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Fixed point theorems of nonlinear contractions on *p*-quasi-cone metric space

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ABSTRACT

In this paper we have proved some results of fixed point on *p*-quasi cone metric spaces. The *p*-quasi cone metric space is a generalization cone metric space. Kiany and Amini-Harandi[1] have given a generalization of Ciric contraction[2]. In this paper we give a generalization of result [2] for *p*-quasi-cone metric space.

Keywords : ***p*-quasi-cone metric space; nonlinear contractive mapping; fixed point**

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INTRODUCTION

There has been a number of generalizations of metric space. One such generalization is a cone metric space. Huang and Zhang [3] have introduced the concept of cone metric spaces replacing the set of real numbers by an ordered Banach space. Abdeljaward and Karapinar [5] and Sonmez [6] have given a definition of quasi-cone metric spaces. Shaddadand Noorani [4] have established four kinds of Cauchy sequences in this space and proved some fixed point theorems in quasi-cone metric spaces without using normality condition.

In this paper, we have introduced the concept of*p*-quasi-cone metric space for*p* ≥1 which is a generalization of quasi-metric spaces where *p*=1. Also we have proved some new fixed point results in *p*-quasi-metric spaces using contractive conditions which generalize the results of Kiany and Amini-Harandi[1] and Lj.B. Ciric[2]. We don't take the cone normal and we don't use the continuity of the function.

Now we recall some known notions, definitions and results which are used in this paper.

PRELIMINARIES

Definition 1.[3] Let E be a real Banach space and P be a subset of E. P is called a *cone* if and only if

(i) P is closed, $P\neq\emptyset$, $P\neq \{0\}$;

(ii) for all x, y□$P\Rightarrow\alpha x + \beta y\in P$, where α, $\beta\in R^+$;

(iii) x□P and–x□P□$x = 0$.

For a given cone $P\subset E$, we can define a partial ordering with respect to P by x□y if and only if $y–x$□P. The denote$x<y$ will stand for x□y and x□y, while x□y will stand for $y–x$□ int$P$, where int$P$ denotes the interior of P.

The cone P is called normal if there is a number $k>0$ such that $0\leq x\leq y\Rightarrow ||x||\leq k||y||$, for all x, $y\in E$. The least positive k satisfying this is called the normal constant of P. The cone P is called regular if every increasing sequence which is bounded above is convergent; that is if x_n is a sequence such that $x_1\leq x_1\leq x_2\leq\cdots\leq x_n\leq\cdots\leq y$, for some $y\in E$, then there is $x\in E$ such that $||x_n–x|| \to 0$ as $n\to\infty$. Equivalently, the cone P is regular if every sequence which is bounded below is convergent.

Definition 2.[3] Let Xbe a nonempty set. Suppose the mapping $d:X\times X\to E$ satisfies

(i) $0\leq d(x, y)$ for all x, $y\in P$, and $d(x, y) = 0$ iff$x = y$;

(ii) $d(x, y) = d(y, x)$ for all x, $y\in X$;

(iii) $d(x, y)\leq d(x, z) + d(z, y)$, for all x, y, z□X.

Then, d is called a *cone metric* on X and (X, d) is called a *cone metric space*.

Definition 3.[5], [6] Let X be a nonempty set. Suppose the mapping $q:X\times X\to E$ satisfies

(i) $0\leq q(x, y)$ for all x, $y\in X$,

(ii) $q(x, y) = 0$ if and only if $x = y$;

(iii) $q(x, y)\leq q(x, z) + q(z, y)$ for all x, y, z□X.

Then q is called a *quasi-cone metric* on X, and (X, q) is called a *quasi-cone metric space*.

Now, we state our definition which is more general than quasi-cone metric space.

Definition 4. Let Xbe a nonempty set and $p\geq 1$. Suppose the mapping $q_p:X\times X\to E$ satisfies

(i) $0\leq q_p(x, y)$ for all x, y, □ □X,

(ii) $q_p(x, y) = 0$ if and only if $x = y$,

(iii) $q_p(x, z)\leq p(q_p(x, y) + q_p(y, z))$ for all x, y, z□X.

Then q_p is called a p-quasi-cone metric on X, and (X, q_p) is called a *p-quasi-cone metric space*.

Example 1. LetLet$X = (0, \infty)$, $E = R^2$, $P = \{(x, y), x, y\in R^+\}$and $q_1 : X\times X\to E$ defined by

$$q_1(x,y)=\begin{cases}(x-y,\alpha(x-y)),\ x>y\\ (0,0),\ x<y\end{cases}\text{, where}\alpha\in R^+;.$$

Remark1. Note that any cone metric space is a *p*-quasi-cone metric space. Some of definitions in *p*-quasi-cone metric space are restrictions of definition in cone metric space.

Now we introduce the appropriate generalization in *p*-quasi-cone metric spaces by considering the established notions in quasi metric spaces.

Definition 5. [4] Let Let (X, q_p) be a p-quasi-cone metric space. A sequence $\{x_n\}$ in X is called

(i) p-bi Cauchy if for each $c \in \mathrm{int}P$, there is $n_0 \in N$ such that $q_p(x_n, x_m) \ll c$ for all m, $n \geq n_0$.

(ii) p-right (left) Cauchy if for each $c \in \mathrm{int}P$, there is $n_0 \in N$ such that $q_p(x_n, x_m) \ll c$ ($q_p(x_m, x_n) \ll c$ resp.) for all $n \geq m \geq n_0$;

(iii) p-weakly right (left) Cauchy if for each $c \in \mathrm{int}P$, there is $n_0 \in N$ such that $q_p(x_n, x_{n_0}) \ll c$ ($q_p(x_{n_0}, x_n) \ll c$ resp.) for all $n \geq n_0$;

(iv) p-right (left) q_p-Cauchy if for each $c \in \mathrm{int}P$, there exist $x \in X$ and $n_0 \in N$ such that $q_p(x_n, x) \ll c$ ($q_p(x, x_n) \ll c$ resp.) for all $n \geq n_0$.

Remark 2. These notions in p-quasi-cone metric space are related in this way:

(i) p-bi-Cauchy $\Rightarrow p$-right (left) Cauchy $\Rightarrow p$-weakly right (left) Cauchy $\Rightarrow p$-right (left) q_p-Cauchy

(ii) a sequence is p-bi-Cauchy if and only if it is both p- left and p-right Cauchy.

We use the notion of p-right Cauchy in this paper.

Definition 6. Let (X, q_p) be a p-quasi-cone metric space. Let $\{x_n\}_{n \in N}$ be a sequence in X. We say that the sequence $\{x_n\}_{n \in N}$ *p-right converges* to $x \in X$ if $q_p(x, x_n) \to 0$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$.

Definition 7. A p-quasi-cone metric space (X, q_p) is called p- *complete* if every p- Cauchy sequence in X converges.

Definition 8. Let (X, q_p) be a p-quasi-cone metric space. A function $T: X \to X$ is called

(i) *continuous* if for any p-right convergent sequence $\{x_n\}_{n \in N}$ in X with $\lim_{n\to\infty} x_n = x$, the sequence $\{Tx_n\}_{n \in N}$ is right convergent and $\lim_{n\to\infty} Tx_n = Tx$.

Definition 9. Let $O(x) = \{x, Tx, T^2x, \ldots\}$ where $x \in X$. The set $O(x)$ is called *orbit* of x.

Definition 10. Let $M \subseteq X$. $\delta(M) = \sup\{q_p(x, y), x, y \in M\}$ is called *diameter* of M.

The orbit $O(x)$ is called bounded if there exist a $c \in P$, $\delta(O(x)) \leq c$.

MAIN RESULTS

In this section, we prove some fixed point results in p-quasi-cone metric space. Firstly we have given a theorem which is a generalization of Kiany and Amini-Harandi [1] due to p-quasi-cone metric space. In this theorem we don't use the normality of cone and we don't take the function $T: X \to X$ continuous.

Lemma 1. Let (X, q_p) be a p-quasi-cone metric space and $\{x_n\}_{n \in N}$ a sequence in X. Suppose there exist a sequence $\{[\alpha(c)]^n\}_{n \in N}$, where$\alpha$: $P \to \left(0, \frac{1}{p}\right)$ such that $\lim_{n\to\infty}[\alpha(c)]^n = 0$ in which $q_p(x_{n+1}, x_n) \leq [\alpha(c)]^n M$ for some $M \in P$, and for all $n \in N$. Then the sequence $\{x_n\}_{n \in N}$ is p-right Cauchy in (X, q_p).

Proof. For $n \in N$ we get

$$q_p(x_{n+k}, x_n) \leq p^k q_p(x_{n+k}, x_{n+k-1}) + p^{k-1} q_p(x_{n-k-1}, x_{n-k-2}) + p^{k-2} q_p(x_{n-k-2}, x_{n-k-3}) + \ldots + p q_p(x_{n+1}, x_n) \leq$$

$$\leq p^k[\alpha(c)]^{n+k} M + p^{k-1}[\alpha(c)]^{n+k-1} M + \ldots + p^2[\alpha(c)]^{n+1} M + p[\alpha(c)]^n M$$

$$= M(p^k[\alpha(c)]^{n+k-1} + p^{k-1}[\alpha(c)]^{n+k-2} + \ldots + p^2[\alpha(c)]^{n+1} + p[\alpha(c)]^n)\frac{p^{n-1}}{p^{n-1}}$$

$$= \frac{M}{p^{n-1}}\{(p[\alpha(c)])^{n+k-1} + (p[\alpha(c)])^{n+k-2} + \ldots + (p[\alpha(c)])^{n+1} + (p[\alpha(c)])^n\}$$

$$= \frac{M}{p^{n-1}} \frac{(p[\alpha(c)])^n(1-(p[\alpha(c)])^k)}{1-p\alpha(c)} \leq \frac{Mp}{1-p\alpha(c)}[\alpha(c)]^n \leq A[\alpha(c)]^n$$

where $A = \dfrac{Mp}{1-p\alpha(c)}$.

Let $a \in \mathrm{int}P$ and choose $\delta > 0$ such that $a + N_\delta(0) \subset P$, where$N_\delta(0) = \{y \in E, \|y\| < \delta\}$. Since, $\lim_{n\to\infty} [\alpha(c)]^n = 0$ there exist a natural number n_0 such that for$n \geq n_0$, $A[\alpha(c)]^n \in N_\delta(0)$, also $-A[\alpha(c)]^n \in N_\delta(0)$.

Since $a + N_\delta(0)$ is open, therefore $a + N_\delta(0) \subset P$, that is $a - A[\alpha(c)]^n \in \text{int}P$. Thus $A[\alpha(c)]^n$ ≪ a for $n \geq n_0$ and so $q_p(x_{n+k}, x_n)$ ≪ a for $n \geq n_0$. Thus, $\{x_n\}_{n\in N}$ is p-right Cauchy sequence.

Definition7. A p-quasi-cone metric space (X, q_p) is Hausdorff if for each pair x_1, x_2 of distinct points of X, there exist neighborhoods V_1, V_2 of x_1, x_2 respectively, they are disjoint.

Now we state a fixed point theorem using a nonlinear contraction condition.

Theorem1. Let (X, q_p) be a p- complete Hausdorff p-quasi-cone metric space and let $X\to X$ be a function that satisfies the nonlinear contraction condition:

(1) $q_p(T(x),T(y)) \leq \alpha(q_p(x,y)) \max\left\{q_p(x,y), q_p(T(x),x), q_p(T(y),y), q_p(T(x),y), q_p(x,T(y))\right\}$

for all $x, y \in X$, where $\alpha: P \to \left(0, \frac{1}{p}\right)$ such that:

1. $\lim_{n\to\infty}[\alpha(c)]^n = 0$, for every $c \in P$, $p \geq 1$.

Let $x_0 \in X$ such that $O(x_0)$ is bounded. Then T has a unique fixed point $x^* \in X$ and the sequence $\{T^n x_0\}_{n\in N}$ is convergent to x^*.

Proof. We have that the orbit $O(x_0)$ is bounded. So $\delta(O(x_0)) \leq c \in P$. We prove now that $q_p(T^{n+1}(x_0), T^n(x_0)) \leq c[\alpha(c)]^n$.

For $n=1$, $q_p(T^2x_0, Tx_0) = q_p(T(T(x_0)), T(x_0))$

$$\leq \alpha(q_p(Tx_0, x_0)) \max\left\{q_p(Tx_0,x_0), q_p(T^2x_0,Tx_0), q_p(Tx_0,x_0), q_p(T^2x_0,x_0), q_p(Tx_0,Tx_0)\right\}$$

$$= \alpha(q_p(Tx_0, x_0)) \max\{q_p(Tx_0,x_0), q_p(T^2x_0,Tx_0), q_p(T^2x_0,x_0)\}.$$

Due to $O(x_0)$ is bounded, we have that:

$$\max\{q_p(Tx_0,x_0), q_p(T^2x_0,Tx_0), q_p(T^2x_0,x_0)\} \leq c.$$

Also, as $q_p(Tx_0,x_0) \leq c \Rightarrow \alpha(q_p(Tx_0,x_0)) \leq \alpha(c)$. So $q_p(T^2x_0,Tx_0) \leq c[\alpha(c)]^1$.

Suppose it is true for $k<n$, $q_p(T^{k+1}x_0, T^kx_0) \leq c[\alpha(c)]^k$.

Let it prove for n.

$$q_p(T^{n+1}x_0, T^nx_0) = q_p(T(T^nx_0), T(T^{n-1}x_0))$$

$$\leq \alpha(q_p(T^nx_0, T^{n-1}x_0)) \max\{q_p(T^nx_0,T^{n-1}x_0), q_p(T^{n+1}x_0,T^nx_0), q_p(T^nx_0,T^{n-1}x_0), q_p(T^{n+1}x_0,T^{n-1}x_0), q_p(T^nx_0,T^nx_0)\}$$

$$= \alpha(q_p(T^nx_0, T^{n-1}x_0)) \max\{q_p(T^nx_0,T^{n-1}x_0), q_p(T^{n+1}x_0,T^nx_0), q_p(T^{n+1}x_0,T^{n-1}x_0)\}$$

Case 1. $\max\{q_p(T^nx_0,T^{n-1}x_0), q_p(T^{n+1}x_0,T^nx_0), q_p(T^{n+1}x_0,T^{n-1}x_0)\} = q_p(T^nx_0,T^{n-1}x_0)$.

$$q_p(T^{n+1}x_0,T^nx_0) \leq \alpha(q_p(T^nx_0,T^{n-1}x_0))q_p(T^nx_0,T^{n-1}x_0) \leq \alpha(q_p(T^nx_0,T^{n-1}x_0))c[\alpha(c)]^{n-1} \leq c[\alpha(c)]^n.$$

Case2. $\max\{q_p(T^nx_0,T^{n-1}x_0), q_p(T^{n+1}x_0,T^nx_0), q_p(T^{n+1}x_0,T^{n-1}x_0)\} = q_p(T^{n+1}x_0,T^nx_0)$

$$q_p(T^{n+1}x_0,T^nx_0) \leq \alpha(q_p(T^nx_0,T^{n-1}x_0))q_p(T^{n+1}x_0,T^nx_0) \leq q_p(T^{n+1}x_0,T^nx_0).$$

Case3. $\max\{q_p(T^nx_0,T^{n-1}x_0), q_p(T^{n+1}x_0,T^nx_0), q_p(T^{n+1}x_0,T^{n-1}x_0)\} = q_p(T^{n+1}x_0,T^{n-1}x_0)$.

$$q_p(T^{n+1}x_0, T^{n-1}x_0) = q_p(T(T^n x_0), T(T^{n-2}x_0))$$
$$\leq \alpha(q_p(T^n x_0, T^{n-2}x_0)) \max\{q_p(T^n x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^n x_0), q_p(T^{n-1}x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^{n-2}x_0), q_p(T^n x_0, T^{n-1}x_0)\}$$

Case3/1. $\max\{q_p(T^n x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^n x_0), q_p(T^{n-1}x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^{n-2}x_0), q_p(T^n x_0, T^{n-1}x_0)\} = q_p(T^{n+1}x_0, T^n x_0)$

Case3/2.

$$\max\{q_p(T^n x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^n x_0), q_p(T^{n-1}x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^{n-2}x_0), q_p(T^n x_0, T^{n-1}x_0)\} = q_p(T^{n-1}x_0, T^{n-2}x_0)$$

These two cases are trivial.

Case3/3. $\max\{q_p(T^n x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^n x_0), q_p(T^{n-1}x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^{n-2}x_0), q_p(T^n x_0, T^{n-1}x_0)\} = q_p(T^n x_0, T^{n-1}x_0)$

$$q_p(T^n x_0, T^{n-1}x_0) \leq c[\alpha(c)]^{n-1} \Rightarrow q_p(T^{n+1}x_0, T^n x_0) \leq c[\alpha(c)]^{n+1} \leq c[\alpha(c)]^n .$$

Case3/4.

$$\max\{q_p(T^n x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^n x_0), q_p(T^{n-1}x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^{n-2}x_0), q_p(T^n x_0, T^{n-1}x_0)\} = q_p(T^n x_0, T^{n-2}x_0)$$

Case3/5

$$\max\{q_p(T^n x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^n x_0), q_p(T^{n-1}x_0, T^{n-2}x_0), q_p(T^{n+1}x_0, T^{n-2}x_0), q_p(T^n x_0, T^{n-1}x_0)\} = q_p(T^{n+1}x_0, T^{n-2}x_0)$$

The cases 3/4 and 3/5 can be proved in the same iterative manner.

So by lemma1 the sequence $\{T^n x_0\}$ is right Cauchy. We see that since by the space is complete and Hausdorff, we have $\lim_{x\to\infty} T^n x_0 = x^*$.

Now we prove that $x^*$is a fixed point of$T$: $X\to X$.

$$q_p(Tx^*, T^n x_0) = q_p(Tx^*, T(T^{n-1}x_0))$$
$$\leq \alpha(q_p(x^*, T^{n-1}x_0)) \max\{q_p(x^*, T^{n-1}x_0), q_p(Tx^*, x^*), q_p(T^n x_0, T^{n-1}x_0), q_p(Tx^*, T^{n-1}x_0), q_p(x^*, T^n x_0)\}$$
$$\leq \alpha(q_p(x^*, T^{n-1}x_0)) \max\{q_p(Tx^*, T^{n-1}x_0), q_p(Tx^*, x^*)\}$$

If $\max\{q_p(Tx^*, T^{n-1}x_0), q_p(Tx^*, x^*)\} = q_p(Tx^*, T^{n-1}x_0)$, we have

$$q_p(Tx^*, T^n x_0) \leq \alpha(c) q_p(Tx^*, T^{n-1}x_0) \leq [\alpha(c)]^2 q_p(Tx^*, T^{n-2}x_0) \leq \ldots \leq [\alpha(c)]^{n-1} q_p(Tx^*, x_0) .$$

Taking the limit of both sides when$n\to\infty$, we have $q_p(Tx^*, x^*) = 0 \Rightarrow Tx^* = x^*$.

If $\max\{q_p(Tx^*, T^{n-1}x_0), q_p(Tx^*, x^*)\} = q_p(Tx^*, x^*)$ then

$$q_p(Tx^*, T^n x_0) \leq \alpha(c) q_p(Tx^*, x^*) \Rightarrow q_p(Tx^*, x^*) \leq \alpha(c) q_p(Tx^*, x^*) \Rightarrow q_p(Tx^*, x^*) = 0 \Rightarrow Tx^* = x^* .$$

Now we prove the uniqueness of the fixed point for T. Suppose there is another pointy^*that$Ty^* = y^*$.

$$q_p(x^*, y^*) = q_p(Tx^*, Ty^*) \leq \alpha(q_p(x^*, y^*)) q_p(x^*, y^*) .$$

Since $0<\alpha(c) < 1$, we have $q_p(x^*, y^*) = 0 \Rightarrow x^* = y^*$.

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Example2. Let $X=[0,1]$, $E=R^2$, $P=\{(x,y)\in E, x,y\geq 0\}$.

Define $q(x,y)=\begin{cases}(0,0), & x\geq y\\ (\frac{1}{2}y,y), & x\leq y\end{cases}$. We take the function $T:X\to X$, $Tx=\begin{cases}\frac{x}{4}, & x\in[0,\frac{1}{2}]\\ \frac{1}{10}, & x\in(\frac{1}{2},1]\end{cases}$ and

$\alpha:P\to(0,1)$, $\alpha(x,y)=\dfrac{4|y-x|}{1+8|y-x|}$.

Case1.

For every $x,y\in\left[0,\frac{1}{2}\right]$, $x\leq y$ we have $q(Tx,Ty)=q(\frac{x}{4},\frac{y}{4})=(\frac{y}{8},\frac{y}{4})$, $\alpha(q(x,y))=\alpha(\frac{y}{2},y)=\frac{2y}{1+4y}$.

$\max\{q(x,y),q(\frac{x}{4},x),q(\frac{y}{4},y),q(\frac{x}{4},y),q(x,\frac{y}{4})\}=(\frac{y}{2},y)\Rightarrow(\frac{y}{8},\frac{y}{4})\leq\frac{2y}{1+4y}(\frac{y}{2},y)$. So we are in conditions of theorem.

Case2.

For every $x,y\in\left(\frac{1}{2},1\right]$, $x\leq y$ we have $q(Tx,Ty)=q(\frac{1}{10},\frac{1}{10})=(\frac{1}{20},\frac{1}{10})$.

$$\max\{q(x,y),q(\frac{1}{10},x),q(\frac{1}{10},y),q(\frac{1}{10},y),q(x,\frac{1}{10})\}=(\frac{y}{2},y)\Rightarrow(\frac{1}{20},\frac{1}{10})\leq\frac{2y}{1+4y}(\frac{y}{2},y).$$

So we are in conditions of theorem.

Case3.

For every $x\in\left[0,\frac{1}{2}\right]$ and $y\in\left(\frac{1}{2},1\right]$. It is clear that the conditions of theorem true in this case.

So, the function $T:X\to X$, $Tx=\begin{cases}\frac{x}{4}, & x\in[0,\frac{1}{2}]\\ \frac{1}{10}, & x\in(\frac{1}{2},1]\end{cases}$ has a fixed point $x = 0$.

The following result is a generalization Theorem of Ciric in metric space.

Corollary 1. Let (X, q_p) be a p-complete, Hausdorff, p-quasi-cone metric space and let T: $X\to X$ be a function that satisfies the nonlinear contraction condition:

$$q_p(T(x),T(y))\leq h\max\left\{q_p(x,y),q_p(T(x),x),q_p(T(y),y),q_p(T(x),y),q_p(x,T(y))\right\}$$

forall $x,y\in X$, where $h\in(0,\frac{1}{p})$. Let $x_0\in X$ such that O(x_0) is bounded. Then T has a unique fixed point $x^*\in X$ and $\lim_{n\to\infty}T^n(x_0)=x^*$

Proof. If we take

$\alpha(t) = h \in \left]0, \frac{1}{p}\right[$ for $t \in P$, we are in condition of Theorem1. So the Corollary1 is true.

The following result is a generalization of Banach contraction metric space.

Corollary2. Let (X, q_p) be a p-complete, Hausdorff, p-quasi-cone metric space and let $T: X \to X$ be a function that satisfies the nonlinear contraction condition:

$$q_p(Tx, Ty) \le hq_p(x, y) \text{ for all } x, y \in X, \text{ where } h \in (0, \frac{1}{p}).$$

Let $x_0 \in X$ such that O(x_0) is bounded. Then T has a unique fixed point $x^* \in X$ and $\lim_{n \to \infty} T^n(x_0) = x^*$

Proof. We take $\alpha(x, y) = h$ and we have

$$q_p(Tx, Ty) \le hq_p(x, y) \le \alpha(q_p(x, y)) \max\{q_p(x, y), q_p(Tx, x), q_p(Ty, y), q_p(Tx, y), q_p(x, Ty)\}$$

So we are in condition of our theorem and the corollary 2 is true.

CONCLUSIONS

In this paper, we have given some generalization of some important results. Theorem 1 is a generalization of result [1], because a p-cone metric space is a generalization of a metric space. Corollary 1 is a generalization of [2] and Corollary2 is a generalization of Banach contraction for p-cone metric space because of [7].

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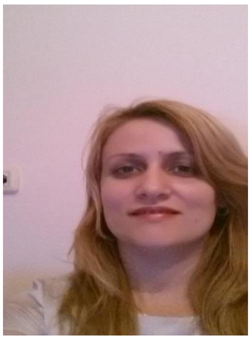

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