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A STUDY ON STRUCTURE OF PO-TERNARY SEMIRINGS

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ABSTRACT

This paper is divided into two sections. In section 1, the notion of a PO-ternary semiring was introduced and examples are given. Further the terms commutative PO-ternary semiring , quasi commutative PO-ternary semiring, normal PO-ternary semiring, left pseudo commutative PO-ternary semiring, lateral pseudo commutative PO-ternary semiring, right pseudo commutative PO-ternary semiring and pseudo commutative PO-ternary semiring are introduced and characterized them. Further the terms left singular, right singular and singular with respect to addition and left singular, right singular, lateral singular, singular with respect to ternary multiplication and two sided singular are introduced and made a study on them.

In section 2, the terms; PO-ternary subsemiring, PO-ternary subsemiring of T generated by a set A, cyclic PO-ternary subsemiring and cyclic PO-ternary semiring are introduced. It is proved that T be a PO-ternary semiring and A be a non-empty subset of T. Then (A) = $\{a_1a_2....a_{n-1}a_n : n \in N, a_1, a_2....a_n \in A\}$ is a smallest PO-ternary subsemiring of T. Let T be a PO-ternary semiring and A be a non-empty subset of T. < A > = the intersection of all PO-ternary subsemirings of T containing A.

Indexing terms/Keywords

PO-Ternary semi ring; commutative; quasi commutative; normal; pseudo commutative singular; cyclic.

Academic Discipline And Sub-Disciplines

Mathematics, Algebra, ternary semirings.

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INTRODUCTION

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces, and the like. The theory of ternary algebraic systems was introduced by D. H. Lehmer [6]. He investigated certain ternary algebraic systems called triplexes which turn out to be commutative ternary groups. D. Madhusudhana Rao[8] characterized the primary ideals in ternary semigroups. about T. K. Dutta and S. Kar [4] introduced and studied some properties of ternary semirings which is a generalization of ternary rings. Our main purpose in this paper is to introduce the notion of some special class of partially ordered ternary semirings.

1. PO-TERNARY SEMIRING

We now introduce the notion of ternary semiring which is due to DUTTA T. K and KAR. S[4].

DEFINITION 1.1 : A nonempty set T together with a binary operation called addition and a ternary multiplication denoted by [ ] is said to be a ***ternary semiring*** if T is an additive commutative semigroup satisfying the following conditions :

i) $[[abc]de] = [a[bcd]e] = [ab[cde]]$,

ii) $[(a + b)cd] = [acd] + [bcd]$,

iii) $[a(b + c)d] = [abd] + [acd]$,

iv) $[ab(c + d)] = [abc] + [abd]$ for all $a; b; c; d; e \in$ T.

NOTE 1.2 : For the convenience we write $x_1x_2x_3$ instead of $\left[x_1x_2x_3\right]$

NOTE 1.3: Let T be a ternary semiring. If A, B and C are three subsets of T , we shall denote the set ABC = $\left\{\Sigma abc : a \in A, b \in B, c \in C\right\}$.

NOTE 1.4: Let T be a ternary semiring. If A, B are two subsets of T , we shall denote the set A + B = $\left\{a + b : a \in A, b \in B\right\}$ and 2A = { $a + a$: $a \in$ A}.

NOTE 1.5 : Any semiring can be reduced to a ternary semiring.

EXAMPLE 1.6 : Let T be an semigroup of all $m \times n$ matrices over the set of all non negative rational numbers. Then T is a ternary semiring with matrix multiplication as the ternary operation.

We now introduce the notion of partially ordered ternary semiring.

DEFINITION 1.7: A ternary semiring T is said to be a **partially ordered ternary semiring** or simply **PO Ternary Semiring** or **Ordered Ternary Semiring** provided T is partially ordered set such that a ≤ b then

(1) $a + c \leq b + c$ and $c + a \leq c + b$,

(2) $acd \leq bcd$, $cad \leq cbd$ and $cda \leq cdb$ for all $a, b, c, d \in$ T.

Throughout T will denote as PO-ternary semiring unless otherwise stated.

NOTE 1.8 : Some times we write $a \geq b$ for $a \leq b$. That is " ≥" is the dual relation of "≤".

In the following some examples of PO-ternary semiring are given

EXAMPLE 1.9: The set of natural numbers under addition, ternary multiplication and ordering is a PO-ternary semiring.

EXAMPLE 1.10: Consider the set T = {0,1,2,3,….} with $m + n$ = max (m, n) or min(m, n), $mn = m + n$. where the addition in the ternary multiplication is the usual addition, for all m, n in S and the order being the usual order relation . Then (T, +, [ ], ≤) is a PO-ternary semiring.

Now we introduce the notion of positively partial ordering in PO-ternary semiring.

DEFINITION 1.11: In a PO-ternary semiring (T, +, [ ], ≤), the set (T, +, ≤) is said to be ***positively partially ordered*** (p.p.o.), if $a \leq a + b$, and $b \leq a + b$ for all a, b in T.

DEFINITION 1.12: In a PO-ternary semiring (T, +, [ ], ≤), the set (T, [ ], ≤) is said to be ***positively partially ordered*** (p.p.o.), if $a \leq [abc]$, $b \leq [abc]$ and $c \leq [abc]$ for all a, b, c in T.

DEFINITION 1.13: A totally ordered semiring (T, +, [ ], ≤) is said to be a ***positively ordered*** in the strict sense if both (T, +, ≤) and (T, [ ], ≤) are positively ordered in the strict sense.

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DEFINITION 1.14: In a PO-ternary semiring (T, +, [ ], ≤), the set (T, +) is said to be ***right naturally partially ordered*** (r. n. p. o) if (T, +) is positively ordered in the strict sense and if $a < b$ implies $b = a + c$ for some c in T.

DEFINITION 1.15: In a PO-ternary semiring (T, +, [ ], ≤), the set (T, +) is said to be ***left naturally partially ordered*** (l. n. p. o) if (T, +) is positively ordered in the strict sense and if $a < b$ implies $b = c + a$ for some c in T.

DEFINITION 1.16: In a PO-ternary semiring (T, +, [ ], ≤), the set (T, [ ]) is said to be ***right naturally partially ordered*** (r. n. p. o) if (T, [ ]) is positively ordered in the strict sense and if $a < b$ implies $b = acd$ for some c, d in T.

DEFINITION 1.17: In a PO-ternary semiring (T, +, [ ], ≤), the set (T, [ ]) is said to be ***lateral naturally partially ordered*** (la. n. p. o) if (T, [ ]) is positively ordered in the strict sense and if $a < b$ implies $b = cad$ for some c, d in T.

DEFINITION 1.18: In a PO-ternary semiring (T, +, [ ], ≤), the set (T, [ ]) is said to be ***left naturally partially ordered*** (l. n. p. o) if (T, [ ]) is positively ordered in the strict sense and if $a < b$ implies $b = cda$ for some c, d in T.

We now introduce the notion of commutative ternary semiring.

DEFINITION 1.19: A PO-ternary semiring T is said to be ***commutative PO-ternary semiring*** provided $abc = bca = cab = bac = cba = acb$ for all $a,b,c \in$ T.

EXAMPLE 1.20 : (Z^0, +, ., ≤) is a PO-ternary semiring of infinite order which is commutative.

EXAMPLE 1.21 : The set 2I of all even integers is a commutative PO-ternary semiring with respect to ordinary addition, ternary multiplication [ ] defined by $[abc] = abc$ for all a, b, $c \in$ T and usual partial order relation.

NOTE 1.22 : The set M of all $n \times n$ matrices with their elements as real numbers (rational numbers, complex numbers, integers) is a non-commutative PO-ternary semiring, with respect to addition and ternary multiplication of matrices as the two ternary semiring compositions and usual partial order relation.

In the following we introduce a quasi commutative ternary semiring.

DEFINITION 1.23 : A PO-ternary semiring T is said to be ***quasi commutative PO-ternary semiring*** provided T is a quasi commutative ternary semiring.

NOTE 1.24: A PO-ternary semiring T is ***quasi commutative*** provided for each $a, b, c \in$ T, there exists an odd natural number n such that $abc = b^n ac = bca = c^n ba = cab = a^n cb$.

EXAMPLE 1.25: The set of all natural numbers under usual addition, ternary multiplication and usual partial ordering is a quasi-commutative PO-ternary semiring.

THEOREM 1.26: If T is a commutative PO-ternary semiring then T is a quasi commutative PO-ternary semiring.

In the following we introduce the notion of a normal partially ordered ternary semiring.

DEFINITION 1.27 : PO- ternary semiring T is said to be ***normal PO- ternary semiring*** provided T is normal ternary semiring.

NOTE 1.28: A PO-ternary semiring T is said to be ***normal*** provided abT = Tab $\forall\ a,b \in$ T.

THEOREM 1.29 : If T is a quasi commutative PO-ternary semiring then T is a normal PO-ternary semiring.

Proof **:** Suppose that T is quasi commutative PO-ternary semiring. Then T is quasi commutative ternary semiring. Let a, $b \in$ T. If $x \in ab$T. Then $x = abc$ where $c \in$ T. Since T is quasi commutative $abc = c^n ab \in$ Tab. Therefore $x \in$ Tab. Thus abT ⊆ Tab. Similarly Tab ⊆ abT and hence abT = Tab $\forall\ a,b \in$ T.

COROLLARY 1.30 : Every commutative PO-ternary semiring is a normal PO-ternary semiring.

In the following we are introducing left pseudo commutative PO-ternary semiring.

DEFINITION 1.31 : A PO-ternary semiring T is said to be ***left pseudo commutative PO-ternary semiring*** provided T is left pseudo commutative ternary semiring.

NOTE1.32: A PO-ternary semiring T is **left pseudo commutative PO- ternary semiring** provided $abcde = bcade = cabde = bacde = cbade = acbde$ $\forall\ a,b,c,d,e \in$ T.

EXAMPLE 1.33: (Z^0, +, ., ≤) is a left pseudo commutative PO-ternary semiring of infinite order.

THEOREM 1.34: If T is a commutative PO-ternary semiring, then T is a left pseudo commutative PO-ternary semiring.

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Proof : Suppose that T is a commutative PO-ternary semiring.

Then $abcde = (abc)de = (bca)de = (cab)de = (bac)de = (cba)de = (acb)de$ $\forall$ $a,b,c,d,e \in$ T. $abcde = bcade = cabde = bacde = cbade = acbde$. Therefore T is a left pseudo commutative PO-ternary semiring.

NOTE 1.35 : The converse of the above theorem is not true. i.e T is a left pseudo commutative PO-ternary semiring then T need not be a commutative PO-ternary semiring.

EXAMPLE 1.36 : Let T = { *a*, *b*, *c*, *d*, *e*}. Define the binary operation + and a ternary operation [ ] on T as [*abc*] = *a.b.c* where the binary operation "." defined as follows :

+	a	b	c	d	e
a	a	b	c	d	e
b	b	d	b	d	e
c	c	b	a	d	c
d	d	c	b	a	e
e	e	d	c	e	e

.	a	b	c	d	e
a	a	a	a	a	a
b	b	a	a	a	a
c	a	a	a	a	a
d	a	a	a	a	a
e	a	b	c	d	e

Then it is easy to see that T is a PO-ternary semiring. Now define the ordering as $a \leq b \leq c \leq d \leq e$, then T is a left pseudo commutative PO-ternary semiring. But T is not a commutative PO-ternary semiring.

In the following we are introducing the notion of lateral pseudo commutative ternary semiring.

DEFINITION 1.37: A PO-ternary semiring T is said to be ***lateral pseudo commutative PO-ternary semiring*** provided T is lateral pseudo commutative ternary semiring.

NOTE 1.38 : A PO-ternary semiring T is said to be a ***lateral pseudo commutative PO- ternary semiring*** provide $abcde = acdbe = adbce = acbde = adcbe = abdce$ for all $a,b,c,d,e \in$ T.

THEOREM 1.39 : If T is a commutative PO-ternary semiring then T is a lateral pseudo commutative PO-ternary semiring.

Proof : Similar to 1.34.

NOTE 1.40 : The converse of the above theorem is not true i.e. T is a lateral pseudo commutative PO-ternary semiring then T need not be a commutative PO-ternary semiring.

EXAMPLE 1.41: Consider the PO-ternary semiring in example 1.36, T is a lateral pseudo commutative PO-ternary semiring.

In the following we are introducing the notion of right pseudo commutative PO-ternary semiring.

DEFINITION 1.42 : A PO-ternary semiring T is said to be ***right pseudo commutative PO-ternary semiring*** provided T is right pseudo commutative ternary semiring.

NOTE 1.43 : A PO-ternary semiring T is said to be ***right pseudo commutative PO-ternary semiring*** provided $abcde = abdec = abecd = abdce = abedc = abced$ $\forall$ $a,b,c,d,e \in$ T.

THEOREM 1.44: If T is a commutative PO-ternary semiring then T is a right pseudo commutative PO-ternary semiring.

Proof : Similar to 1.34.

NOTE 1.45 : The converse of the above theorem is not true i.e. If T is a right pseudo commutative PO-ternary semiring, then T need not be a commutative ternary semiring.

EXAMPLE 1.46 : Consider the PO-ternary semiring in example 1.36, T is a right pseudo commutative. But T is not a commutative PO-ternary semiring.

In the following we introducing the notion of pseudo commutative ternary semiring.

DEFINITION 1.47: A PO-ternary semiring T is said to be ***pseudo commutative***, provided T is a left pseudo commutative, right pseudo commutative and lateral pseudo commutative PO-ternary semiring.

THEOREM 1.48 : If T is a commutative PO-ternary semiring, then T is a pseudo commutative PO-ternary semiring.

NOTE 1.49 : The converse of the above theorem is not true i.e. if T is a pseudo commutative PO-ternary semiring , then T need not be a commutative PO-ternary semiring.

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EXAMPLE 1.50 : Consider the ternary semiring in example 1.36, T is a pseudo commutative. But T is not a commutative PO-ternary semiring.

DEFINITION 1.51 : A semigroup (T, +) is said to be ***left singular*** provided $a + b = a$ for all $a, b \in$ T.

DEFINITION 1.52 : A semigroup (T, +) is said to be ***right singular*** provided $a + b = b$ for all $a, b \in$ T.

DEFINITION 1.53 : A semigroup (T, +) is said to be ***singular*** provided it is both left as well as right singular.

DEFINITION 1.54 : A ternary semigroup (T, [ ]) is said to be ***left singular*** provided $ab^2 = a$ for all $a, b \in$ T.

DEFINITION 1.55 : A ternary semigroup (T, [ ]) is said to be ***lateral singular*** provided $aba = a$ for all $a, b \in$ T.

DEFINITION 1.54 : A ternary semigroup (T, [ ]) is said to be ***right singular*** provided $b^2a = a$ for all $a, b \in$ T.

DEFINITION 1.55 : A ternary semigroup (T, [ ]) is said to be ***two sided singular*** provided it is both left as well as right singular.

DEFINITION 1.56 : A ternary semigroup (T, [ ]) is said to be ***singular*** provided it is left, lateral and right singular.

2. PO-TERNARY SUB SEMIRING

DEFINITION 2.1: Let T be PO-ternary semiring. A non empty subset 'S' is said to be a PO-***ternary subsemiring*** of T if

(i) S is an additive subsemigroup of T,

(ii) $abc \in$ S for all $a, b, c \in$ S.

(iii) $t \in$ T, $s \in$ S, $t \leq s \Rightarrow t \in$ S.

NOTE 2.2: A non empty subset S of a ternary semiring T is a ternary subsemiring if and only if S + S ⊆ S, SSS $\subseteq$ S and $t \in$ T, $s \in$ S, $t \leq s \Rightarrow t \in$ S.

EXAMPLE 2.3. Let $T = M_2\,(Z_0^{\,-})$ and define the ordering as $a_{ii} \leq b_{ii}$. Then T be the PO-ternary semiring of the set of all 2x2 square matrices over $Z_0^{\,-}$, the set of all non-positive integers. Let $S = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} / a \in Z_0^{\,-} \right\}$. Then S is a PO-ternary subsemiring of T.

EXAMPLE 2.4 : Consider the PO-ternary semiring Z^- under the multiplication, then S = Z^- \{-1} is PO-ternary subsemiring of Z^-.

NOTATION 2.5 : Let T be PO-ternary semiring and S be a nonempty subset of T. If H is a nonempty subset of S, we denote {$s \in$ S : $s \leq h$ for some $h \in$ H} by $(H]_S$.

NOTATION 2.6 : Let T be PO-ternary semiring and S be a nonempty subset of T. If H is a nonempty subset of S, we denote {$s \in$ S : $h \leq s$ for some $h \in$ H} by $[H)_S$.

NOTE 2.7 : $(H]_T$ and $[H)_T$ are simply denoted by (H] and [H) respectively.

NOTE 2.8 : For an element a of a PO-ternary semiring T define $(a]$ = { $t \in$ T : $t \leq a$} and for a subset A of a PO-ternary semiring T, (A] = $\bigcup_{a \in A} (a]$

NOTE 2.9 : A nonempty subset S of a PO-ternary semiring T is a PO-ternary subsemiring of T iff (1) S + S ⊆ S, (ii) SSS ⊆ S, (2) (S] ⊆ S.

THEOREM 2.10 : Let T be a po-ternary semiring and A ⊆ T, B ⊆ T and C ⊆ T. Then (i) A ⊆ (A], (ii) ((A]] = (A], (iii) (A](B](C] ⊆ (ABC] and (iv) A ⊆ B ⇒ A ⊆ (B] and (v) A ⊆ B ⇒ (A] ⊆ (B], (vi) (A ∩ B] = (A] ∩ (B], (vii) (A ∪ B] = (A] ∪ (B].

Proof : (i) Let $x \in$ A. $x \in$ A ⇒ $x \in$ T and $x \leq x \Rightarrow x \in$ (A]. Therefore A ⊆ (A].

(ii) Let $x \in$ ((A]] ⇒ $x \leq y$ for some $y \in$ (A] ⇒ $y \leq z$ for some $z \in$ A. $x \leq y$, $y \leq z \Rightarrow x \leq z$. Thus $x \in$ (A].

Therefore ((A]] ⊆ (A] and from (i) A ⊆ (A] ⇒ (A] = ((A]] and hence ((A]] = (A].

(iii) Let $x \in$ (A] (B](C] ⇒ $x \leq abc$ for some $a \in$ A, $b \in$ B, $c \in$ C

⇒ $x \leq abc$ for some $abc \in$ ABC ⇒ $x \in$ (ABC]. Therefore (A](B](C] ⊆ (ABC].

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(iv) From (i) B ⊆ (B] ⇒ A ⊆ B ⊆ (B]. Therefore A ⊆ B ⇒ A ⊆ (B].

(v) A ⊆ B ⇒ A ⊆ (B] ⇒ (A] ⊆ ((B]] = (B] Therefore (A] ⊆ (B].

(vi) we know that A ∩ B ⊆ A and A ∩ B ⊆ B ⇒ (A ∩ B] ⊆ (A], (A ∩ B] ⊆ (B]

⇒ (A ∩ B] ⊆ (A] ∩ (B].

Now let x ∈ (A] ∩ (B] ⇒ x ∈ (A] and x ∈ (B] then there exist s ∈ A such that $x \le s$ for some s ∈ A and t ∈ B

⇒ $x \le t$ for some t ∈ B ⇒ x ∈ (s] ⊆ A, x ∈ (t] ⊆ B ⇒ x ∈ A ∩ B ⊆ (A ∩ B]. Therefore (A ∩ B] = (A] ∩ (B]

(vii) Let x ∈ (A ∪ B], there exist an element t ∈ A ∪ B such that $x \le t$. t ∈ A ∪ B

⇒ t ∈ A or t ∈ B. Hence $x \le t$ for some t ∈ A or $x \le t$ for some t ∈ B ⇒ x ∈ (A] or x ∈ (B]

⇒ x ∈ (A] ∪ (B]. Therefore (A ∪ B] ⊆ (A] ∪ (B].

Now A ⊆ A ∪ B and B ⊆ A ∪ B ⇒ (A] ⊆ (A ∪ B], (B] ⊆ (A ∪ B] ⇒ (A] ∪ (B] ⊆ (A ∪ B] and hence

(A ∪ B] = (A]∪ (B].

THEOREM 2.11: The non-empty intersection of two PO-ternary subsemirings of a PO-ternary semiring T is a PO-ternary subsemiring of T.

Proof : Let S_1, S_2 be two PO-ternary subsemirings of T . Let *a, b, c* ∈ $S_1 \cap S_2$

a, b, c $\in S_1 \cap S_2 \Rightarrow$ *a, b, c* $\in S_1$ and *a, b, c* $\in S_2$

a, b, c $\in S_1$, S_1 is a PO-ternary subsemiring of T ⇒ $a + b \in S_1$, $abc \in S_1$ and $(S_1] \subseteq S_1$.

a, b, c $\in S_2$, S_2 is a PO-ternary subsemiring of T $\Rightarrow$ $a + b \in S_2$, $abc \in S_2$ and $(S_2] \subseteq S_2$.

$a + b \in S_1$, $a + b \in S_2$, $abc \in S_1$, $abc \in S_2 \Rightarrow a + b \in S_1 \cap S_2$, $abc \in S_1 \cap S_2$.

and $S_1 \cap S_2 \subseteq S_1$, $S_1 \cap S_2 \subseteq S_2 \Rightarrow (S_1 \cap S_2] \subseteq (S_1] = S_1$, $(S_1 \cap S_2] \subseteq (S_2] = S_2 \Rightarrow (S_1 \cap S_2] \subseteq S_1 \cap S_2$

$\Rightarrow (S_1 \cap S_2] = S_1 \cap S_2$ and hence $S_1 \cap S_2$ is a PO-ternary semiring of T.

Therefore $S_1 \cap S_2$ is a PO-ternary subsemiring of T.

THEOREM 2.12: The non-empty intersection of any family of PO-ternary subsemirings of a PO-ternary semiring T is a PO-ternary subsemiring of T.

In the following we are introducing a PO-ternary subsemiring which is generated by a subset and a cyclic PO-ternary subsemiring of PO-ternary semiring.

DEFINITION 2.13: Let T be a PO-ternary semiring and A be a non-empty subset of T. The smallest PO-ternary subsemiring of T containing A is called a ***PO-ternary subsemiring of T generated by A***. It is denoted by (A).

THEOREM 2.14 : Let T be a PO-ternary semiring and A be a non-empty subset of T. Then

(A) = $\left\{ \sum_{r=1}^{n} a_1 a_2 \ldots . a_{r-1} a_r : n \in N, a_1, a_2 \ldots . a_r \in A \right\}$ **is a smallest PO-ternary subsemigroup of T.**

Proof : Let S = $\left\{ \sum_{r=1}^{n} a_1 a_2 \ldots . a_{r-1} a_r : n \in N, a_1, a_2 \ldots . a_r \in A \right\}$

Let *a, b, c* ∈ S. $a \in$ S $\Rightarrow a = \sum_{m=1}^{n} a_1 a_2 a_3 \ldots . a_m$ where $a_1, a_2 \ldots . a_m \in A$

$b \in$ S $\Rightarrow$ b = $\sum_{p=1}^{n} b_1 b_2 b_3 \ldots . b_p$ where $b_1, b_2 \ldots . b_n \in A$

$c \in$ S $\Rightarrow$ c = $\sum_{r=1}^{n} c_1 c_2 c_3 \ldots . c_r$ where $c_1, c_2 \ldots . c_r \in A$

Now $a + b = \sum_{m=1}^{n} a_1a_2a_3....a_m + \sum_{p=1}^{n} b_1b_2b_3....b_p \in S \Rightarrow a + b \in S$

and $abc = \sum_{m=1}^{n} a_1a_2a_3....a_m \sum_{p=1}^{n} b_1b_2b_3....b_p \sum_{r=1}^{n} c_1c_2c_3....c_r \in S$

where $a_1, a_2....a_m, b_1, b_2....b_n, c_1, c_2....c_r \in A$ and $m, n, r \in N$ i.e. $abc \in S$.

Now $s \in S$ and $t \in T$ such that $t \leq s$. $s \in S \Rightarrow s = \sum_{q=1}^{n} s_1s_2s_3.....s_q$. Since $t \leq s$, then t can be represented as $t = \sum_{r=1}^{n} t_1t_2t_3.....t_r$ and hence $t \in S$. Therefore S is a PO-ternary subsemiring of T.

Let K be a ternary subsemiring of T such that $A \subseteq K$

Let $a \in S$ then $a = \sum_{m=1}^{n} a_1a_2a_3....a_m$ where $a_1, a_2....a_m \in A$

$a_1, a_2....a_m \in A$, $A \subseteq K \Rightarrow a_1, a_2....a_m \in K$

$a_1, a_2....a_m \in K$, K is a PO-ternary subsemiring i.e. $\sum_{m=1}^{n} a_1a_2a_3....a_m \in K \Rightarrow a \in K$.

Therefore $S \subseteq K$. So S is the smallest PO-ternary subsemiring of T containing A.

Hence (A) = S.

THEOREM 2.15 : Let T be a PO-ternary semiring and A be a non-empty subset of T. Then (A) = the intersection of all PO-ternary subsemiring of T containing A.

DEFINITION 2.16 : Let T be a PO-ternary semiring. A PO-ternary subsemiring S of T is said to be a ***cyclic PO-ternary subsemiring*** of T if S is generated by a single element subset of T.

DEFINITION 2.17: A PO-ternary semiring T is said to be a ***cyclic PO-ternary semiring*** if T is cyclic PO-ternary subsemiring of T itself.

CONCLUSION :

In this paper mainly we studied about quasi commutative, normal and pseudo commutative ternary semirings.

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