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AN ELEMENTARY PROOF OF GILBREATH'S CONJECTURE

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ABSTRACT

Given the fact that the Gilbreath's Conjecture has been a major topic of research in Aritmatic progression for well over a Century,and as bellow:

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61

1 2 2 4 2 4 2 4 6 2 6 4 2 4 6 6 2

1 0 2 2 2 2 2 2 4 4 2 2 2 2 0 4

1 2 0 0 0 0 0 2 0 2 0 0 0 2 4

1 2 0 0 0 0 2 2 2 2 0 0 2 2

1 2 0 0 0 2 0 0 0 2 0 2 0

1 2 0 0 2 2 0 0 2 2 2 2

1 2 0 2 0 2 0 2 0 0 0

1 2 2 2 2 2 2 2 0 0

1 0 0 0 0 0 0 2 0

1 0 0 0 0 0 2 2

1 0 0 0 0 2 0

1 0 0 0 2 2

1 0 0 2 0

1 0 2 2

1 2 0

1 2

1

The **Gilbreath's conjecture** in a way as easy and comprehensive as possible.

He proposed that these differences, when calculated repetitively and left as bsolute values, would always result in a row of numbers beginning with 1,In this paper we bring elementary proof for this conjecture.

Keywords. Forward difference operator,finite difference method,difference Equation

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1 INTRODUCTION:

Given the fact that the Gilbreath's Conjecture has been a major topic of research in aritmatic progression for well over a century,the Gilbreath conjecture in a way as easy and comprehensive as possible. Hopefully it will help the right person take this conjecture out of the unsolved list and into the list of accomplishments of mathematics.

To begin the story, the anecdote goes that an undergraduate student named normanGilbreath was doodling on a napkin one day in a cafe and found a very interesting characteristic of the list of sequential prime numbers and the

diferences between them. He proposed that these diferences, when calculated repetitively and left as absolute values, would always result in a row of numbers

beginning with 1 (after the first row). No one has been able to prove it. In 1878, eighty years before Gilbreath's discovery, François Proth had, however, published the same observations along with an attempted proof, which was later shown to be false.Andrew Odlyzko verified that d_1^k is 1 for $k \leq n = 3.4 \times 10^{11}$in 1993, but the conjecture remains an open problem. Instead of evaluating *n* rows, Odlyzko evaluated 635 rows and established that the 635th row started with a 1 and continued with only 0's and 2's for the next *n* numbers. This implies that the next *n* rows begin with a 1,see[15]

Notation

We define $d_n^k$is K-th row, n- th,Number, in $d_n^k = |d_{n+1}^{k-1} - d_n^{k-1}|$

We should prove that $d_1^k = 1$,for any k

Theorem: $d_1^k = 1$,for any k

Proof: Assume that the Gilbreath's Conjecture is correct until p_m,that is m-th prime in first row by induction, we prove that this Conjecture is correct for p_{m+1},hence below table is correct by induction

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61…….$p_{m-2}p_{m-1}$ p_m

1 2 2 4 2 4 2 4 6 2 6 4 2 4 6 6 2

1 0 2 2 2 2 2 2 4 4 2 2 2 2 0 4

1 2 0 0 0 0 0 2 0 2 0 0 0 2 4

1 2 0 0 0 0 2 2 2 2 0 0 2 2

1 2 0 0 0 2 0 0 0 2 0 2 0

1 2 0 0 2 2 0 0 2 2 2 2

1 2 0 2 0 2 0 2 0 0 0

1 2 2 2 2 2 2 2 0 0

1 0 0 0 0 0 0 2 0

1 0 0 0 0 0 2 2

1 0 0 0 0 2 0

1 0 0 0 2 2

1 0 0 2 0

1 0 2 2

1 2 0

1 2

1

Notice that in above table for K- th row, n- th number,we have

$d_n^k = |d_{n+1}^{k-1} - d_n^{k-1}| < 3^n \leq 3^m$,Now we prove that this table is correct for p_{m+1},

2 LEMMA:

For simplicity this conjecture we state some Lemmas as below:

Lemma 1: if p_m to be m-th prime ,so $p_m < 3^m$ for $m \geq 1$

Proof: According to [1] ,this is Correct

Lemma 2: the Second row is correct ,i.e, $|p_{m+1} - p_m| < p_m < 3^m$

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Proof, this is correct by refer to [1]

Lemma 3: the third row is correct ,i.e, $d_{m-1}^{3} = \left||p_{m+1} - p_m| - |p_m - p_{m-1}|\right| < p_{m-1} < 3^{m-1}$,

Proof :this is correct by refer to [1]

Lemma 4: k-th row is correct,$4 \leq k \leq m+1$,i.e $d_{m-(k-2)}^{k} = \left|d_{m-(k-3)}^{k-1} - d_{m-(k-2)}^{k-1}\right| < 3^{m-(k-2)}$

Proof: we assume that this is not hold for $k \geq 4$,notice that from $k=4$ to $k=m+1$, we have $d_{m-(k-2)}^{k} = |d_{m-(k-3)}^{k-1} -$ *dm−k−2k−1≥3m−k−2*

So for simplicity we write abbreviation as below:

$$a_1 = a - b \geq 3^{m-2}$$

$$a_2 = a_1 - b_1 \geq 3^{m-3}$$

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$$a_{m-2} = a_{m-3} - b_{m-3} \geq 3^{(m-1)-(m-2)}$$

We add above formula ,hence:

$$a_1 + a_2 + \cdots + a_{m-2} \geq 3 + 3^2 + \cdots 3^{m-2}$$

But each item is smaller than$a$, and $a < p_{m-1}$,So:

Or $\quad p_{m-1} > \frac{3^{m-1}-3}{2(m-2)}$

According to [1], there are constants numbers $c_1 \& c_2$ such that:

$$c_1(m-1)\log(m-1) < p_{m-1} < c_2(m-1)\log(m-1)$$

So this is Contradiction for $m \geq 3$

Lemma 5: suppose the s-th row is correct $s \geq 4$,so , $s+1 \leq k \leq m+1$, we prove that $d_{m-(k-2)}^{k} = |d_{m-(k-3)}^{k-1} -$ *dm−k−2k−1<3m−k−2*

Proof : this proof is similar to Lemma 4, by substitute $m-s$ instead m

So ,
$$p_{m-s-1} > \frac{3^{m-s-1}-2}{m-s-2}$$

This is Contradiction for $m-s \geq 3$

Corollary : By refer to above Lemmas,assume that we have some equations as below:

$$p_{k-2} \leq a_1 = a - b < 3^{m-2}$$

$$p_{k-3} \leq a_2 = a_1 - b_1 < 3^{m-3}$$

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$$2 = p_1 \leq a_{m-2} = a_{m-3} - b_{m-3} < 3^{(m-1)-(m-2)}$$

Then,$a_{m-2} = 2$,and this is contradiction ,because a_{m-2},is odd

Notice that ,if from s-th row, we have equations like above , we reach to similar conclusion .

If in above equations , g-th row to be changed,i.e ,
$3^{k-g} \leq a_{g-1} = a_{g-2} - b_{g-2} < p_{k-g}$,this is contradiction too.

3 MAIN THEOREM:

Theorem: $d_1^k = 1$,for any k

Proof:According to above Lemmas this theorem is hold, for $k = m + 1$ since $d_1^{m+1} < 3$ then $d_1^{m+1} = 1$,therefore we proved this Theorem.

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