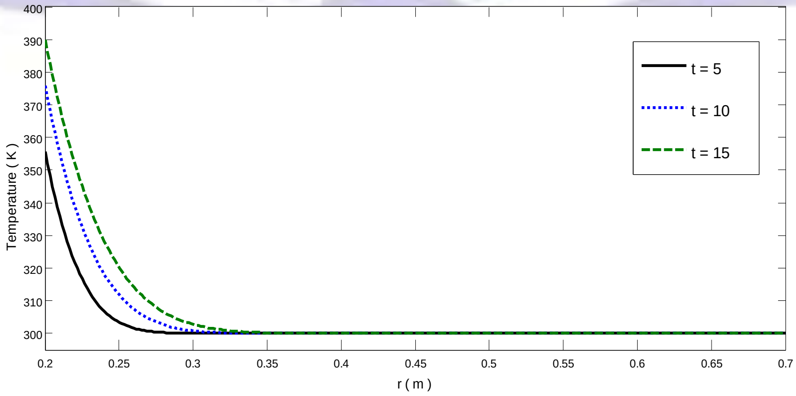

Fig 2: Temperature distribution for $t = 5,10,15(s)$, **heat flux** $Q_0 = 100\, J/sm^2$

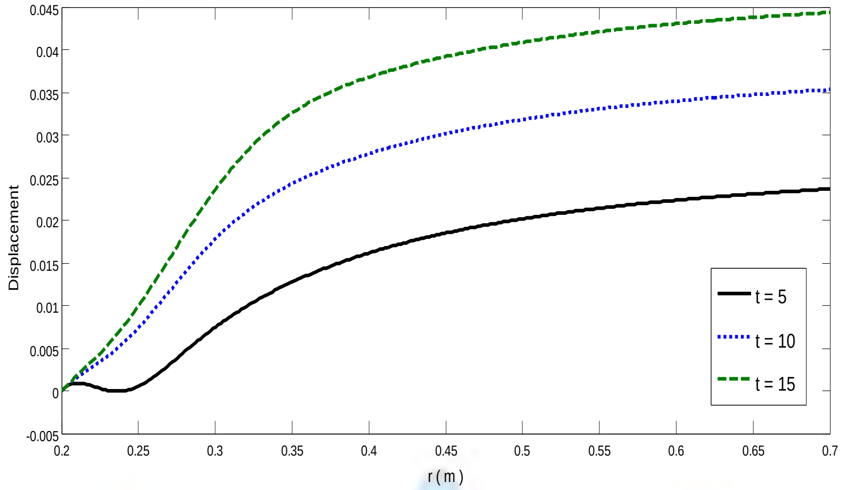

Fig 3: Radial Displacement for $t = 5,10,15(s)$, heat flux $Q_0 = 100\ J / sm^2$

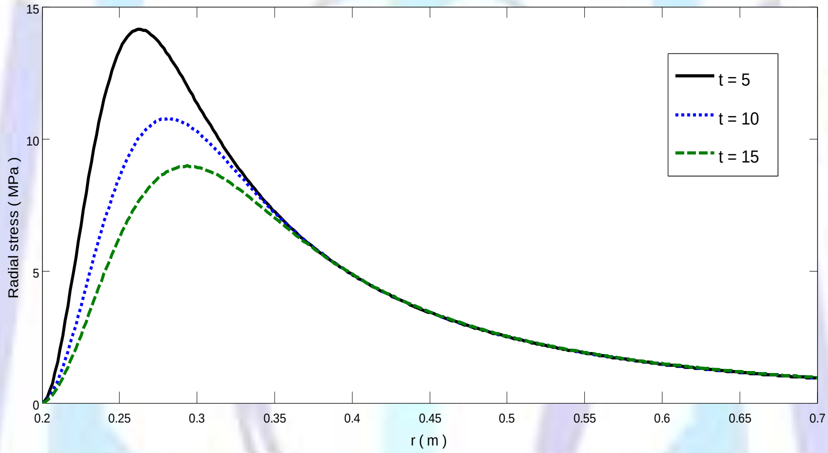

Fig 4: Radial stress distribution for $t = 5,10,15(s)$, heat flux $Q_0 = 100\ J / sm^2$

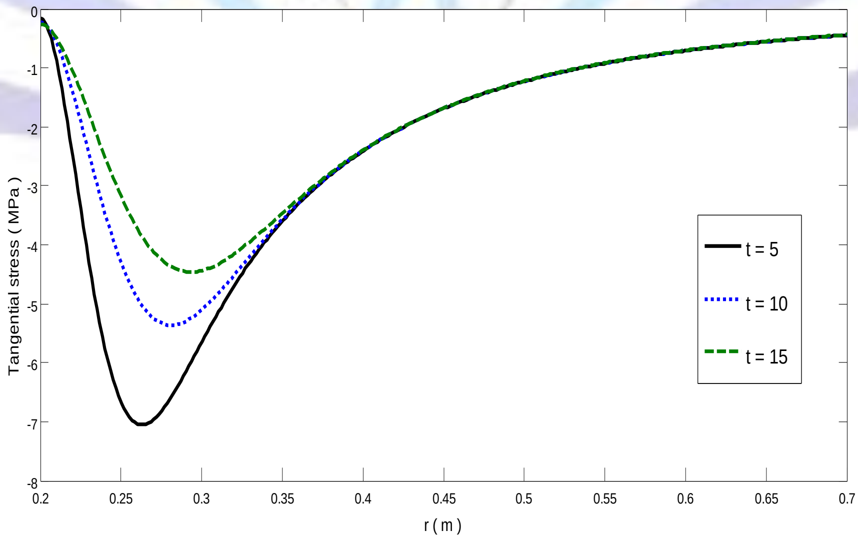

Figure 5 : Tangential Stress distribution for $t = 5,10,15(s)$, heat flux $Q_0 = 100\ J / sm^2$

Case 2: While considering the time dependent heat flux on boundary surface of infinite body with spherical cavity the observations are made in terms of temperature, displacement and the thermal stresses are shown in following figures and illustrated as follows.

Fig. 6 shows change in temperature with radius for the time dependent heat flux in present form. It is observed that there is a radial position where the temperature is identical irrespective of time. It is also observed that the temperature raise with time is small. The temperature rapidly decreases to initial value as radial distance increases from inner boundary surface. **Fig. 7** represents displacement distribution. The change of displacement is observed in the region near to cavity and it is interesting to note that displacement decreases with time. **Fig.8** shows the radial stress distribution and it is compressive in the region near to the surface. It increases with time for heat flux in the present form. **Fig. 9** shows the tangential stress distribution and is compressive in the region as shown. It is observed that the stress is very small on the surface where the cavity is occurring.

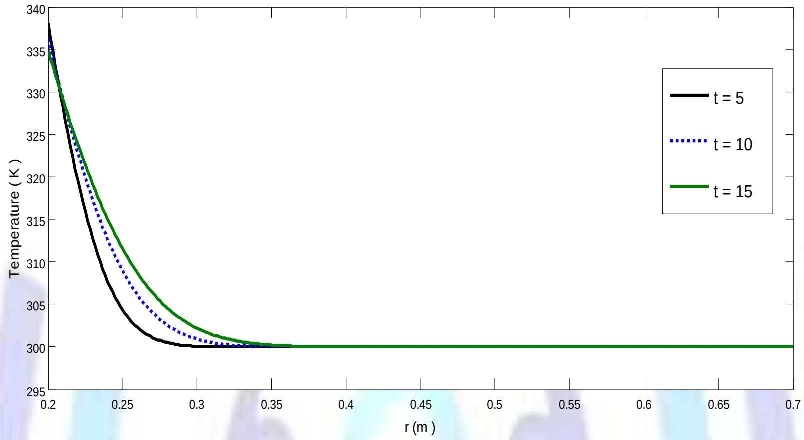

Fig 6: Temperature distribution for $t=5,10,15(s)$, **heat flux** $Q(t)=100\,t^{-\frac{1}{2}}\ J/sm^2$

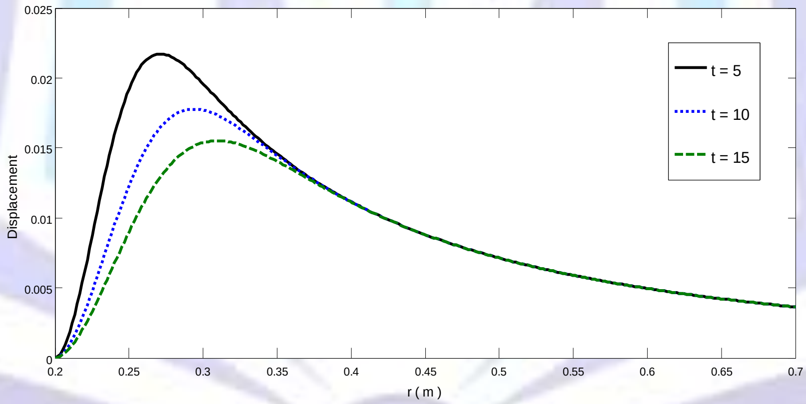

Fig 7: Radial Displacement for $t=5,10,15(s)$, **heat flux** $Q(t)=100\,t^{-\frac{1}{2}}\ J/sm^2$

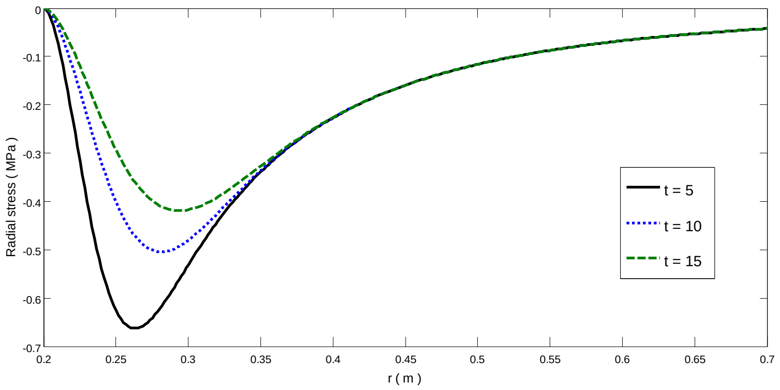

Fig 8: Radial stress distribution for for $t = 5,10,15(s)$**, heat flux** $Q(t) = 100\,t^{-\frac{1}{2}}\ J/sm^2$

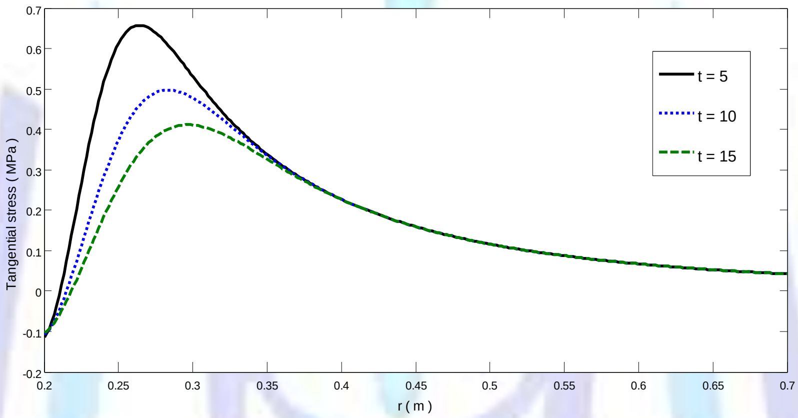

Fig 9: Tangential stress distribution for $t = 5,10,15(s)$**, heat flux** $Q(t) = 100\,t^{-\frac{1}{2}}\ J/sm^2$

4. CONCLUSION

In this study the exact analytical solutions are obtained for temperature, displacement and thermal stresses for isotropic and homogeneous infinite body with a spherical cavity heated with arbitrary heat flux at the boundary surface. As a special case, mathematical model is constructed for copper (pure) large body with spherical cavity with a material properties as specified in the numerical calculations.

The problem is discussed independently with two cases of constant and time dependent heat flux. The temperature distribution with radius for these cases gives very significant results. For constant heat flux, the temperature attains considerable varying values for prescribed time on the boundary surface between $355\,K\ to\ 388\,K$ and it decreases along the radial direction. For time dependent heat flux the variation on the boundary surface is very small and there is a radial position in the region near the boundary surface where the temperature is identical for various prescribed times.

The displacement distribution is studied for these cases and it is observed that for constant heat flux it increases for small span of time. While for time dependent heat flux in the present form it decreases. The radial stress is tensile for constant flux and compressive for time dependent flux. Tangential stress changes from compressive to tensile with change in the flux from constant to time dependent. The stress plots are mirror reflection of each other. The effect of displacement, radial and tangential stresses diminishes along the radial direction away from boundary surface of body. The results are obtained for massive copper material and can be modified by changing the form of flux function in the field of engineering for fabrication of different massive materials.

5. APPENDIX

Case1

$$\vartheta(r,t) = T - T_i = \left(\frac{a^2 Q_0}{k\alpha}\right)\frac{1}{r}\left[-e^{\frac{r-a}{a}}\, e^{\frac{\alpha t}{a^2}}\, erfc\left(\frac{\sqrt{\alpha t}}{a} + \frac{r-a}{2\sqrt{\alpha t}}\right) + erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)\right]$$

The value of $F(r,t)=\int_a^r \vartheta r^2 dr$ is obtained as,

$$F(r,t)=\frac{a^2Q_0}{k\alpha}\int_a^r\left[-e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)\right]rdr$$

$$=\left(\frac{a^2Q_0}{k\alpha}\right)\left[-e^{\frac{\alpha t}{a^2}}\int_a^r re^{\frac{r-a}{a}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)dr+\int_a^r rerfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)dr\right] \quad (5.1)$$

Now consider the first integral in the bracket on R. H. S. of (5.1)

$$\int_a^r re^{\frac{r-a}{a}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)dr \quad (5.2)$$

Put $\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)=m$, $dr=2\sqrt{\alpha t}dm$, $r=a\rightarrow m=\frac{\sqrt{\alpha t}}{a}$ and

$r=r\rightarrow m=\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}$ and $r=a+2\sqrt{\alpha t}\left(m-\frac{\sqrt{\alpha t}}{a}\right)$, Thus (5.2) becomes

$$\int_{\frac{\sqrt{\alpha t}}{a}}^{\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}}\left[a+2\sqrt{\alpha t}\left(m-\frac{\sqrt{\alpha t}}{a}\right)\right]e^{\frac{2\sqrt{\alpha t}\left(m-\frac{\sqrt{\alpha t}}{a}\right)}{a}}erfc(m)2\sqrt{\alpha t}dm$$

$$=2\sqrt{\alpha t}e^{-\frac{2\alpha t}{a^2}}\int_{\frac{\sqrt{\alpha t}}{a}}^{\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}}\left[\left(a-\frac{2\alpha t}{a}\right)+2\sqrt{\alpha t}m\right]e^{\frac{2\sqrt{\alpha t}}{a}m}erfc(m)dm$$

$$2\sqrt{\alpha t}e^{-\frac{2\alpha t}{a^2}}\left[\left(a-\frac{2\alpha t}{a}\right)\int_{\frac{\sqrt{\alpha t}}{a}}^{\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}}e^{\frac{2\sqrt{\alpha t}}{a}m}erfc(m)dm+2\sqrt{\alpha t}\int_{\frac{\sqrt{\alpha t}}{a}}^{\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}}me^{\frac{2\sqrt{\alpha t}}{a}}erfc(m)dm\right] \quad (5.3)$$

Now consider the first integral in the bracket on R.H.S. of eq. (5.3) and using the integral in [7], one obtains

$$\left[\int_{\frac{\sqrt{\alpha t}}{a}}^{\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}}e^{\frac{2\sqrt{\alpha t}}{a}m}erfc(m)dm\right]$$

$$=\frac{a}{2\sqrt{\alpha t}}\left[e^{\frac{2\alpha t}{a^2}+\frac{r-a}{a}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+e^{\frac{\alpha t}{a^2}}erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{2\alpha t}{a^2}}erfc\left(\frac{\sqrt{\alpha t}}{a}\right)\right] \quad (5.4)$$

Now consider the second integral in the bracket on the R. H. S. of eq.(5.3)

$$\int_{\frac{\sqrt{\alpha t}}{a}}^{\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}} m\, e^{\frac{2\sqrt{\alpha t}}{a}m}\, erfc(m)dm$$

$$=\frac{a}{2\sqrt{\alpha t}}\left[erfc(m)e^{\frac{2\sqrt{\alpha t}}{a}m}\left(m-\frac{a}{2\sqrt{\alpha t}}\right)+e^{\frac{\alpha t}{a^2}}\left\{\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)erf\left(m-\frac{\sqrt{\alpha t}}{a}\right)-\frac{1}{\sqrt{\pi}}e^{-\left(m-\frac{\sqrt{\alpha t}}{a}\right)^2}\right\}\right]$$

Putting the value of m and solving one gets

$$=\frac{a}{2\sqrt{\alpha t}}\left[\begin{array}{l}\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}-\frac{a}{2\sqrt{\alpha t}}\right)erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+e^{\frac{\alpha t}{a^2}}\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)\\ -e^{\frac{\alpha t}{a^2}}\frac{1}{\sqrt{\pi}}e^{-\frac{(r-a)^2}{4\alpha t}}-e^{\frac{2\alpha t}{a^2}}\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right)erfc\left(\frac{\sqrt{\alpha t}}{a^2}\right)+\frac{1}{\sqrt{\pi}}e^{\frac{\alpha t}{a^2}}\end{array}\right] \tag{5.5}$$

Now consider the following integral from equation (5.1)

$$\int_a^r r\, erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)dr \tag{5.6}$$

Put $\frac{r-a}{2\sqrt{\alpha t}}=m$, $dr=2\sqrt{\alpha t}\,dm$

When $r=a$, $m=0$ and $r=r, m=\frac{r-a}{2\sqrt{\alpha t}}, r=a+2\sqrt{\alpha t}m$

Integral (5.6) becomes,

$$\int_0^{\frac{r-a}{2\sqrt{\alpha t}}} 2\sqrt{\alpha t}\left(a+2\sqrt{\alpha t}m\right)erfc(m)dm$$

$$=2\sqrt{\alpha t}\left[a\int_0^{\frac{r-a}{2\sqrt{\alpha t}}} erfc(m)dm+2\sqrt{\alpha t}\int_0^{\frac{r-a}{2\sqrt{\alpha t}}} m\,erfc(m)dm\right] \tag{5.7}$$

Consider first integral in the bracket on R. H. S. of (5.7)

$$a\int erfc(m)dm$$

$$=a[merfc(m)]-\frac{1}{\sqrt{\pi}}e^{-m^2}$$

Using the limits of integration

$$a\left[\frac{r-a}{2\sqrt{\alpha t}}erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)+\frac{1}{\sqrt{\pi}}\left(1-e^{-\frac{(r-a)^2}{4\alpha t}}\right)\right] \tag{5.8}$$

Now consider the second integral in the bracket on R. H. S. of (5.7)

$$\int_0^{\frac{r-a}{2\sqrt{\alpha t}}} m\,erfc(m)dm = \left[\frac{1}{2}m^2 erfc(m) + \frac{1}{4}erf(m) - \frac{m}{2\sqrt{\pi}}e^{-m^2}\right]_{m=0}^{m=\frac{r-a}{2\sqrt{\alpha t}}}$$

$$= \frac{1}{2}\frac{(r-a)^2}{4\alpha t}erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right) + \frac{1}{2}erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right) - \frac{r-a}{4\sqrt{\alpha t}\sqrt{\pi}}e^{-\frac{(r-a)^2}{4\alpha t}} \quad (5.9)$$

Using (5.8) and (5.9) in (5.7) and simplifying

$$2\sqrt{\alpha t}\left[a\int_0^{\frac{r-a}{2\sqrt{\alpha t}}} erfc(m)dm + 2\sqrt{\alpha t}\int_0^{\frac{r-a}{2\sqrt{\alpha t}}} m\,erfc(m)dm\right]$$

$$= 2\sqrt{\frac{\alpha t}{\pi}}\left(1 - e^{-\frac{(r-a)^2}{4\alpha t}}\right) + \left[a(r-a) + \frac{(r-a)^2}{2}\right]erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right) \quad (5.10)$$

$$+\alpha t\, erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right) - \sqrt{\frac{\alpha t}{\pi}}(r-a)e^{-\frac{(r-a)^2}{4\alpha t}}$$

Using (5.4) and (5.5) in (5.3),

$$2\sqrt{\alpha t}e^{-\frac{2\alpha t}{a^2}}\left[\left(a - \frac{2\alpha t}{a}\right)\int_{\frac{\sqrt{\alpha t}}{a}}^{\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}} e^{\frac{2\sqrt{\alpha t}}{a}m} erfc(m)dm + 2\sqrt{\alpha t}\int_{\frac{\sqrt{\alpha t}}{a}}^{\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}} m e^{\frac{2\sqrt{\alpha t}}{a}} erfc(m)dm\right] =$$

$$= \left(a^2 - 2\alpha t\right)\left[e^{\frac{r-a}{a}} erfc\left(\frac{\sqrt{\alpha t}}{a} + \frac{r-a}{2\sqrt{\alpha t}}\right) + e^{-\frac{\alpha t}{a^2}} erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right) - erfc\left(\frac{\sqrt{\alpha t}}{a}\right)\right]$$

$$+2a\sqrt{\alpha t}\left[\begin{array}{l} e^{\frac{r-a}{a}}\left(\frac{\sqrt{\alpha t}}{a} + \frac{r-a}{2\sqrt{\alpha t}} - \frac{a}{2\sqrt{\alpha t}}\right)erfc\left(\frac{\sqrt{\alpha t}}{a} + \frac{r-a}{2\sqrt{\alpha t}}\right) - \left(\frac{\sqrt{\alpha t}}{a} - \frac{a}{2\sqrt{\alpha t}}\right)erfc\left(\frac{\sqrt{\alpha t}}{a}\right) \\ + e^{-\frac{\alpha t}{a^2}}\left(\frac{\sqrt{\alpha t}}{a} - \frac{a}{2\sqrt{\alpha t}}\right)erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right) + \frac{1}{\sqrt{\pi}}e^{-\frac{\alpha t}{a^2}}\left(1 - e^{-\frac{(r-a)^2}{4\alpha t}}\right)\end{array}\right] \quad (5.11)$$

Therefore using (5.10)and(5.11) in(5.1) one gets,

$$F(r,t) = \left(\frac{a^2 Q_0}{k\alpha}\right)\left[-e^{\frac{\alpha t}{a^2}}\int_a^r re^{\frac{r-a}{a}} erfc\left(\frac{\sqrt{\alpha t}}{a} + \frac{r-a}{2\sqrt{\alpha t}}\right)dr + \int_a^r r erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)dr\right]$$

$$
= \frac{a^2 Q_0}{k\alpha}\left[\begin{array}{l}
-\left(a^2-2\alpha t\right)\left\{e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}\, erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{\alpha t}{a^2}}\, erfc\left(\frac{\sqrt{\alpha t}}{a}\right)\right\} \\
-2a\sqrt{\alpha t}\left\{\begin{array}{l}
e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}-\frac{a}{2\sqrt{\alpha t}}\right) erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right) \\
-e^{\frac{\alpha t}{a^2}}\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right) erfc\left(\frac{\sqrt{\alpha t}}{a}\right)+\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right) erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right) \\
+\frac{1}{\sqrt{\pi}}\left(1-e^{-\frac{(r-a)}{4\alpha t}}\right)
\end{array}\right\} \\
+2\sqrt{\frac{\alpha t}{\pi}}\left(1-e^{-\frac{(r-a)^2}{4\alpha t}}\right)+erfc\left(\frac{r-a}{2\sqrt{\alpha t}}\right)\left[a(r-a)+\frac{(r-a)^2}{2}\right]+\alpha t\, erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right) \\
-\sqrt{\frac{\alpha t}{\pi}}(r-a)e^{-\frac{(r-a)^2}{4\alpha t}}
\end{array}\right] \tag{5.12}
$$

CASE 2

$$
\vartheta(r,t)=T-T_i=\frac{aQ_0}{k}\sqrt{\frac{\pi}{\alpha}}\frac{1}{r}\left[e^{\frac{r-a}{a}}e^{\frac{\alpha t}{a^2}}erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)\right]
$$

$$
F(r,t)=\int_a^r \vartheta r^2 dr=\frac{aQ_0}{k}e^{\frac{\alpha t}{a^2}}\sqrt{\frac{\pi}{\alpha}}\int_a^r re^{\frac{r-a}{a}}\, erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)dr \tag{5.13}
$$

The solution of integral in (5.13) already obtained in case 1 by equation (5.3) and hence,

$$
F(r,t)=\frac{aQ_0}{k}\sqrt{\frac{\pi}{\alpha}}\left[\begin{array}{l}
\left(a^2-2\alpha t\right)\left\{e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}\, erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right)+erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right)-e^{\frac{\alpha t}{a^2}}\, erfc\left(\frac{\sqrt{\alpha t}}{a}\right)\right\} \\
+2a\sqrt{\alpha t}\left\{\begin{array}{l}
e^{\frac{\alpha t}{a^2}+\frac{r-a}{a}}\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}-\frac{a}{2\sqrt{\alpha t}}\right) erfc\left(\frac{\sqrt{\alpha t}}{a}+\frac{r-a}{2\sqrt{\alpha t}}\right) \\
-e^{\frac{\alpha t}{a^2}}\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right) erfc\left(\frac{\sqrt{\alpha t}}{a}\right)+\left(\frac{\sqrt{\alpha t}}{a}-\frac{a}{2\sqrt{\alpha t}}\right) erf\left(\frac{r-a}{2\sqrt{\alpha t}}\right) \\
+\frac{1}{\sqrt{\pi}}\left(1-e^{-\frac{(r-a)^2}{4\alpha t}}\right)
\end{array}\right\}
\end{array}\right] \tag{5.14}
$$

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