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THE EFFECT OF FRANK-KAMENESTKII PARAMETER ON NEWTONIAN FLUIDS AND NON-NEWTONIAN FLUIDS.

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ABSTRACT

An unsteady state study of the effect of Frank-Kamenestkii parameter when the flow behaviour index is 1 and 2 is made. Transformation and power series solution of the governing energy equation is carried out and its analysis showed that a decrease in temperature is observed in both cases as a result of increase in Frank-Kamenestkii parameter, though more decrease in magnitude of temperature is observed in non-Newtonian fluid.

KEYWORDS: Frank-Kamenestkii parameter; Newtonian fluids; non-Newtonian fluids; Flow behaviour index; Power series solution

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INTRODUCTION

In scientific parlance, a fluid consists of liquids, gases and of recent plasma. Though the inclusion of plasma is not very popular, there has been misuse of the word fluid by some scientists. Many use it as if it is liquid alone. A typical example is the Ebola virus where it is explained in Nigeria by some doctors and health care providers that the virus is not airborne but can only be spread through body fluids. However, McDonough (2003), defined fluid as any substance that deforms continuously when subjected to a shear stress no matter how small. The shear stress in an empirical relation depicts a fluid as either Newtonian or non-Newtonian as the flow behaviour index varies. Most studies of fluid, concentrated on Newtonian fluid because they obey the simple relationship between shear stress and shear strain. However, many common fluids are Non-Newtonian. Examples are solutions of various polymers, drilling mud used in well drilling, paints and many more. According to Hughes and Brighton (1999), the properties of non-Newtonian fluids do not lend themselves to the elegant and precise analysis that has been developed for Newtonian fluids, but the flow of non-Newtonian fluids does possess some interesting, useful and even exciting characteristics. For example in the fracturing treatment of oil wells, materials have been added which when added to water will increase the viscosity of the fluid so that it will suspend sand, glass or metal pellets. Yet the same fluid can be pumped down a well through tubing at enormous rates with less than half the friction loss of water. Some studies have been carried out on the effect of Frank-kamenestkii parameter on the temperature distribution of fluids. For example Ayeni et al (2005), examined the effect of thermal radiation on the critical Frank-Kamenestkii parameter of a thermal ignition in a combustion gas containing fuel droplets and deduced that temperature is decreased as a result of increase in Frank-Kamenestkii parameter. Lamidi and Ayeni (2007), Lamidi et al (2008) and Adegbie (2008) all contributed to the decrease of temperature as the Frank-Kamenestkii parameter increases. A departure from known results was also reported by Ajadi and Gol'dshtein (2010) in the theoretical study on the thermal explosion characteristics associated with heat release due to the forces of internal friction on the Frank-Kamenestkii parameter in the stationary theory and Semenov parameter in the non-stationary theories. Recently, Ngiangia et al (2013), examined the approximation of power law exponent to Newtonian fluids in reactions pathway and showed that the Frank-Kamenestkii parameter and the semenov parameter decrease the minimum temperature of the reaction, thereby delaying the initiation of thermal explosion. In all the literatures cited so far, limitation was made to Newtonian fluids and in most cases steady state description was only considered. Our aim is to extend the study to unsteady state and non-Newtonian fluids and investigate the effect or otherwise of the presence of flow behaviour index and Frank-Kamenestkii parameter. This in our view will assist in broadening our knowledge of fluid flow visualization and description.

MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

The formulation of the problem under investigation is based on the relation that the velocity gradient is a function of temperature and the flow behaviour index is not constant. With these assumptions, the energy equation takes the form

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \varphi\left(\frac{\partial u}{\partial x}\right)^{\alpha} \qquad (1)$$

With boundary condition T(0) = 0, T(-1) = 1

where T(x,t) is temperature of the fluid, k is thermal conductivity, α is flow behaviour index (power law exponent) and φ is consistency index and u is fluid velocity.

It has been established by Hughes and Brighton (1999) that

$$\varphi\left(-\frac{\partial u}{\partial x}\right)^{\alpha} = -\frac{x}{2}\left(\frac{\partial p}{\partial x}\right) \qquad (2a)$$

$$-\frac{\partial u}{\partial x} = \frac{1}{2^{\alpha}\varphi}\left(-\frac{\partial p}{\partial x}\right)^{-\alpha} x^{-\alpha} \qquad (2b)$$

where p is fluid pressure

The equation of state for an ideal fluid is given by

$$p = \rho RT \qquad (2c)$$

where ρ density of fluid and R is universal fluid constant.

If we put equation (2c) into (2b) and the result in equation (1), we get

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \frac{1}{2^\alpha \varphi}\left(\frac{\partial T}{\partial x}\right)^{-\alpha}(\rho Rx)^{-\alpha} \qquad (3)$$

For dimensional homogeneity of equation (3), using Buckingham $\pi -$ method, it is convenient to use the dimensionless variables

$$t' = \frac{t}{t_0},\ w = \frac{T}{T_0},\ x' = \frac{ut}{x},\ \rho' = \frac{\rho u^2 x}{p},\ R' = \frac{Rpx^2}{T},\ \delta = \frac{\varphi}{k},\ \beta = kRT_0$$

and equation (3) can be rewritten as

$$\frac{1}{\beta}\frac{\partial w}{\partial t'} = \frac{\partial^2 w}{\partial x'^2} + \frac{1}{2\beta\delta}\left(\frac{\partial w}{\partial x'}\right)^{-\alpha}(\rho' R' x')^{-\alpha} \qquad (4)$$

where β is dimensionless thermal conductivity, T_0 is characteristic temperature, δ is Frank-Kamenestkii parameter and t_0 is characteristic time.

METHOD OF SOLUTION

Case 1: The Newtonian fluid case is considered which implies α is equal to 1 and equation (4) transform into

$$\frac{1}{\beta}\frac{\partial w}{\partial t'} = \frac{\partial^2 w}{\partial x'^2} + \frac{1}{2\beta\delta}\left(\frac{\partial w}{\partial x'}\right)(\rho' R' x') \qquad (5)$$

To solve equation (5), we assume a solution of the form

$$w(x',t') = \theta(x')e^{-\lambda t'} \qquad (6)$$

where λ is a decay constant and with the boundary conditions $\theta(1) = 0, \theta(-1) = e^{\lambda t'}$

Substitute equation (6) into equation (5) results in

$$\theta''(x') + \frac{1}{2\beta\delta}\theta(x')\left(\theta'(x')\rho' R' + \frac{\lambda}{\beta}\right) = 0 \qquad (7)$$

Applying power series solution, we assume a solution of the form

$$\theta(x') = \sum_{i=0}^{\infty} a_i x^i \qquad (8)$$

If we put equation (8) into equation (7) and simplify, we get

$$a_{i+2}(i+2)(i+1) + \frac{\rho' R'}{2\beta\delta}a_{i+1}(i+1)a_i + \frac{\lambda}{\beta}a_i = 0 \qquad \text{i = 0, 1, 2......} \qquad (9)$$

$$a_2 = -\frac{\left(\frac{\rho' R'}{2\beta\delta}a_1 + \frac{\lambda}{\beta}\right)a_0}{2} \qquad \text{i = 0} \qquad (10)$$

$$a_3 = -\frac{\left(\frac{\rho' R'}{\beta\delta}a_2 + \frac{\lambda}{\beta}\right)a_1}{6} \qquad \text{i = 1} \qquad (11)$$

$$a_4 = -\frac{\left(\frac{3\rho' R'}{2\beta\delta}a_3 + \frac{\lambda}{\beta}\right)a_2}{12} \qquad \text{i = 2} \qquad (12)$$

$$a_5 = -\frac{\left(\frac{2\rho' R'}{\beta\delta}a_4 + \frac{\lambda}{\beta}\right)a_3}{20} \qquad i = 3 \qquad (13)$$

We expand equation (8) to get

$$\theta(x') = a_0 + a_1 x' + a_2 x'^2 + a_3 x'^3 + a_4 x'^4 + a_5 x'^5 . + ... \qquad (14)$$

For brevity, we terminate $\theta(x')$ at x', since $a_2, a_3, a_4, a_5......$ can be expressed as a_0, a_1, then applying the boundary conditions $\theta(1) = 0, \theta(-1) = e^{\lambda t'}$ we get

$$a_0 = \frac{e^{\lambda t'}}{2}, a_1 = -\frac{e^{\lambda t'}}{2} \qquad (15)$$

If we put equation (15) and equation (6) into equation (8) and terminate at x^2, we obtain

$$w(x') = \frac{1}{2} - \frac{x'}{2} - \frac{1}{4}\left(\frac{\rho' \mathrm{R}' \mathrm{e}^{\lambda t'}}{4\beta\delta} + \frac{\lambda}{\beta}\right)x'^2 + ... \qquad (16)$$

In order to get physical insight and numerical validation of the problem, a typical value of universal fluid constant 8.31 and fluid density of water 1000 is chosen. The values of other parameters made use of are $t' = 1, \beta = 1.5, R' = 1.5, \lambda = 0.0035. \rho' = 1000$ and $\delta = 0.4, 0.8, 1.2, 1.6, 2.0$

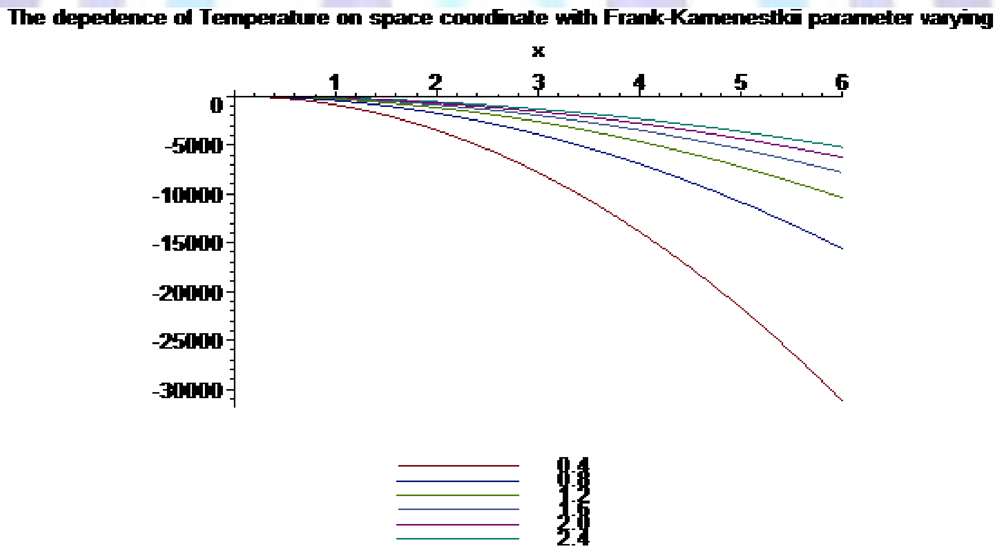

Figure 1: Showing increase of Frank-Kamenestkii parameter (δ) on temperature of Newtonian fluid.

Case 2.

The non-Newtonian fluid case is considered and that implies $\alpha = 2$ and above but we restrict the study to $\alpha = 2$ and equation (4) transform into

$$\frac{1}{\beta}\frac{\partial w}{\partial t'} = \frac{\partial^2 w}{\partial x'^2} + \frac{1}{4\beta\delta}\left(\frac{\partial w}{\partial x'}\right)^{0.5}(\rho' R' x')^{0.5} \qquad (17)$$

Following the same procedure for the Newtonian case, we get

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$$a_{i+2}(i+2)(i+1)+\frac{(\rho'R')^{0.5}}{4\beta\delta}\left(a_{i+1}(i+1)\right)^{0.5}(a_i)^{0.5}+\frac{\lambda}{\beta}a_i=0 \qquad (18)$$

Using equation (14) and applying the boundary conditions, equation (15) is also obtained and we get

$$w(x')=\frac{1}{2}-\frac{x'}{2}-\left(\frac{6.0056}{\delta}+0.001167\right)x'^2+.......... \qquad (19)$$

Having used 2500 for density (ρ') of non-Newtonian fluid

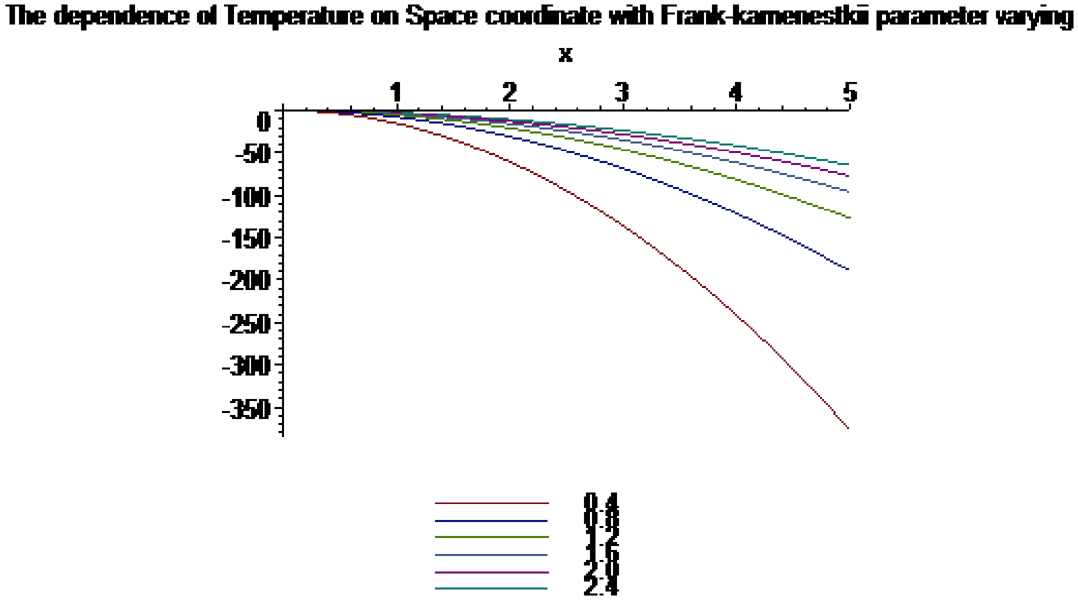

Figure 2: Showing increase of Frank-Kamenestkii parameter (δ) on temperature of non- Newtonian fluid

DISCUSSIONS

From figure 1, increase in Frank-Kamenestkii parameter results in a corresponding decrease in temperature for the unsteady case of Newtonian fluids and laid credence to earlier studies (Ngiangia at al(2003), Ayeni et al (2005), Lamidi and Ayeni (2007), Lamidi et al (2008) and Adegbie (2008)). For the unsteady non-Newtonian case as shown in figure 2 , increase in Frank-Kamenestkii parameter decreases the temperature of the fluid but in greater magnitude.

CONCLUSION

Increase in viscosity as a result of decrease in temperature of the fluid which distinguished Newtonian fluids from non-Newtonian fluids is facilitated by the increase of Frank-Kamenestkii parameter in a given fluid. This assertion was corroborated by all the previous studies in different configurations.

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Appendix

Figure 1 maple plot

```
> plot([1/2-x/2-(347.4639983/0.4+0.002333333)*x**2,1/2-x/2-(347.4639983/0.8+0.002333333)*x**2,1/2-x/2-(347.4639983/1.2+0.002333333)*x**2,1/2-x/2-(347.4639983/1.6+0.002333333)*x**2,1/2-x/2-(347.4639983/2.0+0.002333333)*x**2,1/2-x/2-(347.4639983/2.4+0.002333333)*x**2],x=0..6,title="The depedence of Temperature on space coordinate with Frank-Kamenestkii parameter varying");
```

Figure 2 maple plot

```
> plot([1/2-x/2-(6.0056/0.4+0.001167)*x**2,1/2-x/2-(6.0056/0.8+0.001167)*x**2,1/2-x/2-(6.0056/1.2+0.001167)*x**2,1/2-x/2-(6.0056/1.6+0.001167)*x**2,1/2-x/2-(6.0056/2.0+0.001167)*x**2,1/2-x/2-(6.0056/2.4+0.001167)*x**2],x=0..5,title="The dependence of Temperature on Space coordinate with Frank-kamenestkii parameter varying");
```