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ON ANTI-FUZZY IDEALS OF MΓ-GROUPS

Nagaraju Dasari

Manipal University Jaipur, Jaipur-Ajmer Expressway, Jaipur-, INDIA.

dasari.nagaraju@gmail.com

ABSTRACT

We derive results related to level sets, cosets with respect to anti-fuzzy ideals in MΓ-groups

Keywords

MΓ-Group, Anti-fuzzy Ideal, Anti-fuzzy coset.

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1. INTRODUCTION

Γ-near-ring, introduced by Satyanarayana [8] generalizes both near-ring and Γ-ring. Booth and Grenewald [2], Satyanarayana [9] were studied and developed the concept of MΓ-groups. Given a Γ-near-ring M, an additive group G is said to be a *Γ-near-ring-module over M* (or *MΓ-group* or *MΓ-module*) if the following two conditions hold:

(i) $(m_1 + m_2)\gamma_1 g = m_1\gamma_1 g + m_2\gamma_1 g$; and

(ii) $(m_1\gamma_1 m_2)\gamma_2 g = m_1\gamma_1(m_2\gamma_2 g)$ for $m_1, m_2 \in M$, $\gamma_1, \gamma_2 \in \Gamma$ and $g \in G$.

Fuzzy sets introduced by Zadeh [15], has created interest among the researchers and motivated them to introduce and develop the concept of fuzziness in several mathematical systems. Studies on anti-fuzzy sets in algebraic systems were started in the 1990's with Biswas [1]. Fuzzy MΓ-subgroups were studied by Jun, Kwon and Park [3], later on Kim, Jun and Yon [4] were studied Anti-fuzzy ideals in near-rings. Srinivas, Nagaiah and Narasimha Swamy [13] were studied Anti-fuzzy ideals in Γ-near-rings. Fuzzy ideals of MΓ-groups were studied by Nagaraju, Satyanarayana, Babu Prasad and Venkatachalam [6], Satyanarayana, Vijaya Kumari, Godloza & Nagaraju [12].

2. Anti-fuzzy Ideals

2.1 Definition: A mapping $\nu: G \to [0, 1]$ is said to be an *anti-fuzzy ideal* of G if it satisfy the following axioms:

(i) $\nu(x - y) \le \max\{\nu(x), \nu(y)\}$;

(ii) $\nu(x + y - x) \le \nu(y)$;

(iii) $\nu(m\gamma(a + x) - m\gamma a) \le \nu(x)$ for all $m \in M$, $\gamma \in \Gamma$ and $a, x, y \in G$.

2.2 Example: If $\nu: \mathbb{Z} \to [0, 1]$ defined by

$$\nu(x) = \begin{cases} 0.2, & if\ x = 4n, n \in Z\ or\ x = 0 \\ 0.6, & if\ x = 2m, where\ m \in Z\ but\ not\ of\ the\ form\ x = 4k,\ for\ some\ k \in Z \\ 1, & otherwise \end{cases},$$

then ν is an anti-fuzzy ideal of $\mathbb{Z}$.

2.3 Note: If ν is an anti-fuzzy ideal of G, then (i) $\nu(0) \le \nu(g)$; (ii) $\nu(0) = \inf_{g \in G} \nu(g)$; (iii) $\nu(-g) = \nu(g)$; (iv) $\nu(g_1 + g_2) = \nu(g_2 + g_1)$ for all $g, g_1, g_2 \in G$.

2.4 Thorem: Let G be an MΓ-group. Then a fuzzy set ν is an anti-fuzzy ideal of G if and only if ν^c is a fuzzy ideal of G.

2.5 Theorem: Let ν be an anti-fuzzy ideal of the MΓ-group G. If $\nu(x - y) = \nu(0)$, then $\nu(x) = \nu(y)$.

2.6 Note: The converse of the above theorem is not true. If we consider two odd integers for x and y in example 2.2, then $\nu(x) = 1 = \nu(y)$, which means $\nu(x) = \nu(y)$ whereas $\nu(x - y)$ may not be equal to $\nu(0)$.

2.7 Result: If ν is an anti-fuzzy ideal of MΓ-group G, and $x, y \in G$ with $\nu(x) \ne \nu(y)$, then $\nu(x - y) = \max\{\nu(x), \nu(y)\}$.

Proof: Without loss of generality suppose that $\nu(x) < \nu(y)$. By the definition $\nu(x - y) \le \max\{\nu(x), \nu(y)\} = \nu(y)$ … (i)

Now $\nu(y) = \nu(y + x - x) \le \max\{\nu(y + x), \nu(x)\} = \max\{\nu(x + y), \nu(x)\}$. Suppose if $\nu(x - y) \le \nu(x)$, then $\nu(y) \le \nu(x)$, a contradiction

If $\nu(x - y) \ge \nu(x)$, then $\nu(y) \le \nu(x - y)$….(ii) From (i) and (ii), we have $\nu(x - y) = \nu(y) = \max\{\nu(x), \nu(y)\}$.

2.8 Result: If $\{\nu_i / i \in I\}$ is a family of anti-fuzzy ideals of MΓ-group G, then $\bigvee_{i \in I} \nu_i$ is also an anti-fuzzy ideal of G.

2.9 Definition: Let G be a MΓ-group and ν be any anti-fuzzy ideal of G. For any $t \in [\nu(0), 1]$, the set $\nu_t = \{x \in G / \nu(x) \le t\}$ is called *anti level subset* of ν.

2.10 Theorem: A fuzzy mapping defined on MΓ-group is an anti-fuzzy ideal of G if and only if ν_t is an ideal of G.

Proof: Suppose $\nu: G \to [0, 1]$ is an anti-fuzzy ideal of G. Let t be such that $\nu(0) \le t \le 1$. Since $\nu(0) \le t$, we have that $0 \in \nu_t = \{x / \nu(x) \le t\}$. So ν_t is a non-empty subset of G. Let $x, y \in \nu_t$. Then $\nu(x) \le t$ and $\nu(y) \le t$ and so $\nu(x - y) \le \max\{\nu(x), \nu(y)\} \le \min\{t, t\} = t$ which implies $x - y \in \nu_t$. So $(\nu_t, +)$ is a subgroup of (G, +). Let $g \in G$. Now $\nu(g + x - g) \le \nu(x) \le t$ which implies $g + x - g \in \nu_t$. So $(\nu_t, +)$ is a normal subgroup of (G, +). Let $m \in M$, $\gamma \in \Gamma$. Now $\nu[m\gamma(g + x) - m\gamma g] \le \nu(x) \le t$ and so $m\gamma(g + x) - m\gamma g \in \nu_t$. Hence ν_t is an ideal of the MΓ-group G.

Now suppose that ν_t is an ideal of G. Let x, y $\in$ G and write t = max{ν(x), ν(y)}. Now x, y $\in \nu_t$ which implies x – y $\in \nu_t$ and so ν(x - y) ≤ t. Therefore ν(x - y) ≤ max{ν(x), ν(y)}. Write t = ν(x) implies that x $\in \nu_t$ and g + x – g $\in \nu_t$ for any g $\in$ G. Thus ν(g + x - g) ≤ t = ν(x) implies that ν(g + x - g) ≤ ν(x). Since x $\in \nu_t$ and ν_t is an ideal of G, it follows that mγ(g + x) – mγg $\in \nu_t$ for any m $\in$ M, $\gamma \in \Gamma$, g $\in$ G. So ν[mγ(g + x) – mγg] ≤ t = ν(x) for m $\in$ M, $\gamma \in \Gamma$, x, g $\in$ G.

2.11 Proposition: Let ν be an anti-fuzzy ideal of the MΓ-group G, and ν_t, ν_s (with s < t) be two anti level ideals of ν. Then the following two conditions are equivalent:

(i) $\nu_t = \nu_s$; and

(ii) there is no x $\in$ G such that s < ν(x) ≤ t.

Proof: (i) $\Rightarrow$ (ii) In a contrary way, suppose that there exists an element x $\in$ G such that s < ν(x) ≤ t. Then x $\in \nu_t$ and x $\notin \nu_s$, a contradiction.

(ii) $\Rightarrow$ (i) Let x $\in \nu_s \Rightarrow \nu$(x) ≤ s < t $\Rightarrow \nu$(x) < t $\Rightarrow$ x $\in \nu_t$.

Now take x $\in \nu_t \Rightarrow \nu$ (x) ≤ t, by the assumption there is no y $\in$ G such that s < ν(y) ≤ t and so ν(x) ≤ s $\Rightarrow$ x $\in \nu_s$. Hence $\nu_t = \nu_s$.

2.12 Notation: FI(x) denote the family of all ideals of the MΓ-group G which contain x.

2.13 Theorem: Let ν be an anti-fuzzy ideal of the MΓ-group G.

(i). If FI(x) $\subset$ FI(y), then ν(y) ≤ ν(x)

(ii). If FI(x) = FI(y), then ν(y) = ν(x)

Proof: (i) Suppose FI(x) $\subset$ FI(y) also suppose that ν(y) > ν(x). By letting t = ν(x), x $\in \nu_t$ but y $\notin \nu_t$ means that $\nu_t \in$ FI(x) and $\nu_t \notin$ FI(y), a contradiction. (ii) follows from (i).

2.14 Theorem: If I is an ideal of MΓ-group G, then for each t $\in$ [ν(0), 1], there exists an anti-fuzzy ideal ν of G such that ν_t = I.

Proof: Let G be a MΓ-group and I an ideal of G. For any t $\in$ [ν(0), 1] define ν: G $\rightarrow$ [0, 1] by $\nu(x) = \begin{cases} t, & if\, x \in I \\ s, & otherwise \end{cases}$ for all x $\in$ G, where t < s. Clearly ν_t = I for all t $\in$ [ν(0), 1). For x, y $\in$ G, if x, y $\in$ I, then x - y $\in$ I and so ν(x - y) = t ≤ max{t, t} = max{ν(x), ν(y)}. If x $\in$ I and y $\notin$ I, then x - y $\notin$ I and so ν(x - y) = s ≤ max{t, s} = max{ν(x), ν(y)}. Similarly, we can observe that ν(x - y) ≤ max{ν(x), ν(y)} in the case x $\notin$ I and y $\in$ I. If x $\notin$ I, y $\notin$ I, then ν(x - y) ≤ s = max{s, s} = max{ν(x), ν(y)}. Take x $\in$ I. Since I is an ideal of G, we have that y + x - y $\in$ I and so ν(y + x - y) = t = ν(x). If ν(y + x - y) = s, then y + x - y $\notin$ I and so x $\notin$ I. This shows that ν(y + x - y) = s = ν(x). So ν(y + x - y) = ν(x) for all x, y $\in$ G. Take x $\in$ I, g $\in$ G, $\gamma \in \Gamma$ and m $\in$ M. Since I is an ideal of G, we have that mγ(g + x) – mγg $\in$ I.

Therefore ν(mγ(g + x) - mγg) = t = ν(x). If x $\notin$ I, then ν(mγ(g + x) – mγg) ≤ s = ν(x). Therefore for all x $\in$ G, we have that ν[mγ(g + x) – mγg] ≤ ν(x).

2.15 Notation: For an anti-fuzzy ideal ν of G, we write G_ν = {x $\in$ G/ ν(x) = ν(0)}. Clearly G_ν is an ideal of G.

2.16 Definition: Let G and G^1 be two sets and h a function from G into G^1. Let ν and ν^1 be fuzzy sets on G and G^1 respectively. Then

(i). the *image of* ν under h, h(ν) is a fuzzy set in G^1 and is defined as $h(\nu)(y^1) = \begin{cases} \inf\limits_{h(x)=y^1} \nu(x) & \text{if } h^{-1}(y^1) \neq \phi \\ 0 & \text{if } h^{-1}(y^1) = \phi \end{cases}$ for all $y^1 \in G^1$; and

(ii). $h^{-1}(\nu^1)$ the *pre-image* of ν^1 under h is a fuzzy set in G and it is defined as $h^{-1}(\nu^1)(x) = \nu^1(h(x))$ for all x $\in$ G.

2.17 Definition: Let G and G^1 be two sets, ν is a fuzzy set on G and h: G $\rightarrow G^1$ a function. Then ν is called *h-invariant* if h(x) = h(y) implies ν(x) = ν(y) for all x, y $\in$ G.

2.18 Note: (i) If ν is h-invariant, then $h^{-1}(h(\nu)) = \nu$.

(ii) If h is onto, then $h(h^{-1}(\gamma)) = \gamma$, where γ is anti-fuzzy ideal of G^1.

2.19 Theorem: If ν is anti-fuzzy ideal of the MΓ-group G, then h(ν) is anti-fuzzy ideal of the MΓ-group G^1; and if ν^1 is anti-fuzzy ideal of the MΓ-group G^1, then $h^{-1}(\nu^1)$ is anti-fuzzy ideal of the MΓ-group G which is constant on ker h.

Proof: Assume that ν is an anti-fuzzy ideal of the MΓ-group G. We prove that $h(\nu)$ is anti-fuzzy ideal of the MΓ-group G^1. It is known that $h(\nu)(x + y) \le \max\{h(\nu)(x), h(\nu)(y)\}$; $h(\nu)(x + y - x) \le h(\nu)(y)$. Consider $h(\nu)(n\gamma(a + x) - n\gamma a) = \inf_{h(z)=n\gamma(a+x)-n\gamma a} \nu(z) \le \nu(n^1\gamma(a^1 + x^1) - n^1\gamma a^1)$. Since h is onto there exists n^1, a^1, x^1 such that $h(n^1) = n$, $h(a^1) = a$, $h(x^1) = x$ and so $h(n^1\gamma(a^1 + x^1) - n^1\gamma a^1) = h(n^1)\gamma(h(a^1) + h(x^1)) - h(n^1)\gamma h(a^1) = n\gamma(a + x) - n\gamma a) \le \nu(x^1)$. Therefore $h(\nu)(n\gamma(a + x) - n\gamma a) \le \inf_{h(x^1)=x} \nu(x^1) = h(\nu)(x)$. Hence $h(\nu)$ is anti-fuzzy ideal of the MΓ-group G^1.

Suppose that ν^1 is anti-fuzzy ideal of the MΓ-group G^1. To show $h^{-1}(\nu^1)$ is anti-fuzzy ideal of the MΓ-group G. Consider $h^{-1}(\nu^1)(n\gamma(a + x) - n\gamma a) = \nu^1(h(n\gamma(a + x) - n\gamma a)) = \nu^1(n\gamma h(a + x) - n\gamma h(a)) = \nu^1(n\gamma(h(a) + h(x)) - n\gamma h(a)) \le \nu^1(h(x)) = h^{-1}(\nu^1(x)) = h^{-1}(\nu^1)(x)$. One can easily verify the other conditions of fuzzy ideal. Therefore $h^{-1}(\nu^1)$ is anti-fuzzy ideal of the MΓ-group G. For any $x \in \ker h$, we have that $h^{-1}(\nu^1)(x) = \nu^1(h(x)) = \nu^1(0)$. This shows that $h^{-1}(\nu^1)$ is constant on ker h.

2.20 Lemma: If ν is a fuzzy ideal of G; h: G → G^1 an onto homomorphism, such that ν is constant on ker h, then ν is h-invariant.

Proof: Suppose $h(x) = h(y)$ for some $x, y \in G$. Then $h(x - y) = 0$ and so $x - y \in \ker h$. Since $0, x - y \in \ker h$, we have that $\nu(0) = \nu(x - y)$. It follows that $\nu(x) = \nu(y)$.

2.21 Theorem: The mapping $\nu \to h(\nu)$ defines a one-to-one correspondence between the set of all h-invarinat anti-fuzzy ideal of the MΓ-group G and the set of all anti-fuzzy ideals of the MΓ-group G^1, where h: G → G^1 is an epimorphism.

2.22 Theorem: Let ν and ν^1 be anti-fuzzy ideals of the MΓ-group G and G^1 respectively such that $Im(\mu) = \{t_0, t_1, \ldots, t_n\}$ with $t_0 < t_1 < \ldots < t_n$, and $Im(\nu) = \{s_0, s_1, \ldots, s_m\}$ with $s_0 < s_1 < \ldots < s_m$. Then (i). $Im(h(\nu)) \subset Im(\nu)$ and the chain of level ideals of $h(\nu)$ is $h(\nu_{t_0}) \subseteq h(\nu_{t_1}) \subseteq \ldots \subseteq h(\nu_{t_n})$; and

(ii). $Im(h^{-1}(\nu^1)) = Im(\nu^1)$ and the chain of level ideals of $h^{-1}(\nu^1)$ is $h^{-1}(\nu^1_{s_0}) \subseteq h^{-1}(\nu^1_{s_1}) \subseteq \ldots \subseteq h^{-1}(\nu^1_{s_m})$.

Proof: (i). $h(\nu)(y^1) = \inf_{h(x)=y^1} \nu(x) \in Im\,\nu \Rightarrow Im\,h(\nu) \subseteq Im\,\nu$. $y^1 \in G^1$, $y^1 \in h(\nu_{t_i}) \Leftrightarrow$ there exists $x \in \nu_{t_i}$ such that $h(x) = y^1 \Leftrightarrow \nu(x) \le t_i$ and $h(x) = y^1 \Leftrightarrow \inf_{h(x)=y^1} \nu(x) \le t_i \Leftrightarrow (h(\nu))(y^1) \le t_i \Leftrightarrow y^1 \in (h(\nu))_{t_i}$. Therefore $h(\nu_{t_i}) = (h(\nu))_{t_i}$. Since $t_0 < t_1 < .. < t_n$ and $Im\,h(\nu) \subseteq \{t_0, t_1, ..., t_n\}$, it follows that $(h(\nu))_{t_0} \subseteq (h(\nu))_{t_1} \subseteq ... \subseteq (h(\nu))_{t_n}$ is a sequence of the anti-level ideals of $h(\nu)$. Since $h(\nu_{t_i}) = (h(\nu))_{t_i}$, we can conclude that $h(\nu_{t_0}) \subseteq h(\nu_{t_1}) \subseteq ... \subseteq h(\nu_{t_n})$ is the sequence (of maximum length) of the anti-level ideals of the anti-fuzzy ideal $h(\nu)$.

(ii) Since $h^{-1}(\nu^1)(x) = \nu^1(h(x))$ for all $x \in G$, and since h is onto we have $Im(h^{-1}(\nu^1)) = Im(\nu^1)$. $x \in h^{-1}(\nu^1_{s_i}) \Leftrightarrow$ there exists $y \in \nu^1_{s_i}$ such that $h^{-1}(y) = x \Leftrightarrow \nu^1(y) \le s_i$ and $y = h(x) \Leftrightarrow \nu^1(h(x)) \le s_i \Leftrightarrow h^{-1}(\nu^1)(x) \le s_i \Leftrightarrow x \in (h^{-1}(\nu^1))_{s_i}$. Therefore $h^{-1}(\nu^1_{s_i}) = (h^{-1}(\nu^1))_{s_i}$. Now $s_0 < s_1 < ... < s_m$ and $Im(\nu^1) = \{s_0, s_1, ..., s_m\}$, implies that $(h^{-1}(\nu^1))_{s_0} \subseteq (h^{-1}(\nu^1))_{s_1} \subseteq ... \subseteq (h^{-1}(\nu^1))_{s_m}$. Since $h^{-1}(\nu^1_{s_i}) = (h^{-1}(\nu^1))_{s_i}$, we conclude that $h^{-1}(\nu^1_{s_0}) \subseteq h^{-1}(\nu^1_{s_1}) \subseteq ... \subseteq h^{-1}(\nu^1_{s_m}) = G$ is a chain of anti-level ideals of $h^{-1}(\gamma)$.

3. Anti-fuzzy Cosets

3.1 Definition: Let ν: G → [0, 1] be an anti-fuzzy ideal of the MΓ-group G, and $y \in G$. Then the fuzzy subset $y + \nu$: G → [0, 1] defined by $(y + \nu)(x) = \nu(x - y)$ is called an *anti-fuzzy coset* of the anti-fuzzy ideal ν.

3.2 Theorem: Let ν be an anti-fuzzy ideal of G. Then for all $y_1, y_2 \in G$, $y_1 + \nu \le y_2 + \nu$ implies $\nu(y_1) = \nu(y_2)$.

Proof: Suppose $y_1 + \nu \le y_2 + \nu$. That is $(y_1 + \nu)(x) \le (y_2 + \nu)(x)$ for all $x \in G$. Now $\nu(y_2 - y_1) = (y_1 + \nu)(y_2) \le (y_2 + \nu)(y_2) = \nu(y_2 - y_2) = \nu(0)$. Which implies that $\nu(y_2 - y_1) \le \nu(0)$ and hence $\nu(y_2 - y_1) = \nu(0)$. Thus $\nu(y_2) = \nu(y_1)$.

3.3 Theorem: Let ν be an anti-fuzzy ideal of G. Then $\nu(y_2 - y_1) = \nu(0)$ implies that $y_1 + \nu = y_2 + \nu$ for all $y_1, y_2 \in G$.

Proof: Consider $(y_1 + \nu)(x) = \nu(x - y_1) = \nu[(x - y_2) - (-y_2 + y_1)] \le \max\{\nu(x - y_2), \nu(y_2 - y_1)\} = \max\{\nu(x - y_2), \nu(0)\} = \nu(x - y_2) = (y_2 + \nu)(x)$. Which implies that $y_1 + \nu \le y_2 + \nu$. Similarly we can show that $y_2 + \nu \le y_1 + \nu$.

3.4 Corollary: Let ν be an anti-fuzzy ideal of G. Then $\nu(y_2 - y_1) = \nu(0)$ if and only if $y_1 + \nu = y_2 + \nu$ for all $y_1, y_2 \in$ G.

3.5 Notation: We write $G/\nu = \{x + \nu \mid x \in G\}$, the set of all anti-fuzzy cosets of ν. Define $(x + \nu) + (y + \nu) = (x + y) + \nu$ and $m\gamma(x + \nu) = m\gamma x + \nu$ for $m \in M$, $\gamma \in \Gamma$ and $x \in G$. Then the set G/ν becomes an MΓ-group with respect to the operations defined. The MΓ-group G/ν is called as the quotient group with respect to the anti-fuzzy ideal ν.

3.6 Definitions: (i) If ν is an anti-fuzzy ideal of a MΓ-group G, then we define a fuzzy set θ_ν on G/ν corresponding to ν by $\theta_\nu(y + \nu) = \nu(y)$ for all $y \in G$.

(ii) If θ is an anti-fuzzy ideal of the MΓ-group G/ν, then we define a fuzzy set σ_θ on G by $\sigma_\theta(y) = \theta(y + \nu)$ for all $y \in G$.

3.7 Theorem: If ν is an anti-fuzzy ideal of the MΓ-group G, then the fuzzy set θ_ν is an anti-fuzzy ideal of G/ν.

Proof: Let $a + \nu, x + \nu, y + \nu \in G/\nu$ and $m \in M$. $\theta_\nu((y + \nu) - (x + \nu)) = \theta_\nu((y - x) + \nu) = \nu(y - x) \leq \max\{\nu(y), \nu(x)\} = \max\{\theta_\nu(y + \nu), \theta_\nu(x + \nu)\}$. $\theta_\nu((y + \nu) + (x + \nu) - (y + \nu)) = \theta_\nu((y + x - y) + \nu) = \nu(y + x - y) \leq \nu(x) = \theta_\nu(x + \nu)$ $\theta_\nu(m\gamma((a + \nu) + (y + \nu)) - m\gamma(a + \nu)) = \theta_\nu(m\gamma((a + y) + \nu) - (m\gamma a + \nu)) = \theta_\nu([m\gamma(a + y) - m\gamma a] + \nu) = \nu(m\gamma(a + y) - m\gamma a) \leq \nu(y) = \theta_\nu(y + \nu)$.

3.8 Corollary: If ν and σ are two anti-fuzzy ideals of G such that $\sigma \subseteq \nu$ and $\sigma(0) = \nu(0)$, then the mapping $h_\sigma: G/\nu \to [0, 1]$ defined by $h_\sigma(y + \nu) = \sigma(y)$ for all $y + \nu \in G/\nu$, is an anti-fuzzy ideal. Also $h_\sigma \subseteq \theta_\nu$ on G/ν and $\theta_\nu(0) = h_\sigma(0)$.

Proof: Let $x + \nu, y + \nu \in G/\nu$ such that $x + \nu = y + \nu$. This implies $\nu(0) = \nu(x - y) \Rightarrow \sigma(0) \leq \sigma(x - y) \leq \nu(x - y) = \nu(0) = \sigma(0)$ $\Rightarrow \sigma(0) = \sigma(x - y) \Rightarrow x + \sigma = y + \sigma \Rightarrow (x + \sigma)(0) = (y + \sigma)(0) \Rightarrow \sigma(0 - x) = \sigma(0 - y) \Rightarrow \sigma(-x) = \sigma(-y) \Rightarrow \sigma(x) = \sigma(y) \Rightarrow h_\sigma(x + \nu) = h_\sigma(y + \nu)$. This shows that h_σ is well defined.

Take $x + \nu, y + \nu, a + \nu \in G/\nu$ and $m \in M$. Now $h_\sigma((x + \nu) - (y + \nu)) = h_\sigma((x - y) + \nu) = \sigma(x - y) \leq \max\{\sigma(x), \sigma(y)\} = \max\{h_\sigma(x + \nu), h_\sigma(y + \nu)\}$. Now $h_\sigma((x + \nu) + (y + \nu) - (x + \nu)) = h_\sigma((x + y - x) + \nu) = \sigma(x + y - x) \leq \sigma(y) = h_\sigma(y + \nu)$. $h_\sigma(m\gamma((a + \nu) + (x + \nu)) - m\gamma(a + \nu)) = h_\sigma(m\gamma((a + x) + \nu) - (m\gamma a + \nu)) = h_\sigma([m\gamma(a + x) - m\gamma a] + \nu) = \sigma(m\gamma(a + x) - m\gamma a) \leq \sigma(x) = h_\sigma(x + \nu)$. It follows that h_σ is an anti-fuzzy ideal of G/ν. Now $h_\sigma(x + \nu) = \sigma(x) \leq \nu(x) = \theta_\nu(x + \nu)$. This shows that $h_\sigma \subseteq \theta_\nu$. Also $\theta_\nu(0) = \nu(0) = \sigma(0) = h_\sigma(0)$.

3.9 Theorem: Let ν be an anti-fuzzy ideal of G, and θ an anti-fuzzy ideal of G/ν such that $\theta \subseteq \theta_\nu$ and $\theta_\nu(0) = \theta(0)$. Then $\sigma_\theta: G \to [0, 1]$ defined by $\sigma_\theta(x) = \theta(x + \nu)$, is an anti-fuzzy ideal of G such that $\sigma_\theta \subseteq \nu$ and $\nu(0) = \sigma_\theta(0)$.

Proof: First we show that σ_θ is an anti-fuzzy ideal of G. For this, take $a, x, y \in G$ and $m \in M$. $\sigma_\theta(x - y) = \theta((x - y) + \nu) = \theta((x + \nu) - (y + \nu)) \leq \max\{\theta(x + \nu), \theta(y + \nu)\} = \max\{\sigma_\theta(x), \sigma_\theta(y)\}$

$\sigma_\theta(x + y - x) = \theta((x + y - x) + \nu) = \theta((x + \nu) + (y + \nu) - (x + \nu)) \leq \theta(y + \nu) = \sigma_\theta(y)$

$\sigma_\theta(m\gamma(a + x) - m\gamma a) = \theta([m\gamma(a + x) - m\gamma a] + \nu) = \theta(m\gamma((a + \nu) + (x + \nu)) - m\gamma(a + \nu)) \leq \theta(x + \nu) = \sigma_\theta(x)$. It follows that σ_θ is an anti-fuzzy ideal of G. For any $x \in G$, we have that $\sigma_\theta(x) = \theta(x + \nu) \leq \theta_\nu(x + \nu) = \nu(x)$. This shows that $\sigma_\theta \subseteq \nu$. Also $\sigma_\theta(0) = \theta(0 + \nu) = \theta(0) = \theta_\nu(0) = \theta_\nu(0 + \nu) = \nu(0)$. Therefore $\sigma_\theta(0) = \nu(0)$.

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