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M (X)/G/1 WITH TWO PHASE OF HETEROGENEOUS SERVICE UNDER DIFFERENT VACATION POLICY , RESTRICTED ADMISSIBILITY AND SET UP

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ABSTRACT

In this paper, we consider a batch arrival queueing system with two stage of heterogeneous service with different vacation policy subject to Restricted admissibility and set up time is considered. Customers arrive in batches according to compound Poisson process with rate λ and are served one by one in FIFO basis. After first-stage service the server must provide the second stage service. The service times of two phase of heterogenous services follow arbitrary (general) distribution with different vacation policies. Before providing service to a new customer or a batch of customers that joins the system in the renewed busy period, the server enters into a setup time process such that setup time follows exponential distribution. In addition we assume restricted admissibility of arriving batches in which not all batches are allowed to join the system at all times. . The probability generating function for the number of customers in the queue is found using the supplementary variable technique. The mean number of customers in the queue and the system are also found.

Keywords: Batch arrival; Probability Generating Function; Vacation; Restricted Admissibility and Setup.

AMS Subject Classification: 60K25, 60K30

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1. INTRODUCTION

Batch queueing models have been analyzed in the past by several authors. Server vacation models are useful for the systems in which the server wants to utilize the idle time for different purposes. Application of vacation models can be found in production line systems, designing local area networks and data communication systems.

Queueing models with vacations have been investigated by many authors including Keilson and Servi, Cramer Scholl and kleinrock , Doshi, Madan, Choudhury and Madan have studied a queueing system with Bernoulli schedule server vacation. Chae et al., Chang and Takine and Igaki have studied queues with generalized vacations. Vacation queue with *c* servers has been studied by Tian et al,]. Choudhury and Borthakur and Hur and Ahn have studied vacation ueues with batch arrivals. Queue with multiple vacations has been studied by Tian and Zhang .

Recently there have been several contributions considering queueing systems of *M/G/*1 type in which the server may provide a second phase of service. In some queueing systems with batch arrival there is a restriction such that not all batches are allowed to join the system at all time. Choudhury and Madan proposed an *M (x)/G/*1 queueing system with restricted admissibility of arriving batches and Bernoulli schedule server vacation.

Takine and Sengupta , Aissani and Artalejo have studied different queueing systems subject to random breakdowns. Kulkarni and Choi and Wang et al. have studied retrial queues with system breakdowns and repairs. Thangaraj and Vanitha discussed a M/G/1 queue with two stage heterogeneous service compulsory server vacation and random breakdowns.

In this paper, we consider a batch arrival queueing system such that the customers are arriving in batches according to poisson distribution . The server provides two stage of heterogenous service under different vacation policy subject to Restricted admissibility and setup time is considered. Upon completion of a service , the server may remain in the system to serve the next customer with probability β_0 or he may proceed on the ith vacation scheme with probability β_0 (1≤ i≤ M) and $\sum_{i=1}^{M} \beta_i = 1$. In addition we assume restricted admissibility of arriving batches in which not all batches are allowed to join the system at all times and set up time is considered. The probability generating function for the number of customers in the queue is found using the supplementary variable technique. The mean number of customers in the queue and the system are also found.

2. MATHEMATICAL DESCRIPTION OF MODEL

We assume the following to describe the queueing model of our study.

Customers arrive at the system in batches of variable size in a compound Poisson process and they are provided one by one service on a first come - first served basis. Let λc_i dt (i = 1, 2, . ..) be the first order probability that a batch of ' i' customers arrives at the system during a short interval of time (t; t + dt], where0 $\le k_i \le 1$ and $\sum_{i=1}^{\infty} k_i$ = 1 and λ > 0 is the arrival rate of batches . The customers are served according to the first come, first served rule

Each customer undergoes two stages of heterogeneous service provided by a single server on a first come first served basis. The service time of the two stages follow different general (arbitrary) distributions with distribution function B_j *(t*) and the density function *bj*(t), *j* = 1, 2

The random setup time is a random variable called SET variable following exponential distribution with mean set up time being γ.

There is a single server which provides service following a general (arbitrary) distribution with distribution function B_i (v) and density function b(v).Let μ_j (*x*)d*x* be the conditional probability of completion of the j *th* stage of service during the interval (*x, x*+ d*x*] given that elapsed time is *x*, so that

$$\mu_j(x) = \frac{bj\,(x)}{1-B_j\,(x)}, \quad j=1,2 \qquad (1)$$

and therefore,

$$bj(\mathrm{t}) = \mu_j\ (t)e^{-(\int \mu_j(x)dx)}\ , \quad j= 1, 2 \qquad (2)$$

There is a policy restricted admissibility of batches in which not all batches are allowed to join the system at all times. Let α (0≤α≤ 1) and β (0≤ β ≤ 1) be the probability that an arriving batch will be allowed to join the system during the period of server's non- vacation period and vacation period respectively.

The server's vacation time follows a general (arbitrary) distribution with distribution function $V_i(s)$ and density function $v_i(s)$. Let $v_i(x)$dx be the conditional probability of a completion of a vacation during the interval (x; x + dx] given that the elapsed vacation time is x, so that

$$v_i(x) = \frac{v_{i(x)}}{1-V_{i(x)})} \quad i = 1, 2,3,\ldots M \qquad (3)$$

and for $v_i(s) = v_i(s)\, e^{-\int_0^s v_i(x)dIx)}$ i= 1,2,3,..M (4)

On returning from vacation the server instantly starts serving the customer at the head of the queue if any. The server stays in the system for being available if there are no customers.

3. DEFINITIONS AND EQUATIONS GOVERNING THE SYSTEM

Let N_q (t) denote the queue size(excluding one in service) at time t. We introduce the random variable Y(t) as follows

$$Y(t) = \begin{cases} 1, \textit{if the system is busy with first stage of service at time } t \\ 2, \textit{if the system is busy with second stage of service at time } t \\ 3, \textit{if the system is on vacation at time } t \\ 4, \textit{if the system is on setup at time } t \end{cases}$$

We introduce the supplementary variable as,

$$L(t) = \begin{cases} B_1^0(t) \quad if \ Y(t) = 1 \\ B_2^0(t) \quad if \ Y(t) = 2 \\ V^0(t) \quad if \ Y(t) = 3 \\ S^0(t) \quad if \ Y(t) = 4 \end{cases}$$

where

$B_1^0(t)$ = elapsed service time for the first stage of service at time t,

$B_2^0(t)$ = elapsed service time for the second stage of service at time t,

$V^0(t)$ = elapsed vacation time of the server at time t.

$S^0(t)$ = elapsed set up time of the server at time t.

The process $\{N_Q(t), L(t)\}$ is a continuous time Markov process. we define

the probabilities for i = 1,2.

$P_n^i(x,t) = Prob\{N_Q(t = n, L(t) = B_i^0 ; x < B_i^0 \le x + dx\};\ x > 0, n > 0$

$Vn(x, t) = Prob\{N_Q(t = n, L(t) = V^0(t) ; x < V^0(t) \le x + dx\}, x > 0, n \ge 0$

$Sn(t) = Prob\{N_Q(t = n, L(t) = S^0(t) ; x < S^0(t) \le x + dx\}, x > 0, n \ge 0$

In steady state condition, we have

$P_n^i(x)\,dx = \lim_{t\to\infty} P_n^i(x,t)$, i=1, 2 $x > 0$; $n \ge 0$

$V_n = \lim_{t\to\infty} V_n(t)$; $n \ge 0$

$S_n = \lim_{t\to\infty} S_n(t)$; $n \ge 0$

Assume that

$V_0(0) = 1, V_n(0) = 0$, and for i $= 1,2$, $B_i(0), B_i(\infty) = 1$.Also V(x) and Bi(x) are continuous at x=0.

(i) P_n^i(x, t) = probability that at time 't' the server is active providing (i=1,2,3)service and there are 'n' (n≥1) customers in the queue including the one being served and the elapsed service time for this customer is x. Consequently P_n^i(t) denotes the probability that at time 't' there are 'n' customers in the queue excluding the one customer in i^{th} service irrespective of the value of x.

(ii) $V^{i}{}_{n}(x,t)$ = probability that at time 't', the server is on i th vacation with elapsed vacation time x, and there are 'n'(n≥1) customers waiting in the queue for service. Consequently $V^{i}{}_{n}(t)$ denotes the probability that at time 't' there are 'n' customers in the queue and the server is on i thvacation irrespective of the value of x.

(iii) $S_n(t)$ = probability that at time 't' the server is in setup time while there are "n" (n≥1) customers in the queue.

(iv) Q(t) = probability that at time 't' there are no customers in the system and the server is idle but available in the system ..

The queueing model is then, governed by the following set of differential-difference equations:

$$\frac{d}{dt} S_n(t) = -(\alpha\lambda + \gamma) S_n(t) + \alpha\lambda \sum_{k=1}^{n} c_k S_{n-i}(t) \ (t) + \alpha\lambda c_k Q(t), \quad n \geq 1 \tag{5}$$

$$\frac{\partial}{\partial x} P_n^{(1)}(x,t) + \frac{\partial}{\partial t} P_n^{(1)}(x,t) + (\lambda + \mu_1(x)) P_n^{(1)}(x,t) = \lambda(1-\alpha) P_n^{(1)}(x,t) + \alpha\lambda \sum_{k=1}^{n} c_k P_{n-k}^{(1)}(x,t); n \geq 1 \tag{6}$$

$$\frac{\partial}{\partial x} P_n^{(2)}(x,t) + \frac{\partial}{\partial t} P_n^{(2)}(x,t) + (\lambda + \mu_2(x)) P_n^{(2)}(x,t) = \lambda(1-\alpha) P_n^{(2)}(x,t) + \alpha\lambda \sum_{k=1}^{n} c_k P_{n-k}^{(2)}(x,t) \quad ; n \geq 1 \tag{7}$$

$$\frac{\partial}{\partial x} V_n^{(i)}(x,t) + \frac{\partial}{\partial t} V_n^{(i)}(x,t) + (\lambda + \gamma_i(x)) V_n^{(i)}(x,t) = \lambda(1-\beta) V_n^{(i)}(x,t) + \beta\lambda \sum_{k=1}^{n} c_k V_{n-k}^{(i)}(x,t); n \geq 1, \tag{8}$$

$$\frac{\partial}{\partial x} V_0^{(i)}(x,t) + \frac{\partial}{\partial t} V_0^{(i)}(x,t) + (\lambda + \gamma_i(x)) V_0^{(i)}(x,t) = \lambda(1-\beta) V_n^{(i)}(x,t), \ i = 1,2,3,\ldots M \tag{9}$$

$$\frac{d}{dt} Q(t) = \beta_0(t) \int_0^\infty P_{n+1}^{(2)} \square_2(x)dx + \sum_{i=1}^{M} \int_0^\infty V_0^{(i)}(x,t) \square_i(x)dx - \lambda Q(t) + \lambda(1-\alpha) Q(t); \ i=1,2,3\ldots M \tag{10}$$

Equations (5) to (10) are to be solved subject to the following boundary conditions at x=0.

$$P_n^{(1)}(0,t) = \beta_0 \int_0^\infty P_{n+1}^{(2)}(x,t) \square_2(x)dx + \sum_{k=1}^{M} \int_0^\infty V_{n+1}^{(i)}(t)\, \gamma(x)\, dx + \gamma S_n(t)$$

$$i = 1,2,3\ldots M \quad, n \geq 1, \tag{11}$$

$$P_n^2(0,t) = \int_0^\infty P_n^{(1)}(x,t) \square_1(x)\, dx, \quad n \geq 0 \tag{12}$$

$$V_n^{(i)}(0,t) = \beta_K \int_0^\infty P_{n+1}^{(2)}(t) \square_2(x)dx, \quad n \geq 0. \ i = 1,2,3 \ldots M \quad, n \geq 0 \tag{13}$$

We assume that initially there are no customers in the system and the server is idle. so the initial Conditions are with

$$Q(0)=1, \ V_n^{(j)}(0) = V_0^{(j)} = 0 \text{ and } P_n^{(i)} = 0, \ n \geq 0. \ i=1,2, \ j=1,2,3,\ldots M \tag{14}$$

4. The time dependent solution

Generating function of the queue length .

Define the Laplace transform

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)\, dt, \tag{15}$$

Taking Laplace transform of equation (5) to (13)

$$(S + \lambda + \gamma) \bar{S}_n(s) = \lambda \sum_{k=1}^{n-1} C_k \bar{S}_n(s) + \lambda C_n \bar{Q}(s), \ n \geq 1 \tag{16}$$

$$\frac{\partial}{\partial X} \bar{P}_n^{(1)}(x,s) + (S + \lambda + \square_1(x)) \bar{P}_n^{(1)}(x,s) = \lambda \sum_{k=1}^{n-1} C_k \bar{P}_{n-k}^{(1)}(x,s), \ n \geq 1 \tag{17}$$

$$\frac{\partial}{\partial X} \bar{P}_n^{(2)}(x,s) + (S + \lambda + \square_2(x)) \bar{P}_n^{(2)}(x,s) = \lambda \sum_{k=1}^{n-1} C_k \bar{P}_{n-k}^{(2)}(x,s), \quad n \geq 1 \tag{18}$$

$$\frac{\partial}{\partial X} \bar{V}_n^{(i)}(x,s) + (S + \lambda + \gamma_1(x)) \bar{V}_n^{(i)}(x,s) = \lambda \sum_{k=1}^{n} C_k \bar{V}_{n-k}^{(i)}(x,s), \ n \geq 1 \ , i=1,2,3\ldots M \tag{19}$$

$$\frac{\partial}{\partial X} \bar{V}_0^{(i)}(x,s) + (S + \lambda + \gamma_1(x)) \bar{V}_0^{(i)}(x,s) = 0 \quad , i=1,2,3\ldots M \tag{20}$$

$$(S+\lambda) \bar{Q}(S) = 1 + \beta_0 \int_0^\infty \bar{P}_1^{(2)}(x,s) \square_2(x)dx + \sum_{i=1}^{M} \int_0^\infty \bar{V}_{n+1}^{(i)}(x,s) \gamma_i(x)dx, \ i=1,2,3\ldots M \tag{21}$$

$$\bar{P}_n^{(1)}(0,S) = \lambda C_n \bar{Q}(S) + \beta_0 \int_0^\infty \bar{P}_{n+1}^{(2)}(x,s) \square_2(x)dx + \sum_{K=1}^{M} \int_0^\infty \bar{V}_{n+1}^{(i)}(x,s) \gamma_i(x)dx + \gamma S_n(s)$$

$$i=1,2,3\ldots M, \ n \geq 1, \tag{22}$$

$$\bar{P}_n^{(2)}(0,S) = \int_0^\infty \bar{P}_n^{(1)}(x,s) \square_1(x)dx, \ n \geq 0 \tag{23}$$

$$\bar{V}_n^{(i)}(0,s) = \beta_i \int_0^\infty \bar{P}_{n+1}^{(2)}(x,s) \square_2(x)dx, \ i = 1,2,3,\ldots M \tag{24}$$

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Define the Probability generating functions as follows

$P_q^{(i)}$(x,z,t) = $\sum_{n=0}^{\infty} P_n^{(i)}$(x,t) Z^n; i=1,2

$P_q^{(i)}$(z,t) = $\sum_{n=0}^{\infty} P_n^{(i)}$(t) Z^n; i=1,2

$V_q^{(i)}$(x,z,t) = $\sum_{n=0}^{\infty} V_n^{(i)}$(x,t) Z^n, $V_q^{(i)}$(z,t) = $\sum_{n=0}^{\infty} V_n^{(i)}$(t) Z^n, i= 1,2,3 …M (25)

S_q(z,t) =$\sum_{n=0}^{\infty} S_n$(t)Z^n

C(z) = $\sum_{n=1}^{\infty} C_n Z^n$

Now multiplying equation (16) by Z^n from 1 to ∞and using (25)

(S+λ(1- c(z))+γ)$\bar{S}_q$ (z,s) = λ c(z)$\bar{Q}$ (s) , n≥ 1 (26)

Now multiplying equation (17) and (18) by Z^n respectively and take summation over all possible 'n'

$\frac{\partial}{\partial X}$ $\bar{P}_q^{(1)}$(x,z,s) + (S+λ(1- c(z))+μ_1(x)) $\bar{P}_q^{(1)}$(x,z,s) =0 (27)

$\frac{\partial}{\partial X}$ $\bar{P}_q^{(2)}$(x,z,s) + (S + λ(1- c(z))+μ_2(x)) $\bar{P}_q^{(2)}$(x,z,s) =0 (28)

Multiplying equation (19) and (20) by Z^n respectively and take summation over all possible 'n'

$\frac{\partial}{\partial X}$ $\bar{V}_q^{(i)}$(x,z,s) + (S + λ(1- c(z))+γ_1(x)) $\bar{V}_q^{(i)}$(x,z,s) =0 , i=1,2,3…M (29)

For the boundary condition multiply both sides of equation (22) by z^n , summing over 1 to ∞ by using the definition of probability generating function equation ,we get

Z$\bar{P}_q^{(1)}$(0,z,s) = $\beta_0 \int_0^{\infty}$ $\bar{P}_q^{(2)}$(x,z,s)μ_2(x)dx+) $\sum_i^M$ z $\int_0^{\infty}$ $\bar{V}_q^{(i)}$(x,z,s)γ_i(x)dx+

λ[C(Z)-1]$\bar{Q}$(S)+[1-S$\bar{Q}$(S)]z +z γ$\bar{s}_q$ (z,s) (30)

Performing similar operation on equation (23) to (24) we obtain

$\bar{P}_q^{(2)}$(0,Z,S)= $\int_0^{\infty}$ $\bar{P}_q^{(1)}$(x,z,s)μ_1(x)dx (31)

z $\bar{V}_q^{(i)}$(0,Z,S)= β_k $\int_0^{\infty}$ $\bar{P}_q^{(2)}$(x,z,s)μ_2(x)dx, n≥ 0 (32)

Solving equation (27)

$\bar{P}_q^{(1)}$(x,Z,S) =$\bar{P}_q^{(1)}$(0,Z,S) $e^{-(s+\lambda(1-C(z))x-\int_0^x \mu_1(t)dt}$ (33)

where $\bar{P}_q^{(1)}$(0,Z,S) is given by (30) , Again integrating equation (33) by parts w. r. to x yields

$\bar{P}_q^{(1)}$(Z,S) =$\bar{P}_q^{(1)}$(0,Z,S)$\left[\frac{1-\bar{B}_1(s+\lambda(1-C(z))}{s+\lambda(1-C(z))}\right]$ (34)

where $\bar{B}_1$ (s+ λ -λ C(z))= $e^{\int_0^{\infty}-(s+\lambda(1-\lambda C(z))x}$ dB_1(x) is the laplace transform of the two phase of heterogeneous service $\bar{B}_1$ (x) .

Now multiplying both sides of equation (33) by μ_1 (x) and integrating over x, we get

$\int_0^{\infty} \bar{P}_q^{(1)}(x,Z,S)\mu_1(x)dx = \bar{P}_q^{(1)}(0,Z,S)\ \bar{B}_1\ (s+\lambda(1-C(z))$ (35)

Similarly as Integrating equation (28) to (29) from 0 to λ, we get

$\bar{P}_q^{(2)}$(x,z,s) = $\bar{P}_q^{(2)}$(0,z,s)$e^{-(s+\lambda(1-c(z))x-\int_0^x \mu_2(t)dt}$ (36)

$\bar{V}_q^{(i)}$(x,z,s) = $\bar{V}_q^{(i)}$(0,z,s) $e^{-(s+\lambda(1-c(z))x-\int_0^x v_i(t)dt}$ (37)

Again integrating (36) and (37)

$\bar{P}_q^{(2)}$(Z,S) =$\bar{P}_q^{(1)}$(0,Z,S)$\left[\frac{1-\bar{B}_2(s+\lambda(1-C(z))}{s+\lambda(1-C(z))}\right]$ (38)

$\bar{V}_q^{(i)}$(Z,S) =$\bar{V}_q^{(i)}$(0,Z,S)$\left[\frac{1-\bar{V}_i(s+\lambda(1-C(z))}{s+\lambda(1-C(z))}\right]$ (39)

where $\bar{V}_i$ (s+λ(1-C(z))= $e^{\int_0^{\infty}-(s+\lambda(1-C(z))x}$ dV(x) is the laplace –Stieltjies transform of the i th vacation $V_i\ (x)$

Z$\bar{P}_q^{(1)}$(0,z,s) = $\beta_0\bar{P}_q^{(2)}$(0,Z,S) $\bar{B}_2(s+\lambda-\lambda(1-C(z)$ + $\sum_i^M$ z $\bar{V}_q^{(i)}$(0,z,s)$\bar{V}_K(s+\lambda-\lambda(1-C(z)+$

λ[C(Z)-1]$\bar{Q}$(S)+[1-S$\bar{Q}$(S)]z +z γ$\bar{s}_q$ (z,s) (40)

$\bar{P}_q^{(2)}$ (0,z,s)□$_2$(x)dx = $\bar{P}_q^{(1)}$(0,z,s) $\bar{B}_1$(s+λ-λ(1-c(z)) (41)

Using (36) in(32)

$$\overline{Z V}_q^{(i)}(0,z,s) = \beta_i \bar{P}_q^{(1)}(0,Z,S)\, \bar{B}_1(s+\lambda-\lambda(1-c(z))\, \bar{B}_2(s+\lambda(1-c(z))\,) \quad (42)$$

Solving (40)

$$\bar{P}_q^{(1)}(0,Z,S) = \frac{[(1-sQ(s))z+\lambda z(\gamma c(z)-1)\bar{Q}(S)}{z-\{\beta_0+\sum_i^M \beta_k \bar{V}_i(s+\lambda(1-c(z))\,\bar{B}_1(s+\lambda(1-c(z))\,\bar{B}_2(s+\lambda(1-c(z))} \quad (43)$$

$$\bar{P}_q^{(1)}(Z,S) = \frac{[(1-sQ(s))z+\lambda z(\gamma c(z)-1)\bar{Q}(S)]\left[\frac{(1-\bar{B}_1(s+\lambda(1-\lambda(1-c(z))}{s+\lambda-\lambda c(z)+\alpha}\right]}{z-\{\beta_0+\sum_i^M \beta_i \bar{V}_i(s+\lambda(1-c(z))\,\bar{B}_1(s+\lambda(1-c(z))\bar{B}_2(s+\lambda(1-c(z))} \quad (44)$$

$$\bar{P}_q^{(2)}(0,Z,S) = \frac{[(1-sQ(s))z+\lambda z(\gamma c(z)-1)\bar{Q}(S)]\,\bar{B}_1(s+\lambda(1-\lambda(1-c(z))}{z-\{\beta_0+\sum_i^M \beta_i \bar{V}_i(s+\lambda(1-c(z))\,\bar{B}_1(s+\lambda(1-c(z))\bar{B}_2(s+\lambda(1-c(z))} \quad (45)$$

The equation (45) becomes

$$\bar{P}_q^{(2)}(z,s) = \frac{[(1-sQ(s))z+\lambda z(\gamma c(z)-1)\bar{Q}(S)]\,\bar{B}_1(s+\lambda(1-\lambda(1-c(z))\,\left[\frac{(1-\bar{B}_2(s+\lambda(1-\lambda c(z)}{s+\lambda-\lambda c(z)+\alpha}\right]}{z-\{\beta_0+\sum_i^M \beta_i \bar{V}_i(s+\lambda(1-c(z))\,\bar{B}_1(s+\lambda(1-c(z))\bar{B}_2(s+\lambda(1-c(z))} \quad (46)$$

Now using equation (42), in, (39)

$$V_q^{(i)}(0,z,s) = \frac{\beta_k\,(1-sQ(s))z+\lambda z(\gamma c(z)-1)\bar{Q}(S))\,\bar{B}_1(s+\lambda-\lambda(1-c(z))\;\; \bar{B}_2(s+\lambda-\lambda(1-c(z))}{z-\{\beta_0+\sum_i^M \beta_i \bar{V}_i(s+\lambda(1-\lambda c(z))\,\bar{B}_1(s+\lambda-\lambda c(z)\bar{B}_2(s+\lambda-\lambda(1-c(z))} \quad (47)$$

Using above equation (47) in (39)

$$V_q^{(i)}(z,s) = \frac{\beta_i\,(1-s\bar{Q}(S))z+\lambda z(\gamma c(z)-1)\bar{Q}(S))\,\bar{B}_1(s+\lambda-\lambda(1-c(z))\;\; \bar{B}_2(s+\lambda-\lambda(1-c(z))}{z-\{\beta_0+\sum_i^M \beta_i \bar{V}_i(s+\lambda(1-\lambda c(z))\,\bar{B}_1(s+\lambda-\lambda c(z)\bar{B}_2(s+\lambda-\lambda c(z))}\left[\frac{1-\bar{V}_i(s+\lambda-\lambda(1-c(z))}{(s+\lambda-\lambda(1-c(z))}\right] \quad (48)$$

equation (26) becomes

$$\bar{S}_q(z,s) = \frac{\lambda c(z)\bar{Q}(S)}{s+\lambda-\lambda(1-c(z))+\gamma} \quad (49)$$

5. THE STEADY STATE ANALYSIS

In this section, derive the steady state probability distribution for our queueing model. To define the steady state probabilities, suppress. The argument 't' wherever it appears in the time dependent analysis

$$\lim_{s\to 0} s\bar{f}(s) = \lim_{t\to\infty} f(t) \quad (50)$$

By applying Property (50) and simplifying, we get

$$\bar{S}_q(z) = \frac{\lambda c(z)\boldsymbol{Q}]}{s+\lambda-\lambda(1-c(z))+\gamma} \quad (51)$$

$$\boldsymbol{\bar{P}_q^{(1)}}(z) = \frac{\boldsymbol{(\lambda+\gamma)z[1-\bar{B}_1\,\alpha\lambda(1-}(c(z))\boldsymbol{Q}]}{\boldsymbol{Dr}} \quad (52)$$

$$\boldsymbol{\bar{P}_q^{(2)}}(z) == \frac{\boldsymbol{(\lambda+\gamma)z\bar{B}_1\,\alpha\lambda(1-}(c(z)))\boldsymbol{([1-\bar{B}_2\,\alpha\lambda(1-}(c(z)))\boldsymbol{Q}}{\boldsymbol{Dr}} \quad (53)$$

$$\bar{V}_q^k(z) = \frac{\beta_i(\frac{\alpha}{\beta})\;\; \boldsymbol{\bar{B}_1\alpha\lambda(1-}(c(z)))\boldsymbol{([1-\bar{B}_2\,\alpha\lambda(1-}(c(z))[1-\bar{V}_i(\lambda-\alpha\lambda(1-c(z))Q]}{Dr} \quad (54)$$

Where Dr= f_1 $[\beta_0 + \sum_{i=1}^{M} \beta_i\ \bar{V}_i(\lambda-\alpha\lambda(1-c(z))\ \boldsymbol{\bar{B}_1\alpha\lambda(1-}(c(z)))(\boldsymbol{\bar{B}_2}\ \boldsymbol{\alpha\lambda(1}-(c(z))Q$

where f_1 (z) = s+ λ(1-c(z))+ γ. using equation (51), (52), (53)and (54),

Let W_q(Z) denotes the PGF of queue size irrespective of the state of the system. Then adding

, $\mathbf{W_q(Z)}$ = P_q^1 (z) +P_q^2 (z)+$\sum_{i=1}^{M} V_q^i$ (z) + S_q (z) (55)

In order to obtain Q, using the normalization condition

W_q(1) + Q = 1

We see that for Z=1, $W_q(Z)$ have been completely and explicitly determinate which

where V_i (0)=1, i =1,2, ,M , E(I) is mean batch size if the arriving customers and V(0)=1,$V_0^i = E$(v), $E(V_i)$=i=1,2,3.. M the mean vacation time

$$W_q\,(1) = \frac{\lambda\alpha E(I) Q[E(S_1)+E(S_2)]+\sum_{i=1}^{M}\beta_i E(V_i)}{Dr}$$

Where dr =1-λαE(I) $[E(S_1)+E(S_2)] - \beta_i \sum_{i=1}^{M} E(V_i)$

$$Q = [\frac{1-\lambda\alpha E(I)\,[E(S_1)+E(S_2)] - \lambda\beta\, E(I)\sum_{i=1}^{M} E(V_i)}{1+\lambda(\alpha-\beta)E(I)\sum_{i=1}^{M} E(V_i)}] \quad (56)$$

ρ =1- Q ,Where ρ<1 is the stability condition under which the steady state exists, equation (56) gives the Probability that the server is idle.

6. THE AVERAGE QUEUE SIZE

Let Lq denote the mean number of customers in the queue under the steady state then $\frac{d}{dz}\left[W_q^-(z)\right]_{z=1}$

since this gives $\frac{0}{0}$ form we write $\left[W_q^-(z)\right] = \frac{N(z)}{D(z)}$ where N(Z) & D(Z) are the number and denominator of the right hand side of equation (56) respectively then we use $L_q = \lim_{z\to 1}[\frac{D'N'' - D''N'}{2(D')^2}]$ (57)

$$N'^{(1)} = (\lambda+\gamma)Q\lambda\alpha E(I)\{[E(S_1)+E(S_2)] - \sum_{i=1}^{M}\beta_i E(V_i)\} \quad (58)$$

$$N''(1) = (\lambda+\gamma)Q[(\lambda\alpha E(I))^2]\,[E(S_1)+E(S_2)] + 2\,E(S_1)\,E(S_2) + \sum_{i=1}^{M}\beta_i E(V_i)\,E(S_1) + E(S_2)$$
$$[(\lambda\alpha E(I))^2[E(S_1)^2 + E(S_2)^2] + E(S_1)\,E(S_2) + \lambda E(I)\,\beta^2\sum_{i=1}^{M}\beta_i E(V_i)^2 +$$
$$\lambda\alpha E(I)(I-1)\{[E(S_1)+E(S_2)] + \sum_{i=1}^{M}\beta_i E(V_i) \quad (59)$$

$$D'(1) = \gamma\{\lambda\alpha E(I)[E(S_1)+E(S_2)] + \lambda\beta E(I)\sum_{i=1}^{M}\beta_i E(V_i) - 1\} \quad (60)$$

$$D''(1) = \gamma\{\,[(\lambda\alpha E(I))^2[E(S_1)^2 + E(S_2)^2] + \lambda\alpha E(I)(I-1)\{[E(S_1)+E(S_2)] + \lambda\beta E(I(I-1))\sum_{i=1}^{M}\beta_i E(V_i)$$
$$2(\lambda\alpha E(I))^2]\,[E(S_1)\,E(S_2)]\,\lambda E(I)\beta^2\sum_{i=1}^{M}\beta_i E(V_i)^2 +$$
$$2\lambda^2\,\alpha\beta\,E(I)^2[E(S_1)+E(S_2)]\sum_{i=1}^{M}\beta_i E(V_i) \quad (61)$$

where $E(V_2)$ is the second moment of the vacation time and Q has been found in equation (56). Then if we substitute the values of N'(1), N"(1), D'(1) and D"(1) from equations (58),(59),(60),(61) in to equation(57). we obtain L_q in a closed form.

Mean waiting time of a customer could be found as

$$W_q = \frac{Lq}{\lambda}$$

by using Little's formula.

7. CONCLUSION

In this paper we have studied a batch arrival with two stage of service and different types of vacation ,restricted admissibility of arriving batches and set up time is discussed.. This queueing model can be utilized in large scale manufacturing industries and communication networks.

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