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# MHD free convective heat transfer in fluids with oscillatory suction

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## ABSTRACT

A study on the influence of Magnetohydrodynamic (MHD) flow of convective heat transfer in the presence of oscillatory suction velocity was carried out. Analysis of the solution governing the hydrodynamical equations showed that increase in Grashof number, angular frequency and Prandtl number correspond to an increase in velocity profile of the fluid while increase in magnetic Hartmann number results to a decrease in the velocity profile of the fluid. Increase in Prandtl number and angular frequency also caused a decrease in the temperature profile of the fluid.

**KEYWORDS**: Magnetohydrodynamics; Boussinesq approximation; SuctionVelocity; Buckingham $\pi-$theorem; Convective Heat Transfer.

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## INTRODUCTION

Fluid flow of convective heat transfer is common in nature and has many applications in engineering and sciences. A typical example is the phenomenon of land and sea breezes observed in areas close to seas and large water bodies. Several researchers have investigated such flows and literatures on the properties and phenomenon are abounded. For instance, Soundalgekar (1973) investigated the effects of free-convection currents on the oscillatory type boundary layer flow past an infinite vertical plate with constant suction where the plate temperature differs from the free stream temperature. In another study, Soundalgekar (1974) investigated a two dimensional steady free-convection flow of an incompressible, viscous electrically conducting fluid past an infinite vertical porous plate with constant suction and plate temperature when the difference between the plate temperature and free stream is moderately large to cause free-convection currents. Also, Israel-Cookey and Sigalo (2002) investigated the problem of unsteady MHD past a semi-infinite vertical plate in an optically thin environment with simultaneous effects of radiation, free-convection parameter and time dependent suction. Recently, attention has been on MHD flows. According to Branover (1978), since magnetic field exist everywhere in the world, it follows that MHD phenomena must occur wherever conducting fluids are available. In essence, the flow of an electrically conducting fluids has many applications in engineering problems such as MHD generators, MHD flow meters, MHD pumps and engines, plasma studies, nuclear reactors, geothermal energy extraction, ground water pollution, heat exchangers and much more. The aim of this paper is therefore, to critically examine the effect of magnetic field and suction in free-convection flow regime and the interplay of magnetic fields and heat transfer to flows in particular.

## NOMENCLATURE

u= fluid velocity

$\rho =$ fluid density

$T_0 =$ reserviour temperature

$\sigma_c =$ electrical conductivity

T = temperature of fluid

$B^2 =$ imposed magnetic field

$\omega =$ angular frequency

$z' =$ dimensionless coordinate

z = transverse coordinate

a = thermal diffusivity

$\theta =$ dimensionless temperature

Pr = Prandtl number

g = acceleration due to gravity

$\beta =$ coefficient of volume expansion

d = characteristic distance

$\mu =$ absolute viscosity

Gr = Grashof number

Ha = magnetic Hartmann number

$\omega' =$ dimensionless angular frequency

$t' =$ dimensionless time

t = time

$\mu' =$ dimensionless absolute viscosity

## MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

We consider the unsteady MHD convective heat transfer in fluids with the assumption that the fluid is essentially incompressible. Under Boussinesq approximation and imposed magnetic field, the flow is governed by the following equations

$$\frac{\partial u}{\partial t} + \left(1 + Ae^{i\omega t}\right)\frac{\partial u}{\partial t} = \beta g (T - T_0) + \frac{\mu \partial^2 u}{\rho \partial z^2} - \frac{\sigma_c B^2 u}{\rho} \quad (1)$$

$$\frac{\partial T}{\partial t} + \left(1 + Ae^{i\omega t}\right)\frac{\partial T}{\partial z} = \frac{a^2 \partial^2 T}{\partial z^2} \quad (2)$$

Subject to the boundary conditions

$$u(0)=0, u(d)=1$$
$$T(0)=1, T(\infty)=0 \qquad (3)$$

## Non-dimensional analysis

Using the Buckingham $\pi -$theorem, equations (1) - (3) is transformed into the form as follows

$$\frac{\partial u'}{\partial t'}+\left(1+Ae^{i\omega' t'}\right)\frac{\partial u'}{\partial z'}=Gr\theta+\frac{\mu'\partial^2 u'}{\partial z'^2}-Hau' \qquad (4)$$

$$\frac{\partial \theta}{\partial t'}+\left(1+Ae^{i\omega' t'}\right)\frac{\partial \theta}{\partial z'}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial z'^2} \qquad (5)$$

Subject to the boundary conditions

$$u'(0)=0, u'(d)=1$$
$$\theta(0)=1, \theta(\infty)=0 \qquad (6)$$

where

$$u'=\frac{ud\rho}{\mu}, t'=\frac{tu}{\rho d^2}, \theta=\frac{T}{T_0}, z'=\frac{z}{d}, Gr=\frac{g\beta T d^3\rho^2}{u^2}, Ha=\frac{\sigma_c B^2}{u^2}$$
$$\Pr=\frac{a^2\rho}{\mu}, \omega'=\omega t,, \mu'=\frac{\mu}{\rho}$$

## METHOD OF SOLUTION

We seek solution of equations (4) and (5) by adopting transformations of the form

$u'=H(z')e^{-\alpha t'}$ and $\theta=\varphi(z')e^{-\alpha t'}$ (7)

with the boundary conditions

$$H(0)=0, H(d)=e^{\alpha t'} \qquad (8)$$

$$\varphi(0)=e^{\alpha t'}, \varphi(\infty)=0 \qquad (9)$$

where $\alpha$ is a decay constant

If we substitute equation (7) into equation (5) and after simplification, results in

$$\varphi''(z')-\Pr\left(1+Ae^{i\omega' t'}\right)\varphi'(z')+\alpha\Pr\varphi(z')=0 \qquad (10)$$

The solution of equation (10) and the imposition of the boundary conditions (9) as well as its back substitution, gives us

$$\theta(z')=e^{0.0004153z'}-\left(2.4913\right)e^{5.9558z'} \qquad (11)$$

Having used $A=1, \Pr=0.71, \omega=2, t=1, \alpha=0.0035$

Similarly, we put equation (7) into equation (4) and after simplification, we get

$$H''(z')-\left(1+Ae^{i\omega' t'}\right)H'(z')+\left(\alpha+Ha\right)H(z')=-Gr\theta \qquad (12)$$

Also, we put equation (11) into equation (12), solving the resulting non-homogeneous differential equation and imposing the boundary conditions (8) as well as its back substitution, we get

$$u'(z') = Ae^{\lambda_1 z'} + Be^{\lambda_2 z'} + Gr\left(M_1 e^{0.00043153z'} + M_2 e^{5.9558z'}\right)$$

where

$$A = \frac{Gr(M_1 + M_2) + e^{\alpha t'} - Gr(M_1 e^{0.00043153d} + M_2 e^{5.9558d})}{\left(e^{\lambda_1 d}\right)e^{\alpha t'}}$$

$$B = \frac{-Gr(M_1 + M_2) + e^{\alpha t'} + Gr(M_1 e^{0.00043153d} + M_2 e^{5.9558d})}{\left(e^{\lambda_1 d}\right)e^{\alpha t'}} - Gr\left(M_1 + M_2\right)$$

$$\lambda_1 = \frac{\left(1 + Ae^{i\omega' t'}\right) - \sqrt{\left(1 + Ae^{i\omega' t'}\right)^2 - 4(\alpha + Ha)}}{2}$$

$$\lambda_2 = \frac{\left(1 + Ae^{i\omega' t'}\right) + \sqrt{\left(1 + Ae^{i\omega' t'}\right)^2 - 4(\alpha + Ha)}}{2}$$

$$M_1 = \frac{-1}{1.86218x10^{-07} - 0.00043153\left(1 + Ae^{i\omega' t'}\right) + (\alpha + Ha)}$$

$$M_2 = \frac{2.4913}{35.4715 - 5.9558\left(1 + Ae^{i\omega' t'}\right) + (\alpha + Ha)}$$

The solution is now complete.

## RESULTS

In order to get physical insight and numerical validation of the problem, a typical value of the prandtl number corresponding to an astrophysical body (air) at $25^0 C$ is chosen because it is weakly electrically conducting under assumed circumstances and the problem under study in particular. The values of other parameters made use of are

$$\alpha = 0.0035$$
$$\Pr = 0.00, 0.11, 0.31, 0.51, 0.71$$
$$\omega' = 0.0, 2.0, 3.0, 4.0, 5.0$$
$$Gr = 0.0, 2.0, 4.0, 6.0, 8.0$$
$$Ha = 0.0, 1.0, 3.0, 5.0, 7.0$$
$$t' = 1$$
$$d = 1$$

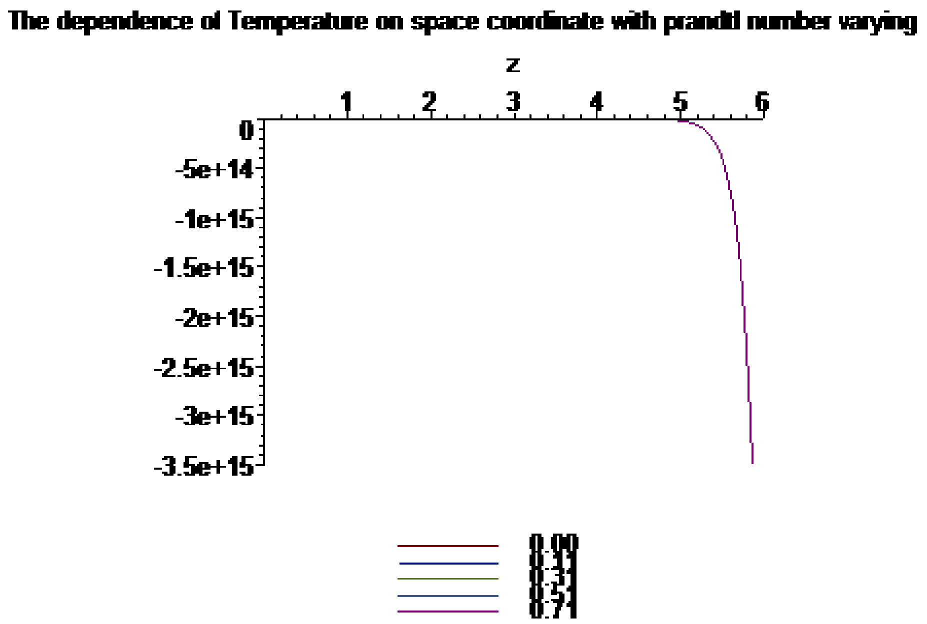


Figure 1: Temperature profile $\theta$ against boundary layer $z'$ for varying Prandtl number.

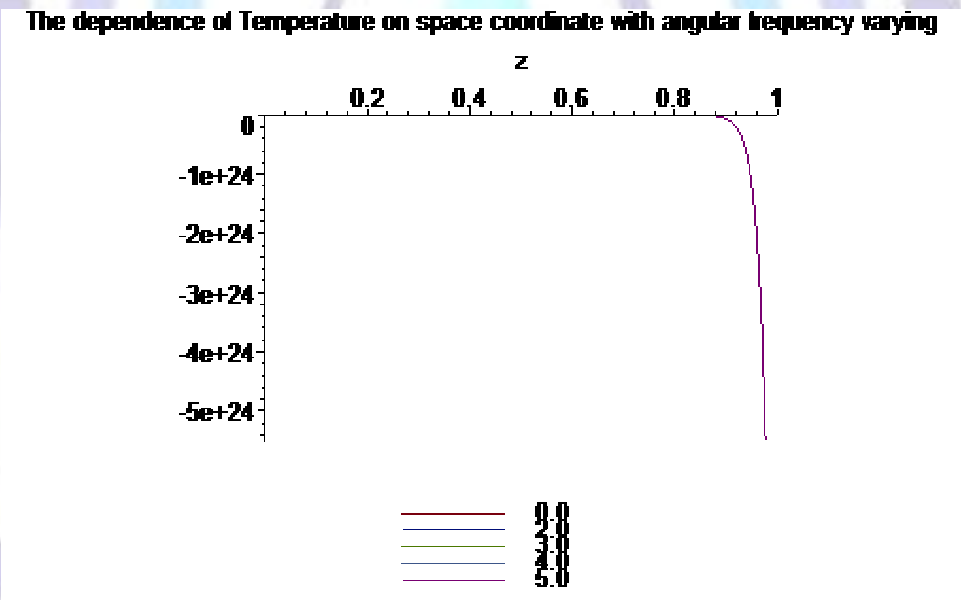


Figure 2: Temperature profile $\theta$ against boundary layer $z'$ for varying Stream frequency.

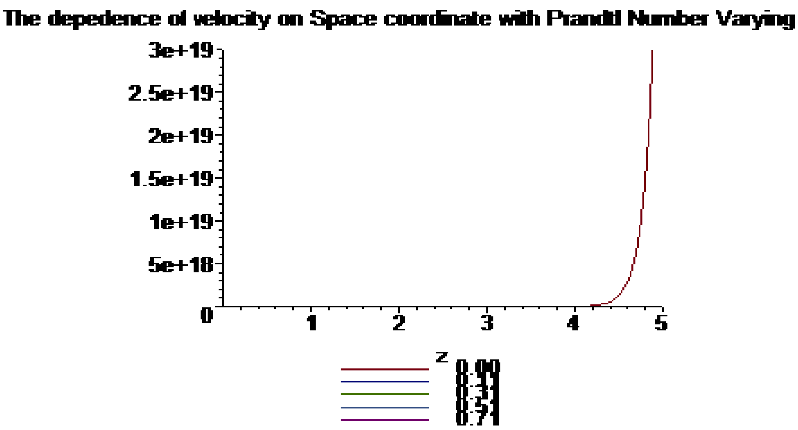


Figure 3: Velocity profile $u'$ against boundary layer $z'$ for varying Prandtl number.

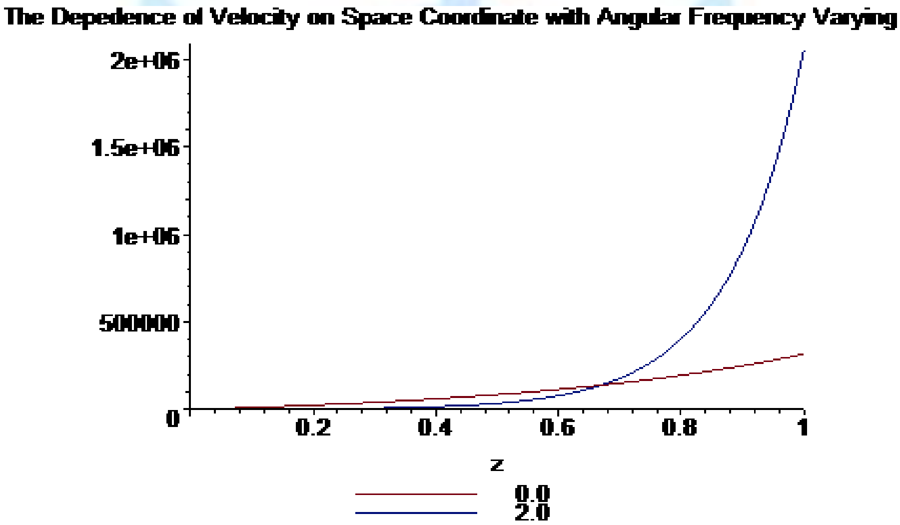


Figure 4: Velocity profile $u'$ against boundary layer $z'$ for varying Stream frequency.

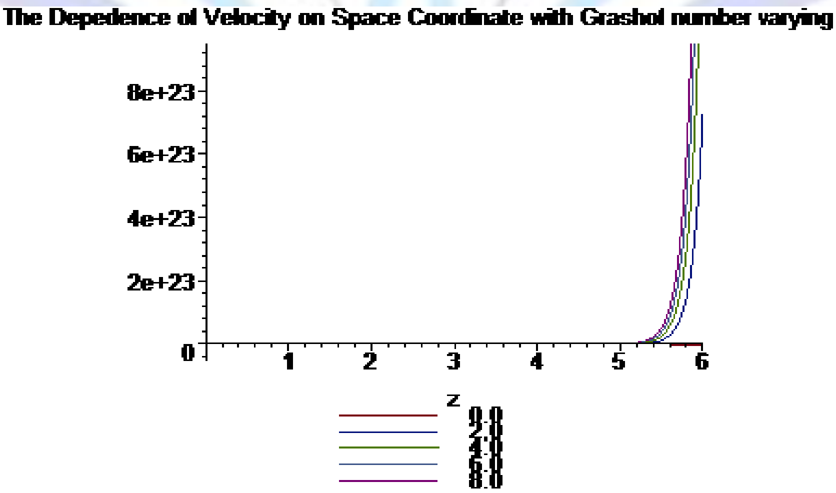


Figure 5: Velocity profile $u'$ against boundary layer $z'$ for varying Grashof number.

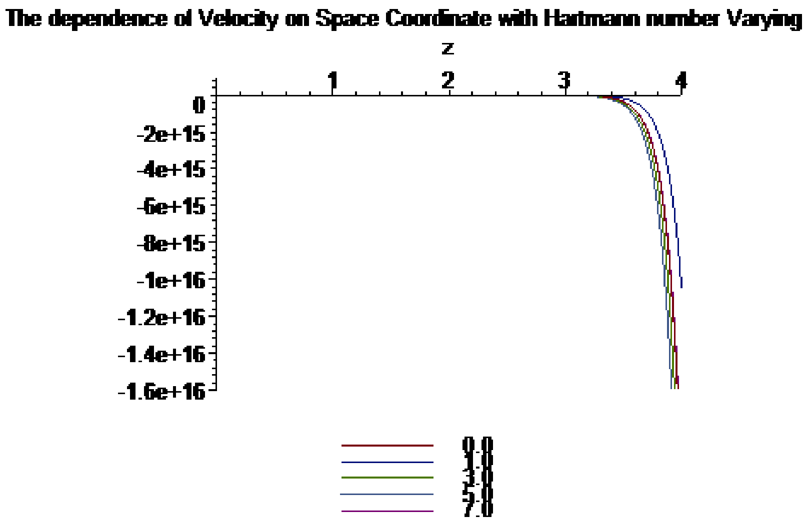


Figure 6: Velocity profile $u'$ against boundary layer $z'$ for varying Hartmann number.

## DISCUSSION

In the analysis, we start with temperature profile. The effect of increase in Prandtl number as shown in Figure 1, results in a decrease in the temperature profile of the fluid which is consistent with the study of Israel-Cookey (2003). Figure 2 showed that an increase in angular frequency gave rise to a decrease in the temperature distribution of the fluid. Increase in Prandtl number caused a corresponding increase in the velocity profile of the fluid as depicted in Figure 3. Figure 4 displayed the effect of increase in angular frequency which results in an astronomical increase in the velocity profile of the fluid.

This observation is in agreement with an earlier study of Ngiangia and Orukari (2013). Increase in Grashof number result in heating of the fluid and caused an increase in the velocity profile of the fluid as shown in Figure 5. Finally, increase in Magnetic Hartmann number as depicted in Figure 6 results in a decrease in the velocity profile of the fluid.

## CONCLUSION

Generally free convection currents and its attendant parameters were investigated in the study and within minimal approximations as well as use of Maple 18 software, the graphs were drawn which showed the effect of the parameters investigated on the temperature and velocity profiles of the fluid.

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