ISSN 2347-1921

LEFT FIXED MAPS AND α-DERIVATIONS OF KU-ALGEBRA

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ABSTRACT

In This paper, we introduce the concept of left fixed maps in a KU-algebra and we discuss some related properties of this concept. Moreover, we study the notion of left-right (resp., right-left) α -derivation in a KU-algebra and establish some results on α -derivation in a KU-algebra.

Keywords : KU-algebra; left fixed map; idempotent map; α -derivation.

SUBJECT CLASSIFICATION

Mathematics Subject Classification: 13A15; 06F35; 13L05.

Council for Innovative Research

Peer Review Research Publishing System

Journal: JOURNAL OF ADVANCES IN MATHEMATICS

Vol .9, No 7

www.cirjam.com , editorjam@gmail.com

1. INTRODUCTION

BCK and BCI-algebras are first introduced by Imai and Is´eki [5, 6]. Later on, Prabpayak and Leerawat [12, 13] introduced a new algebraic structure which is called KU-algebra. They gave the concept of homomorphisms of KU-algebras and investigated some related properties. Many authors studied the left and the right fixed maps; see [3, 7, 9]. The notion of derivation is an important topic in mathematics. Usually, the properties of central derivations were discussed in several papers with respect to the ring structures. In [8] Y.B. Jun and X.L. Xin applied the derivation in ring theory to BCI-algebras, and they introduced a new concept called a regular derivation in BCI-algebras. Also, many research articles are studied the derivation by different ways; see [1, 2, 4, 10, 14]. In this paper, we define the concept of a left fixed map in KU-algebra X and then investigate some related properties. Also, by using the definition of idempotent map, we discuss some properties of idempotent left fixed map of X. Moreover, we study the notion of α-derivation in KU-algebra X and establish some results on α-derivation in a KU-algebra.

2. Preliminaries

Now, we will recall some known concepts related to KU-algebra from the literature which will be helpful in further study of this article.

Definition2.1. **[12, 13]** Algebra(X, $*$, 0) of type (2, 0) is said to be a KU-algebra, if it satisfies the following axioms: For all $x, y, z \in X$,

(ku_1) $(x*y)*[(y*z)*(x*z)]=0$,

(ku_2) $x*0=0$,

(ku_3) $0*x=x$,

(ku_4) $x*y=0$ and $y*x=0$ implies $x=y$,

(ku_5) $x*x=0$.

On a KU-algebra $(X,*,0)$ we can define a binary relation $\leq$ on X by putting:

$x\leq y \Leftrightarrow y*x=0$.

Thus KU-algebra X satisfies the conditions:

For all $x, y, z \in X$,

$(ku_{1'})$ $(y*z)*(x*z)\leq(x*y)$,

$(ku_{2'})$ $0\leq x$,

$(ku_{3'})$ $x\leq y, y\leq x$ implies $x=y$,

$(ku_{4'})$ $y*x\leq x$.

Theorem 2.2. **[11]** In a KU-algebra X, the following axioms are satisfied:

For all $x, y, z \in X$,

(1) $x\leq y$ imply $y*z\leq x*z$,

(2) $x*(y*z)=y*(x*z)$, for all $x, y, z \in X$,

(3) $((y*x)*x)\leq y$.

We will refer to X is a KU-algebra unless otherwise indicated.

Definition 2.3. **[12]** A non-empty subset S of a KU-algebra (X,$*$, 0) is called a KU-sub algebra of X if $x*y\in S$ whenever $x, y \in S$.

Definition2.4. **[12, 13]** A non-empty subset I of a KU -algebra (X,*, 0) is called an ideal of X if for any $x, y \in X$,

(i) $0 \in I$,

(ii) $x * y$, $x \in I$ imply $y \in I$.

Definition 2.5. We define $x \dot{\wedge} y = (y * x) * x$, then a KU-algebra $(X,*,0)$ is said to be KU-commutative if it satisfies: for all x, y in X, $(y * x) * x = (x * y) * y$, i.e. $x \dot{\wedge} y = y \dot{\wedge} x$.

Example 2.6. Let $X = \{0, a, b, c, d, e\}$ be a set with the operation $*$ defined by the following table

*	0	a	b	c	d	e
0	0	a	b	c	d	e
a	0	0	b	c	b	c
b	0	a	0	b	a	d
c	0	a	0	0	a	a
d	0	0	0	b	0	b
e	0	0	0	0	0	0

Using the algorithms in Appendix A, we can prove that $(X,*,0)$ is a KU-algebra and $\{0, a\}, \{0, b, c\}$ are ideals of X.

3. Left fixed maps.

Definition 3.1. A left fixed map α of X is defined to be a self map $\alpha : X \to X$ satisfying $\alpha(x * y) = x * \alpha(y)$ for all $x, y \in X$.

Example 3.2. Let $X = \{0, a, b, c, d\}$ be a set with the operation $*$ defined by the following table:

*	0	a	b	c	d
0	0	a	b	c	d
a	0	0	b	c	d
b	0	a	0	c	c
c	0	0	b	0	b
d	0	0	0	0	0

Using the algorithms in Appendix A, we can prove that $(X,*,0)$ is a KU-algebra and the self map α of X defined by $\alpha(0) = 0$, $\alpha(a) = 0$, $\alpha(b) = b$, $\alpha(c) = 0$ and $\alpha(d) = b$, it is easy to verify that α is a left fixed map of X.

Example 3.3. In Example 2.6. Let $\alpha : X \to X$ be defined by $\alpha(0) = 0$, $\alpha(a) = 0$, $\alpha(b) = b$, $\alpha(c) = c$, $\alpha(d) = b$ and $\alpha(e) = d$. Then α is not a left fixed map of X since $c = \alpha(a * e) \neq a * \alpha(e) = b$.

Lemma 3.4. If α is a left fixed map of X, then

(i) $\alpha(0) = 0$,

(ii) $\forall x \in X, \alpha(x * 0) = 0$,

(iii) $\forall x \in X, \alpha(x) \leq x$,

(iv) $\forall x, y \in X, x \leq y \Rightarrow \alpha(x) \leq y$.

Proof.

(i) $\forall x, y \in X$, we have $\alpha(0) = \alpha(\overbrace{\alpha(0) * 0}^{by\ (KU_2)}) = \alpha(0) * \alpha(0) = 0$.

(ii) $\forall x, y \in X$, we have $\alpha(x * 0) = \alpha(0) = 0$.

(iii) For any $x \in X$, we get $0 = \alpha(0) = \alpha(x * x) = x * \alpha(x)$, hence $\alpha(x) \leq x$.

(iv) Suppose that $x \leq y$ for every $x, y \in X$, then $y * x = 0$ it follows that $0 = \alpha(0) = \alpha(y * x) = y * \alpha(x)$, hence $\alpha(x) \leq y$.

Lemma 3.5. If α and β are left fixed maps of X, then $\alpha \circ \beta$ is a left fixed map of X.

Proof. Let α and β be left fixed maps of X. Then, for all $x, y \in X$ $(\alpha \circ \beta)(x * y) = \alpha(\beta(x * y) = \alpha(x * \beta(y)) = x * \alpha(\beta(y)) = x * (\alpha \circ \beta)(y)$. Hence $\alpha \circ \beta$ is a left fixed map of X.

Definition 3.6. For a left fixed map α of X, the kernel of α denoted by $\ker(\alpha)$, is defined to be the set $\ker(\alpha) = \{x \in X : \alpha(x) = 0\}$.

Lemma 3.7. Let α be a left fixed map of X. Then $\ker(\alpha)$ is subalgebra of X.

Proof. Since $0 \in \ker(\alpha)$, so $\ker(\alpha) \neq \phi$. Let $x, y \in \ker(\alpha)$, then $\alpha(x) = 0$ and $\alpha(y) = 0$. It follows that $\alpha(x * y) = x * \alpha(y) = x * 0 = 0$, hence $x * y \in \ker(\alpha)$. Thus $\ker(\alpha)$ is subalgebra of X.

Theorem 3.8. Let α be a left fixed map of X. Then α is one to one if and only if $\ker(\alpha) = 0$.

Proof. Suppose that α is one to one and $x \in \ker(\alpha)$. Then $\alpha(x) = 0 = \alpha(0)$, and thus $x = 0$, i.e., $\ker(\alpha) = \{0\}$.

Conversely, suppose that $\ker(\alpha) = \{0\}$. Let $x, y \in X$ be such that $\alpha(x) = \alpha(y)$. It follows that $\alpha(x * y) = x * \alpha(y) = x * \alpha(x) = \alpha(x * x) = \alpha(0) = 0$. Hence $x * y \in \ker(\alpha)$, and so $x * y = 0$. Similarly, $y * x = 0$ thus $x = y$. Therefore α is one to one.

Theorem 3.9. Let α be a left fixed map of X. Then α is one to one if and only if α is the identity map.

Proof. Sufficiency is obvious. Suppose that α is one to one. For every $x \in X$, we have

$\alpha(\alpha(x) * x) = \alpha(x) * \alpha(x) = 0 = \alpha(0)$ and so $\alpha(x) * x = 0$, i.e., $x \leq \alpha(x)$. Since $\overbrace{\alpha(x) \leq x}^{from\ Lemma\ 3.4\ (iii)}$ for all $x \in X$, it follows that $\alpha(x) = x$ thus α is the identity map.

Lemma 3.10. Let X be a commutative KU-algebra. If $x \in \ker(\alpha)$ and $y \leq x$, then $y \in \ker(\alpha)$.

Proof. Let $x \in \ker(\alpha)$ and $y \leq x$. Then $\alpha(x) = 0$ and $x * y = 0$.

$\alpha(y) = \alpha(0 * y) = \alpha((x * y) * y) = \alpha((y * x) * x) = (y * x) * \alpha(x) = (y * x) * 0 = 0$, thus $y \in \ker(\alpha)$.

Lemma 3.11. Let α be an endomorphism left fixed map of X. Then $\ker(\alpha)$ is an ideal of X.

Proof. Clearly, $0 \in \ker(\alpha)$. Let $x \in \ker(\alpha)$ and $x * y \in \ker(\alpha)$. Then we have $\alpha(x) = 0$ and $\alpha(x * y) = 0$, thus $0 = \alpha(x * y) = \alpha(x) * \alpha(y) = 0 * \alpha(y) = \alpha(y)$. This implies $y \in \ker(\alpha)$. Hence $\ker(\alpha)$ is an ideal of X.

Theorem 3.12. Let α be a left fixed map of X. If α is idempotent, i.e., $\alpha(\alpha(x)) = \alpha(x)$ for all $x \in X$, then

(i) $\ker(\alpha) \cap \mathrm{Im}(\alpha) = \{0\}$,

(ii) $\alpha(x) = x \Leftrightarrow x \in \mathrm{Im}(\alpha)$, for all $x \in X$.

Proof. (i) If $x \in \ker(\alpha) \bigcap \mathrm{Im}(\alpha)$, then $\alpha(x) = 0$ and $\alpha(y) = x$ for some $y \in X$. It follows that $0 = \alpha(x) = \alpha(\alpha(y)) = \alpha(y) = x$, thus $\ker(\alpha) \bigcap \mathrm{Im}(\alpha) = \{0\}$.

(ii) Sufficiency is obvious. If $x \in \mathrm{Im}(\alpha)$, then $\alpha(y) = x$ for some $y \in X$. Thus $\alpha(x) = \alpha(\alpha(y)) = \alpha(y) = x$.

Denote by $LF(X)$ the set of all left fixed maps of X. Let $\oplus$ be a binary operation on $LF(X)$ defined by $(\alpha \oplus \beta)(x) = \alpha(x) * \beta(x)$ for all $\alpha, \beta \in LF(X)$ and $x \in X$. It is easy to verify that $(LF(X), \oplus)$ is a KU-algebra. Let $ILF(X)$ be the set of all idempotent left fixed maps of X.

Theorem 3.13. For every $\alpha, \beta \in ILF(X)$, if $\alpha \oplus \beta = 0$ in $LF(X)$, then $\mathrm{Im}(\beta) \subset \mathrm{Im}(\alpha)$.

Proof. Let $\alpha, \beta \in ILF(X)$ satisfy $\alpha \oplus \beta = 0$. If $y \in \mathrm{Im}(\beta)$, then by Theorem 3.12 $\beta(y) = y$. Hence $0 = (\alpha \oplus \beta)(y) = \alpha(y) * \beta(y) = \alpha(y) * y$, i.e., $y \leq \alpha(y)$. Combining this with Lemma 3.4(iii), we get $y = \alpha(y) \in \mathrm{Im}(\alpha)$. Hence $\mathrm{Im}(\beta) \subset \mathrm{Im}(\alpha)$.

Theorem 3.14. Let $\alpha, \beta \in ILF(X)$, then

(i) If $\alpha(\beta(x)) = \beta(\alpha(x))$ for all $x \in X$, then $\alpha \oplus \beta \in ILF(X)$,

(ii) If $\mathrm{Im}(\beta) \subset \mathrm{Im}(\alpha)$ and $\alpha(\beta(x)) = \beta(\alpha(x))$ for all $x \in X$, then $\alpha \oplus \beta = 0$ in $LF(X)$,

(iii) $\mathrm{Im}(\beta) \bigcap \ker(\alpha) \subset \mathrm{Im}(\alpha \oplus \beta)$.

Proof. (i) Assume that $\alpha(\beta(x)) = \beta(\alpha(x))$ for all $x \in X$. Then

$$\begin{aligned}(\alpha \oplus \beta)((\alpha \oplus \beta)(x)) &= (\alpha \oplus \beta)(\alpha(x) * \beta(x)) \\ &= \alpha(\alpha(x) * \beta(x)) * \beta(\alpha(x) * \beta(x)) \\ &= (\alpha(x) * \alpha(\beta(x))) * (\alpha(x) * \beta(\beta(x))) \\ &= (\alpha(x) * \beta(\alpha(x))) * (\alpha(x) * \beta(x)) \\ &= \beta(\alpha(x) * \alpha(x)) * (\alpha(x) * \beta(x)) \\ &= \beta(0) * (\alpha \oplus \beta)(x) \\ &= (\alpha \oplus \beta)(x).\end{aligned}$$

That is $(\alpha \oplus \beta)$ is idempotent, hence $\alpha \oplus \beta \in ILF(X)$.

(ii) Suppose that $\mathrm{Im}(\beta) \subset \mathrm{Im}(\alpha)$ and $\alpha(\beta(x)) = \beta(\alpha(x))$ for all $x \in X$.

Since $\beta(x) \in \mathrm{Im}(\beta) \subset \mathrm{Im}(\alpha)$ for all $x \in X$, it follows from theorem 3.12 that

$(\alpha \oplus \beta)(x) = \alpha(x) * \beta(x) = \alpha(x) * \alpha(\beta(x)) = \alpha(x) * \beta(\alpha(x)) = \beta(\alpha(x) * \alpha(x)) = \beta(0) = 0$, for all $x \in X$, hence $\alpha \oplus \beta = 0$.

(iii) If $y \in \mathrm{Im}(\beta) \bigcap \ker(\alpha)$, then $\beta(x) = y$ and $\alpha(y) = 0$ for some $x \in X$. It follows from ku_5 that $y = \beta(x) = 0 * \beta(\beta(x)) = \alpha(y) * \beta(y) = (\alpha \oplus \beta)(y) \in \mathrm{Im}(\alpha \oplus \beta)$. Hence $\mathrm{Im}(\beta) \bigcap \ker(\alpha) \subset \mathrm{Im}(\alpha \oplus \beta)$.

4. The derivations of left fixed maps in KU-algebras.

In this section, we introduce the notion of the derivations fixed maps and investigate their properties in KU-algebra.

In what follows, let α be a left fixed map and d_α be a self map of X.

Definition 4.1. A self map d_α of X is called a $(l,r)_\alpha$ - derivation of X if it satisfies: $d_\alpha(x*y)=(d_\alpha(x)*\alpha(y))\dot\wedge(\alpha(x)*d_\alpha(y))$ for all $x,y\in X$.

If d_α satisfies the following: $d_\alpha(x*y)=(\alpha(x)*d_\alpha(y))\dot\wedge(d_\alpha(x)*\alpha(y))$ for all $x,y\in X$, then it is called a $(r,l)_\alpha$ - derivation of X . If d_α is both a $(l,r)_\alpha$ - derivation and a $(r,l)_\alpha$ - derivation of X , we say that d_α is a α - derivation of X .

In definition above, if α is the identity map, then d_α is denoted by d and is called a (l,r) - derivation (resp. a (r,l) - derivation) of X .

Definition 4.2. A α - derivation of a KU-algebra is called regular if $d_\alpha(0)=0$.

Lemma 4.3. A α - derivation d_α of a KU-algebra is regular.

Proof.

If d_α is a $(l,r)_\alpha$ -derivation of X ,

$$\begin{aligned} d_\alpha(0)=d_\alpha(x*0)&=(d_\alpha(x)*\alpha(0))\dot\wedge(\alpha(x)*d_\alpha(0))\\ &=(d_\alpha(x)*0)\dot\wedge(\alpha(x)*d_\alpha(0))\\ &=0\dot\wedge(\alpha(x)*d_\alpha(0))=[(\alpha(x)*d_\alpha(0))*0]*0=0 \end{aligned}$$

If d_α is a $(r,l)_\alpha$ -derivation of X ,

$$\begin{aligned} d_\alpha(0)=d_\alpha(x*0)&=(\alpha(x)*d_\alpha(0))\dot\wedge(d_\alpha(x)*\alpha(0))\\ &=(\alpha(x)*d_\alpha(0))\dot\wedge(\alpha(x)*0)\\ &=(\alpha(x)*d_\alpha(0))\dot\wedge 0\\ &=[0*(\alpha(x)*d_\alpha(0))]*(\alpha(x)*d_\alpha(0))\\ &=(\alpha(x)*d_\alpha(0))*(\alpha(x)*d_\alpha(0))=0 \end{aligned}$$

Example 4.4. In Example 3.2. Define a map $d_\alpha : X\to X$ by

$$d_\alpha(x)=\begin{cases} 0 & x\in\{0,a,b,c\}\\ b & x=d \end{cases}$$

Then it is easy to show that d_α is both a $(l,r)_\alpha$ and $(r,l)_\alpha$ - derivation of X .

Lemma 4.5. Let d_α be a self map of KU-algebra X , then

(i) If d_α is a $(l,r)_\alpha$ -derivation of X , then $d_\alpha(x)=\alpha(x)\dot\wedge d_\alpha(x)$ for all $x\in X$.

(ii) If d_α is a $(r,l)_\alpha$ -derivation of X , then $d_\alpha(x)=d_\alpha(x)\dot\wedge\alpha(x)$ for all $x\in X$.

Proof. (i) Let d_α be a $(l,r)_\alpha$ -derivation of X , then

$$\begin{aligned} d_\alpha(x)=d_\alpha(0*x)&=(d_\alpha(0)*\alpha(x)\dot\wedge(\alpha(0)*d_\alpha(x))\\ &=(0*\alpha(x))\dot\wedge(0*d_\alpha(x))\\ &=\alpha(x)\dot\wedge d_\alpha(x) \end{aligned}$$

(ii)Let d_α be a $(r,l)_\alpha$ -derivation of X , then

$$d_\alpha(x) = d_\alpha(0*x) = (\alpha(0)*d_\alpha(x)) \dot\wedge (d_\alpha(0)*\alpha(x))$$
$$= (0*d_\alpha(x)) \dot\wedge (0*\alpha(x))$$
$$= d_\alpha(x) \dot\wedge \alpha(x).$$

Lemma 4.6. Let d_α be a $(r,l)_\alpha$-derivation of X, then for all $x, y \in X$,

(1) $d_\alpha(x) \le \alpha(x)$,

(2) $d_\alpha(x*y) \le x*\alpha(y)$.

Proof. Let d_α be a $(r,l)_\alpha$-derivation of X, then

(1) From Lemma 4.5(ii)

$$d_\alpha(x) = d_\alpha(x) \dot\wedge \alpha(x)$$
$$= (\alpha(x)*d_\alpha(x))*d_\alpha(x) \le \alpha(x).$$

(2) from (1) $d_\alpha(x*y) \le \alpha(x*y) = x*\alpha(y)$.

Definition 4.7. Let d_α be a α-derivation of a KU-algebra X. Then, d_α is said to be an isotone α-derivation if $x \le y \Rightarrow d_\alpha(x) \le d_\alpha(y)$ for all $x, y \in X$.

Lemma 4.8. Let X be a KU-algebra and d_α be a α-derivation on X. For all $x, y \in X$, if $d_\alpha(x*y) = d_\alpha(x)*d_\alpha(y)$, then d_α is an isotone α-derivation.

Proof. Let $d_\alpha(x*y) = d_\alpha(x)*d_\alpha(y)$. If $x \le y \Rightarrow y*x = 0$ for all $x, y \in X$. Then, we have

$$d_\alpha(x) = d_\alpha(0*x)$$
$$= d_\alpha((y*x)*x)$$
$$= d_\alpha(y*x)*d_\alpha(x)$$
$$= [d_\alpha(y)*d_\alpha(x)]*d_\alpha(x)$$
$$\le d_\alpha(y)$$

Thus $d_\alpha(x) \le d_\alpha(y)$ which implies that d_α is an isotone α-derivation.

Lemma 4.9. Let X be a KU-algebra with partial order $\le$, and d_α be a α-derivation of X. Then the following hold for all $x, y \in X$:

(i) $d_\alpha(x*y) \le d_\alpha(x)*\alpha(y)$

(ii) $d_\alpha(x*y) \le \alpha(x)*d_\alpha(y)$

(iii) $\ker d_\alpha = \{x \in X : d_\alpha(x) = 0\}$ is a subalgebra of X.

Proof. (i)

$$(d_\alpha(x)*\alpha(y))*d_\alpha(x*y) = (d_\alpha(x)*\alpha(y))*\overbrace{[(d_\alpha(x)*\alpha(y)) \dot\wedge (\alpha(x)*d_\alpha(y))]}^{(l,r)_\alpha - derivation}$$
$$= (d_\alpha(x)*\alpha(y))*\{[(\alpha(x)*d_\alpha(y))*(d_\alpha(x)*\alpha(y))]*(d_\alpha(x)*\alpha(y))\}$$
$$= \overbrace{[(\alpha(x)*d_\alpha(y))*(d_\alpha(x)*\alpha(y))]*\{(d_\alpha(x)*\alpha(y))*(d_\alpha(x)*\alpha(y))\}}^{Th.2(2)}$$
$$= [(\alpha(x)*d_\alpha(y))*(d_\alpha(x)*\alpha(y))]*0 = 0$$

Then $d_\alpha(x*y) \le d_\alpha(x)*\alpha(y)$

Similarly, if d_α is a $(r,l)_\alpha$-derivation of X, then $d_\alpha(x*y) \le d_\alpha(x)*\alpha(y)$.

(ii) If d_α is $(l,r)_\alpha$-derivations of X, we have

$$\begin{aligned}(\alpha(x)*d_\alpha(y))*d_\alpha(x*y) &= (\alpha(x)*d_\alpha(y))*\overbrace{[(d_\alpha(x)*\alpha(y))\,\dot\wedge\,(\alpha(x)*d_\alpha(y))]}^{(l,r)_\alpha-derivation}\\ &= (\alpha(x)*d_\alpha(y))*\{[(\alpha(x)*d_\alpha(y))*(d_\alpha(x)*\alpha(y))]*(d_\alpha(x)*\alpha(y))\}\\ &= \overbrace{[(\alpha(x)*d_\alpha(y))*(d_\alpha(x)*\alpha(y))]*\{(\alpha(x)*d_\alpha(y))*(d_\alpha(x)*\alpha(y))\}}^{Th.2(2)}\\ &= 0\end{aligned}$$

Then $d_\alpha(x*y) \le \alpha(x)*d_\alpha(y)$

Similarly, if d_α is a $(r,l)_\alpha$-derivation of X, then $d_\alpha(x*y) \le \alpha(x)*d_\alpha(y)$.

(iii) We have $d_\alpha(0)=0$, then $\ker d_\alpha \ne \phi$. Let $x, y \in \ker d_\alpha$, then $d_\alpha(x)=0, d_\alpha(y)=0$, $d_\alpha(x*y) = (d_\alpha(x)*\alpha(y))\,\dot\wedge\,(\alpha(x)*d_\alpha(y)) = (0*\alpha(y))\,\dot\wedge\,(\alpha(x)*0) = \alpha(y)\,\dot\wedge\,0 = 0$. Hence $x*y \in \ker d_\alpha$. Therefore, $\ker d_\alpha = \{x \in X : d_\alpha(x)=0\}$ is a subalgebra of X.

Definition 4.10. An ideal A of KU-algebra X is said to be an α-ideal if $\alpha(A) \subseteq A$.

Definition 4.11. Let d_α be a self map of a KU-algebra X. An α-ideal A of X is said to be d_α-invariant if $d_\alpha(A) \subseteq A$.

Theorem 4.12. Let d_α be a regular $(r,l)_\alpha$-derivation of a KU-algebra X, then every α-ideal A of X is d_α-invariant.

Proof. By Lemma 4.6(1), we have $d_\alpha(x) \le \alpha(x)$ for all $x \in X$. Let $y \in d_\alpha(A)$. Then $y = d_\alpha(x)$ for some $x \in A$. It follows that $\alpha(x)*y = \alpha(x)*d_\alpha(x) = 0 \in A$. Since $x \in A$, then $\alpha(x) \in \alpha(A) \subseteq A$ as A is an α-ideal. It follows that $y \in A$ since A is an ideal of X. Hence $d_\alpha(A) \subseteq A$, and thus A is d_α-invariant.

Theorem 4.13. Let d_α be a α-derivation of a KU-algebra X, then d_α is regular if and only if every α-ideal of X is d_α-invariant.

Proof. Let d_α be a α-derivation of a KU-algebra X and assume that every α-ideal of X is d_α-invariant. Then since the zero ideal $\{0\}$ is α-ideal and d_α-invariant, we have $d_\alpha(\{0\}) \subseteq \{0\}$, which implies that $d_\alpha(0)=0$. Thus d_α is regular. Combining this and Theorem 4.12, the proof is complete.

5.Conclusion.

We have introduced the concept of left fixed maps in KU-algebra X. Also, by using the definition of idempotent map, we discussed some properties of idempotent left fixed maps of X. In the present paper, the notion of left-right (resp., right-left) α-derivation is introduced and investigated the useful properties of these types derivations in KU-algebras, for example, we have proved that, if d_α is a regular $(r,l)_\alpha$-derivation of a KU-algebra X, then every α-ideal A of X is d_α-invariant.

In our opinion, these definitions and main results can be similarly extended to some other algebraic systems such as BCH-algebra, Hilbert algebra, BF-algebra, J-algebra, WS-algebra, CI-algebra, SU-algebra, BCL-algebra, BP-algebra, Coxeter algebra, BO-algebra and so forth.

The main purpose of our future work is to investigate the derivations ideals in KU-algebras, which may have a lot of applications in different branches of theoretical physics and computer science.

ISSN 2347-1921

Appendix A. Algorithms

This appendix contains all necessary algorithms

Algorithm for KU-algebras

Input (X :set, $*$:binary operation)

Output (“ X is a KU-algebra or not”)

Begin

If $X=\phi$ then go to (1.);

EndIf

If $0\notin X$ then go to (1.);

EndIf

Stop: =false;

$i:=1;$

While $i\le |X|$ and not (Stop) do

If $x_i * x_i \ne 0$ then

Stop: = true;

EndIf

$j:=1$

While $j\le |X|$ and not (Stop) do

If $((y_j * x_i)*x_i)\ne 0$ then

Stop: = true;

EndIf

EndIf

$k:=1$

While $k\le |X|$ and not (Stop) do

If $(x_i * y_j)*((y_j * z_k)*(x_i * z_k))\ne 0$ then

Stop: = true;

EndIf

EndIf While

EndIf While

EndIf While

If Stop then

(1.) Output (“ X is not a KU-algebra”)

Else

Output (“ X is a KU-algebra”)

EndIf

End

Algorithm for an ideal

Input (X :KU-algebra, I :subset of X);

Output ("I is an ideal of X or not");

Begin

If $I = \phi$ then go to (1.);

EndIf

If $0 \notin I$ then go to (1.);

EndIf

Stop: =false;

$i := 1;$

While $i \le |X|$ and not (Stop) do

$j := 1$

While $j \le |X|$ and not (Stop) do

If $(x_i * y_j) \in I$ and $x_i \in I$ then

If $y_j \notin I$ then

Stop: = true;

EndIf

EndIf

EndIf While

EndIf While

EndIf While

If Stop then

Output ("I is an ideal of X ")

Else

(1.) Output ("I is not ideal of X ")

EndIf

End

ACKNOWLEDGMENTS

The authors are thankful to the referees for a careful reading of the paper and for valuable comments and suggestions.

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