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# Coincidence and common fixed points of Greguš type weakly biased mappings

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## ABSTRACT

In this note, common fixed point theorem of compatible mappings of type($A$) due to Murthy et al.[ P. P. Murthy, Y. J. Cho and B. Fisher, Common fixed points of GreguŠ type mappings, Glasnik Maematicki, Vol.30(50), (1995), 335-341] has extended to weakly biased mappings. Our result also extends the results of Sessa and Fisher [S. Sessa and B. Fisher, Common fixed points of two mappings on Banach spaces, J. Math. Phys. Sci. 18(1984), 353-360] and, Fisher and Sessa[B. Fisher and S. Sessa, On a fixed point theorem of GreguŠ , Internat. J. Math. Math. Sci. 9, (1986), 22-28].

**Keywords**: Compatible maps; weakly compatible maps; biased maps and fixed point.

**Subject Classication**: 47H10, 54H25.

## 1. INTRODUCTION

In 1996, Jungck[5], introduced the concept of compatible maps which is a generalization of commuting mappings[3] and used to extend a theorem of Park and Bae[4]. A pair of self mappings {*A*, *S*} of a metric space (*X*, *d*) is said to be compatible[5] iff $\lim_{n\to\infty} d(SAx_n, ASx_n)=0$ whenever, $\{x_n\}$ is a sequence in *X* such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t\in X$. Noted that *A* and *S* are non-compatible if there exists atleast one sequence $\{x_n\}$ in *X* such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$, for some $t\in X$ but $\lim_{n\to\infty} d(SAx_n, ASx_n)$ is either non-zero or non existence (also see [1], [15], [16] etc.). Murthy et al.[13] introduced the concept of compatible of type (*A*). A pair of self mappings {*A*, *S*} of a metric space (*X*, *d*) is said to be compatible of type (*A*) if $\lim_{n\to\infty} d(ASx_n, S^2x_n) = \lim_{n\to\infty} d(SAx_n, A^2x_n) = 0$ whenever, $\{x_n\}$ is a sequence in *X* such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t\in X$. A pair of self mappings {*A*, *S*} of a metric space (*X*, *d*) is said to be weakly compatible[8] if they commute at their coincidence points, i.e. $At = St$ for some $t\in X$, then $SAt = ASt$. Jungck and Pathak[6] introduced the concepts of weakly biased mappings. A pair of self mappings {*A*, *S*} of a metric space (*X*, *d*) is said to be weakly *S*-biased iff *At* = *St* implies $d(SAt, St)\le d(ASt, At)$. In [18], it has shown that weakly biased is more general then the concept of weakly compatible of two mappings. On the other hand, common fixed point theorems of Greguš type[3] has been obtained by many authors viz. Deviccaro et al. [2], Fisher and Sessa[3], Jungck[6], Mukherjee and Verma[12], Sessa and Fisher[17], etc.

Murthy et al.[13] proved the following theorem for compatible mappings of type(*A*).

**Theorem 1.1** [13]. Let $A, B, S$ and $T$ be mappings of a Banach space $X$ into itself satisfying the following conditions:

(*i*) $A(X)\subseteq T(X)$ and $B(X)\subseteq S(X)$;

$$(ii)\ \|Ax-By\|^p \le \varphi\left(\alpha\|Sx-Ty\|^p + (1-\alpha)\max\left\{\|Sx-Ax\|^p, \|Ty-By\|^p\right\}\right);$$

for all $x, y\in X$, $p\ge 1$, $0<\alpha<1$ and $\varphi$ is a mapping of $[0,+\infty)$ into itself such that $\varphi$ non decreasing, upper semi continuous and $\varphi(t)<t$ for $t>0$. Suppose that one of the mappings $A, B, S$ and $T$ is continuous and that $\{A, S\}$ and $\{B, T\}$ are compatible pairs of type (*A*). Then $A, B, S$ and $T$ have a unique common fixed point in $X$.

Moreover, in [13], it has raised an open question on Theorem 1.1 that "under what conditions the sequence $\{y_n\}$ given in (1) converges if $\varphi$ is removed from condition (*ii*) of Theorem 1.1 ?". The right answer corresponding to this open question is that if we replace the factor $(1-\alpha)$ by another constant say $\beta$ such that $0<\alpha+\beta<1$, in Theorem 1.1, then the sequence $\{y_n\}$ converges.

Now, we give the following theorem without prove.

**Theorem 1.2**. Let $A, B, S$ and $T$ be mappings of a Banach space $X$ into itself satisfying the following conditions:

(*i*) $A(X)\subseteq T(X)$ and $B(X)\subseteq S(X)$;

$$(ii)\ \|Ax-By\|^p \le \alpha\|Sx-Ty\|^p + \beta\max\left\{\|Sx-Ax\|^p, \|Ty-By\|^p\right\};$$

for all $x, y\in X$, $p\ge 1$, $0<\alpha+\beta<1$. for all $x, y\in X$, $p\ge 1$, $0<\alpha<1$ and $\varphi$ is a mapping of $[0,+\infty)$ into itself such that $\varphi$ non decreasing, upper semi continuous and $\varphi(t)<t$ for $t>0$. Suppose that one of the mappings $A, B, S$ and $T$ is continuous and that $\{A, S\}$ and $\{B, T\}$ are compatible pairs of type (*A*). Then $A, B, S$ and $T$ have a unique common fixed point in $X$.

To prove our theorem we need the following lemma.

**Lemma 1.3**[11]. Suppose that $\varphi$ is a mapping of $[0,+\infty)$ into itself such that $\varphi$ non decreasing, upper semi continuous and $\varphi(t)<t$ for all $t>0$. Then, $\lim_{n\to\infty}\varphi^n(t)=0$.

## 2. MAIN RESULTS

We prove the following theorem.

**Theorem 2.1**. Let $A, B, S$ and $T$ be mappings of a Banach space $X$ into itself satisfying the following conditions:

(*i*) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$;

$$(ii)\ \|Ax - By\|^p \le \varphi\left(\alpha \|Sx - Ty\|^p + (1-\alpha)\max \frac{1}{2^p}\left\{\|Sx - Ax\|^p, \|Ty - By\|^p, \|Sx - By\|^p, \|Ty - Ax\|^p\right\}\right);$$

for all $x, y \in X$, $p \ge 1$, $0 < \alpha < 1$ and $\varphi : [0, +\infty) \to [0, +\infty)$ is non decreasing, upper semi continuous, $\varphi(t) < t$ for $t \in (0, +\infty)$ and $\varphi(0) = 0$.

Then, the pairs $\{A, S\}$ and $\{B, T\}$ have coincidence points. Further, if the pairs $\{A, S\}$ and $\{B, T\}$ are weakly *S*- and *T*- biased, then *A*, *B*, *S* and *T* have a unique common fixed point in $X$.

**Proof**. Let $x_0$ be an arbitrary point in $X$. Then by virtue of (*i*), it is guaranteed to choose the points $x_1, x_2 \in X$ such that $Ax_0 = Tx_1$ and $Bx_1 = Sx_2$. Similarly, we choose $x_3, x_4 \in X$ such that $Ax_1 = Tx_2$ and $Bx_2 = Sx_3$. Continuing in this process, we obtain a sequence $\{y_n\} \subseteq X$ such that

$$y_{2n} = Sx_{2n} = Bx_{2n-1},\ y_{2n+1} = Tx_{2n+1} = Ax_{2n}, \text{ for } n = 1, 2, 3, \ldots \qquad (1)$$

Now, we show that $\{y_n\}$ is Cauchy sequence in $X$. By (*ii*) and (1), we obtain

$$\begin{aligned}
\|y_{2n+1} - y_{2n}\|^p &= \|Ax_{2n} - Bx_{2n-1}\|^p \\
&\le \varphi\left(\alpha \|Sx_{2n} - Tx_{2n-1}\|^p + (1-\alpha)\max \frac{1}{2^p}\left\{\|Sx_{2n} - Ax_{2n}\|^p, \|Tx_{2n-1} - Bx_{2n-1}\|^p \right.\right. \\
&\qquad \left.\left. \|Sx_{2n} - Bx_{2n-1}\|^p, \|Tx_{2n-1} - Ax_{2n}\|^p\right\}\right) \\
&= \varphi\left(\alpha \|y_{2n} - y_{2n-1}\|^p + (1-\alpha)\max \frac{1}{2^p}\left\{\|y_{2n} - y_{2n+1}\|^p, \|y_{2n-1} - y_{2n}\|^p, \right.\right. \\
&\qquad \left.\left. \|y_{2n} - y_{2n}\|^p, \|y_{2n-1} - y_{2n+1}\|^p\right\}\right) \\
&\le \varphi\left(\alpha \|y_{2n} - y_{2n-1}\|^p + (1-\alpha)\max \frac{1}{2^p}\left\{\|y_{2n} - y_{2n+1}\|^p, \|y_{2n-1} - y_{2n}\|^p, \right.\right. \\
&\qquad \left.\left. \left[\|y_{2n-1} - y_{2n}\| + \|y_{2n} - y_{2n+1}\|\right]^p\right\}\right) \qquad (2)
\end{aligned}$$

Suppose that $\|y_{2n+1} - y_{2n}\| > \|y_{2n} - y_{2n-1}\|$, then from (2) we obtain

$$\begin{aligned}
\|y_{2n} - y_{2n+1}\| &\le \left(\varphi\left(\alpha \|y_{2n} - y_{2n+1}\|^p + \frac{(1-\alpha)}{2^p}.2^p \|y_{2n} - y_{2n+1}\|^p\right)\right)^{1/p} \\
&< \|y_{2n} - y_{2n+1}\|
\end{aligned}$$

This is a contradiction. Thus, $\|y_{2n+1} - y_{2n}\| > \|y_{2n} - y_{2n-1}\|$. Similarly, we can show that

$\|y_{2n+2} - y_{2n+1}\| > \|y_{2n+1} - y_{2n}\|$. Consequently, we obtain

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$$\|y_{n+1}-y_n\|^p \le \varphi\left(\|y_n-y_{n-1}\|^p\right)$$
$$\le \varphi^n\left(\|y_0-y_1\|^p\right),\ \ n=1,2,3,\ldots$$

By Lemma 1.3, we obtain

$$\lim_{n\to\infty}\|y_{n+1}-y_n\|=0\,. \qquad (3)$$

In order to show that $\{y_n\}$ is Cauchy sequence, it is sufficient to show that $\{y_{2n}\}$ is a Cauchy sequence. Suppose not, then there exists $\varepsilon>0$, $\{n(k)\}$ a sequence of even integers defined inductively with $n(1)=2$ and $n(k+1)$ is the smallest even integers greater than $n(k)$ such that

$$\|y_{n(k+1)}-y_{n(k)}\|>\varepsilon \qquad (4)$$

so that $\|y_{n(k+1)-2}-y_{n(k)}\|\le\varepsilon$ (5)

Using (4), we obtain

$$\varepsilon<\|y_{n(k+1)}-y_{n(k)}\|\le\|y_{n(k+1)}-y_{n(k+1)-1}\|$$
$$+\|y_{n(k+1)-1}-y_{n(k)-2}\|+\|y_{n(k)-2}-y_{n(k)}\|,\ \text{for } k=1,2,3,\ldots$$

By (3) and (5), we obtain

$$\lim_{n\to\infty}\|y_{n(k+1)}-y_{n(k)}\|=\varepsilon \qquad (6)$$

Also, by triangular inequality, we have

$$\left|\|y_{n(k+1)}-y_{n(k)}\|-\|y_{n(k+1)-1}-y_{n(k)}\|\right|\le\|y_{n(k+1)}-y_{n(k+1)-1}\|$$

and $$\left|\|y_{n(k+1)-1}-y_{n(k)+1}\|-\|y_{n(k+1)}-y_{n(k)}\|\right|\le\|y_{n(k+1)}-y_{n(k+1)-1}\|+\|y_{n(k)+1}-y_{n(k)}\|$$

It follows from (3) and (6), we obtain

$$\lim_{n\to\infty}\|y_{n(k)}-y_{n(k+1)-1}\|=\lim_{n\to\infty}\|y_{n(k+1)-1}-y_{n(k)+1}\|=\varepsilon \qquad (7)$$

Now, we have

$$\|y_{n(k+1)}-y_{n(k)}\|\le\|y_{n(k+1)}-y_{n(k)+1}\|+\|y_{n(k)+1}-y_{n(k)}\|$$
$$\le\|Ax_{n(k)}-Bx_{n(k+1)-1}\|+\|y_{n(k)+1}-y_{n(k)}\| \qquad (8)$$

By (ii), we obtain

$$\|Ax_{n(k)}-Bx_{n(k+1)-1}\|^p\le\varphi\Big(\alpha\|Sx_{n(k)}-Tx_{n(k+1)-1}\|^p+(1-\alpha)\max\frac{1}{2^p}\Big\{\|Sx_{n(k)}-Ax_{n(k)}\|^p,$$
$$\|Tx_{n(k+1)-1}-Bx_{n(k+1)-1}\|^p,\|Sx_{n(k)}-Bx_{n(k+1)-1}\|^p,\|Tx_{n(k+1)-1}-Ax_{n(k)}\|^p\Big\}\Big)$$

$$\|y_{n(k)+1}-y_{n(k+1)}\|^p\le\varphi\Big(\alpha\|y_{n(k)}-y_{n(k+1)-1}\|^p+(1-\alpha)\max\frac{1}{2^p}\Big\{\|y_{n(k)}-y_{n(k)+1}\|^p,$$
$$\|y_{n(k+1)-1}-y_{n(k+1)}\|^p,\|y_{n(k)}-y_{n(k+1)}\|^p,\|y_{n(k+1)-1}-y_{n(k)+1}\|^p\Big\}\Big)$$

Using (5), (6), (8) and letting $n\to\infty$, we obtain

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$$\varepsilon \le \left(\varphi\left(\alpha\varepsilon^p + (1-\alpha)\frac{1}{2^p}.\varepsilon^p\right)\right)^{1/p}$$

$$= \left(\varphi\left(\alpha + \frac{(1-\alpha)}{2^p}\right)\varepsilon^p\right)^{1/p},\ 0 < \alpha + (1-\alpha).\frac{1}{2^p} < 1,$$

$$< \left(\varphi(\varepsilon^p)\right)^{1/p} < \varepsilon$$

This is a contradiction. Therefore, $\{y_{2n}\}$ is Cauchy sequence and hence $\{y_n\}$ is also Cauchy sequence in $X$. Since, $X$ is Banach space, the sequence $\{y_n\}$ converges to a point $t \in X$. Moreover, $\{Sx_{2n}\},\{Bx_{2n-1}\}$, $\{Tx_{2n+1}\}$ and $\{Ax_{2n}\}$ are subsequences of $\{y_n\}$ so that $Sx_{2n}, Bx_{2n-1}, Tx_{2n+1}, Ax_{2n} \to t \in X$.

Since $B(X) \subseteq S(X)$, there is a point $u \in X$ such that $Su = t \in X$. We claim that $Au = Su = t$. Suppose not, then there exists $\varepsilon > 0$ such that $\|Au - Su\| > \varepsilon$.

Now by (ii), we have

$$\begin{aligned}\|Au - Su\| &= \|Au - t\| \\ &\le \|Au - Bx_{2n-1}\| + \|Bx_{2n-1} - t\| \\ &\le \varphi\Big(\alpha\|Su - Tx_{2n-1}\|^p + (1-\alpha)\max\frac{1}{2^p}\Big\{\|Su - Au\|^p, \|Tx_{2n-1} - Bx_{2n-1}\|^p, \\ &\qquad \|Su - Bx_{2n-1}\|^p, \|Tx_{2n-1} - Au\|^p\Big\}\Big)^{1/p} + \|Bx_{2n-1} - t\|\end{aligned}$$

Letting $n \to \infty$, we obtain

$$\begin{aligned}\|Au - Su\| &\le \left(\varphi\left(\frac{(1-\alpha)}{2^p}\|Su - Au\|^p\right)\right)^{1/p} \\ &< \left((1-\alpha).\frac{1}{2^p}\|Su - Au\|^p\right)^{1/p},\ 0 < (1-\alpha).\frac{1}{2^p} < 1 \\ &< \|Su - Au\|\end{aligned}$$

Thus, $Au = Su = t \in X$ and hence $u$ is the coincidence point of the pair $\{A,S\}$. Therefore, $u \in C(A,S)$. Since, $A(X) \subseteq T(X)$, there exists a point $v \in X$ such that $Tv = t \in X$. We claim that $Bv = Tv = t \in X$. By (*ii*), we obtain

$$\begin{aligned}\|Tv - Bv\| &\le \|Au - Bv\| \\ &\le \left(\varphi\left((1-\alpha).\frac{1}{2^p}\|Tv - Bv\|^p\right)\right)^{1/p} \\ &< \left(\varphi\left(\|Tv - Bv\|^p\right)\right)^{1/p},\ (1-\alpha) < 2^p \\ &< \|Tv - Bv\|\end{aligned}$$

This is contradiction if $Bv \ne Tv$ and hence $v$ is the coincidence point of the pair $\{B,T\}$. So that $v \in C(B,T)$. Therefore, $Au = Su = Bv = Tv = t \in X$.

Since $u \in C(A,S)$ and $v \in C(B,T)$ for some $u, v \in X$. Consequently, weakly *S*-biased of the pair $\{A,S\}$ implies that

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$$\|SAu - Su\| \le \|ASu - Au\|. \qquad (9)$$

Similarly, by weakly $T$- biased of $\{B,T\}$ implies that

$$\|TBv - Tv\| \le \|BTv - Bv\|. \qquad (10)$$

On the other hand, $u \in C(A,S)$ follows that $Au = Su$, then $AAu = ASu$ and $SAu = SSu$.

Similarly, $v \in C(B,T)$ follows that $Bv = Tv$, then $BBv = BTv$ and $TBv = TTv$. Now, we claim that $Au$ is a common fixed point of $A, B, S$ and $T$.

By (*ii*) and (9), we obtain

$$\|AAu - Au\| = \|AAu - Bv\| \le \Big(\varphi\Big(\alpha\|SAu - Tv\|^p + (1-\alpha)\max\frac{1}{2^p}\Big\{\|SAu - AAu\|^p,$$

$$\|Tv - Bv\|, \|SAu - Bv\|^p, \|Tv - Au\|^p, \Big\}\Big)\Big)^{1/p}$$

$$= \Big(\varphi\Big(\alpha\|SAu - Su\|^p + (1-\alpha)\max\frac{1}{2^p}\Big\{\|SAu - ASu\|^p,$$

$$\|SAu - Su\|^p\Big\}\Big)\Big)^{1/p}$$

$$\le \Big(\varphi\Big(\alpha\|SAu - Su\|^p + (1-\alpha)\max\frac{1}{2^p}\Big\{\big(\|SAu - Su\| + \|ASu - Au\|\big)^p,$$

$$\|SAu - Su\|^p\Big\}\Big)\Big)^{1/p}$$

$$\le \Big(\varphi\Big(\|ASu - Au\|^p\Big)\Big)^{1/p}$$

$$< \|ASu - Au\| = \|AAu - Au\|.$$

This is a contradiction and hence $AAu = Au.$ Since, the pair $\{A,S\}$ is weakly S-biased. It follows that

$$\|SAu - Au\| = \|SAu - Su\| \le \|ASu - Au\| = \|AAu - Au\| = 0.$$

Hence , $AAu = SAu = Au.$

Now, by (*ii*) and (10), we obtain

$$\|Au - BAu\| = \|Au - BBv\| \le \Big(\varphi\Big(\alpha\|Su - TBv\|^p + (1-\alpha)\max\frac{1}{2^p}\Big\{\|Su - Au\|^p,$$

$$\|TBv - BBv\|, \|Su - BBv\|^p, \|TBv - Au\|^p\Big\}\Big)\Big)^{1/p}$$

$$= \Big(\varphi\Big(\alpha\|TBv - Tv\|^p + (1-\alpha)\max\frac{1}{2^p}\Big\{\|TBv - BTv\|,$$

$$\|BTv - Bv\|^p, \|TBv - Tv\|^p\Big\}\Big)\Big)^{1/p}$$

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$$\leq \left(\varphi\left(\alpha \|TBv - Tv\|^p + (1-\alpha)\max \frac{1}{2^p}\left\{\left(\|TBv - Tv\| + \|BTv - Bv\|\right)^{1/p},\right.\right.\right.$$

$$\left.\left.\left.\|BTv - Bv\|^p, \|TBv - Tv\|^p\right\}\right)\right)^{1/p}$$

$$\leq \left(\varphi\left(\|BTv - Bv\|^p\right)\right)^{1/p}$$

$$< \|BTv - Bv\| = \|BAu - Au\|$$

This is a contradiction and hence, $BAu = Au.$ Further, weakly *T*-biased of $\{B,T\}$ follows that $\|TAu - Au\| = \|TBv - Tv\| \leq \|BTv - Bv\| = \|BAu - Au\| = 0$ and hence $TAu = Au$. Therefore, $AAu = BAu = SAu = TAu = Au = t \in X$. This shows that $Au = t \in X$ is a common fixed point of $A, B, S$ and $T$.

Now, we show that $Au = t$ is a unique common fixed point of $A, B, S$ and $T$.

Suppose that $t$ and $t'$ be two points in *X* such that $At = St = Bt = Tt = t$ and $At' = St' = Bt' = Tt' = t'$, then by (*ii*), we have

$$\|t - t'\| = \|At - Bt'\| \leq \left(\varphi\left(\alpha \|St - Tt'\|^p\right.\right.$$

$$\left.\left. + (1-\alpha)\max \frac{1}{2^p}\left\{\|St - At\|^p, \|Tt' - Bt'\|^p, \|St - Bt'\|^p, \|Tt' - At'\|^p\right\}\right)\right)^{1/p}$$

$$\leq \left(\varphi\left(\alpha + (1-\alpha)/2^p\right)\|t - t'\|^p\right)^{1/p}$$

$$< \|t - t'\|.$$

This is a contradiction. This completes the proof.□

**Corollary 2.2**. Let $A, B, S$ and $T$ be mappings of a Banach space $X$ into itself satisfying the following conditions (*i*) and (*ii*) of Theorem 2.1. Then, the pairs $\{A,S\}$ and $\{B,T\}$ have coincidence points. Further, if the pairs $\{A,S\}$ and $\{B,T\}$ are weakly compatible, then *A*, *B*, *S* and *T* have a unique common fixed point in $X$.

**Proof**: It can be proof as in Theorem 2.1..

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