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Edge Jump Distance Graphs

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ABSTRACT

The concept of edge jump between graphs and distance between graphs was introduced by Gary Chartrand et al. in [5]. A graph H is obtained from a graph G by an edge jump if G contains four distinct vertices u, v, w, and x such that uv $\in$ E(G), wx $\notin$ E(G) and H $\cong$ G – uv + wx. The concept of edge rotations and distance between graphs was first introduced by Chartrand et.al [4]. A graph H is said to be obtained from a graph G by a single edge rotation if G contains three distinct vertices u, v, and w such that uv $\in$ E(G) and uw $\notin$ E(G). If a graph H is obtained from a graph G by a sequence of edge jumps, then G is said to be j-transformed into H.

In this paper we consider edge jumps on generalized Petersen graphs $G_p(n,1)$ and cycles. We have also developed an algorithm that gives self-centered graphs and almost self-centered graphs through edge jumps followed by some general results on edge jumps.

Indexing terms/Keywords

Edge jump; Jump distance graphs(JDG); edge jump distance graphs(EJDG); rotation distance graphs(RDG); edge rotation distance graph (ERDG), cycles; self centered graphs; almost self centered graphs; generalized Petersen graphs; tree; path, edge distance graphs(EDG); prism.

Academic Discipline And Sub-Discipline

Graph Theory, Concept of distance, Rotation distance graphs.

SUBJECT CLASSIFICATION

Primary 05C12, Secondary 05C38, 05C05.

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TYPE (METHOD/APPROACH)

Edge Operation on Graphs like rotation, move, jump and slide.

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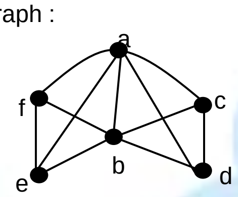
INTRODUCTION

Unless mentioned otherwise, for terminology and notation the reader may refer Buckley and Harary [1] and Chartrand and Ping Zhang [2], new ones will be introduced as and when found necessary.

In this paper, by a graph G we mean a simple, undirected, connected graph without self loops. The order and size are respectively the number of vertices denoted by n and edges denoted by m.

The distance d(u, v) between any two vertices u and v of G is the length of a shortest path between u and v. The eccentricity e(u) of a vertex u is the distance to a farthest vertex from u. If d(u,v) = e(v), (v $\neq$ u) then we say that v is an eccentric vertex of u. The maximum and the minimum eccentricity amongst the vertices of G are respectively called the diameter diam(G) and the radius rad(G). If diam(G) = rad(G), then the graph G is said to be a self-centered graph.A graph G is almost self-centered graph if thecenter of G consists of |V (G)| − 2 vertices, i.e., the graph with almost 2 non-central vertices. In the below quoted example of a almost self-centered graph, the vertices 'a' and 'b' are the central vertices with eccentricity 'one' and the rest of the vertices having eccentricity two.

Example of an almost self-centered graph :



In [1] the eccentric mean or average eccentricity is defined as $\mu_e(G) = \left(\frac{1}{n}\right)\sum_{i=1}^{n} m_i e_i$ taken over all vertices in the graph where m_i's are the multiplicities of the eccentricities e_i.

In [3], the prism of the graph G is defined as the cartesian product G $\times$ K_2 ; that is take two disjoint copies of G and add a matching joining the corresponding vertices in the two copies.

Many distances between graphs work on idea of transformations. In this paper we consider transformations like edge jump and edge rotations which were introduced by Gary Chartrand et al. [4] [5] . Many results on rotations were developed after Zelinka introduced the concept of distance between the isomorphism class of graphs and trees in [6] [7]. This was further extended to various other classes of graphs in [8], [9], [10], [11]. The concept of edge rotations was introduced first and then followed by edge jump.

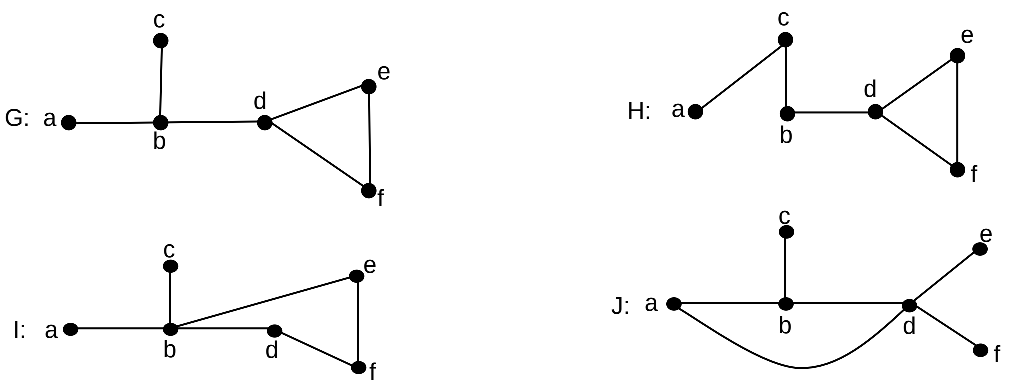
Let G and H be two graphs having the same order and the same size. In [12] a graph G is said to have move transformed into H, if G contains four vertices(not necessarily distinct) u, v, w and x, such that uv $\in$ E(G) and wx $\notin$ E (G) and H $\cong$ G – uv + wx. A graph G is m-transformed into a graph H if H is obtained (isomorphic to the graph) from G by a sequence of edge moves, i.e., if there is a sequence G = G_0, G_1, . . . , G_n = H (n $\geq$ 0) of graphs such that G_{i+1} is obtained from G_i by an edge move for i = 0, 1, 2, ….. n-1.

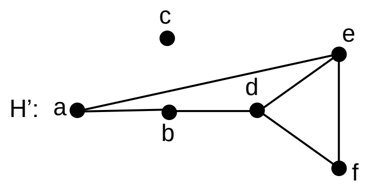
Similarly, in [4] a graph H is said to be obtained from G by an edge rotation if G contains three distinct vertices u, v and w such that uv $\in$ E(G) and uw $\notin$ E(G) and H $\cong$ G – uv + uw. Generally, G is r-transformed into H if H is obtained from G by a sequence of edge rotations.

Now we define the concept of edge jump. In [5], Chartrand et al., defined that a graph H is obtained from a graph G by an edge jump if G contains four distinct vertices u, v, w and x such that uv $\in$ E(G) and wx $\notin$ E(G) and H $\cong$ G – uv + wx. If H is (isomorphic to the graph)obtained from G by an edge jump we say that G is j-transformed into H.

In edge move, it is unrestricited transfer of an edge uv of a graph G to an edge wx, where wx $\notin$ G, whereas in edge rotation and edge jump it is an n restricted edge transfer. In edge move, the vertices u, v, w and x may or may not be distinct. In an edge rotation, the vertices u, v, w and x are not distinct, while in an edge jump the vertices must be distinct.

Example for edge move, rotation and edge jump are as follows:





In the above graphs, in H, we denote edge move without restrictions using only 3 vertices. In the graph I, the edge rotation is shownbetween the edges 'de' and 'be'. In the graph J, we have shown edge jump. In H' we have shown edge move with restriction resulting in a disconnected graph.

The rotation distance between graphs G and H is denoted by $d_r(G, H)$, if there exists a sequence ofgraphs G_1, G_2, …,G_{k-1} such that G_1 is obtained by an edge rotation on G, and for each $1 \le i \le k-1$, G_{i+1} isobtained by an edge rotation on G_i, with H obtained from $G_{k-1}$by one edge rotation. In this case we denote therotation distance from G to H as $d_r(G, H)$ and it is equal to k.

The jump distance between graphs G and H is denoted by $d_j(G, H)$, if there exists a sequence of graphs G_1, G_2, …G_{k-1} such that G_1 is obtained by an edge jump on G, and for each $1 \leq i \leq k-1$, G_{i+1} is obtained by an edge jump on G_i, with H obtained from G_{k-1} by one edge jump. Thus for every two graphsG and H of the same order and the same size, the jump distance graph denoted by $d_j(G, H)$ is defined as the minimum number of edge jumps required to j-transform G into H.

The edge rotation between the graphs G and I can be represented as $d_r(G,I)$, $I \cong G - de + be$.

The edge jump between the graphs G and J can be represented as $d_j (G,J)$, $J \cong G - ef + ad$.

The edge move between the graphs G and H without any restriction can be represented as $d_m(G, H)$, $H \cong G – ab + ac$.

Similarly, the edge move with restriction can be represented as $d_m(G, H')$, $H' \cong G – cb + ae$.

Definition [5]: Let S = {G_1, G_2, . . . , G_k} be a set of graphs all of the same order and the same size (atleast 5). Then the jump distance graph $D_j(S)$ of S has S as its vertex set and G_1 and G_2 in S are adjacent if and only if $d_j(G_1 , G_2) = 1$.

A graph G is a edge jump distance graph (EJDG) if there exists a set S of graphs of the same order and the same size with $D_j(S) = G$ ($D_j(S) \cong G$) .

Chartrand et al.[5] showed that complete graphs, trees, cycles and the complement of the line graph are edge jump distance graphs (EJDG). They also showed the Cartesian product of two jump distance graphs is a jump distance graph. Under the concept of graph operations like join, for 2 graphs G and H of the same order and the same size, such that $d_j(G, H) = 1$, they showed that $d_j(G +K_1 , H+K_1) = 1$.

In this paper we consider the edge jumps for generalized Petersen graphs ($G_p(n,k)$) for k = 1 and show that it is a edge jump distance graph for $n \geq 5$, $n \in N$, k = 1, where N is the set of natural numbers. We have also considered the edge jump operations on the generalized Petersen graphs which results in a almost self-centered graph. Similarly the edge jump on a prism ($C_n \times K_2$) also results in a self-centered /almost self-centered graph depending on value of 'n' being odd or even.

Next, we have considered edge jumps on various classes of graphs like trees and paths. Some results on average eccentricity with respect to edge jumps are also proved. Finally we have developed algorithm to generate almost self-centered graphs from even cycles.

2. EDGE JUMPS FOR GENERALIZED PETERSEN GRAPHS

The generalized Petersen graphs were introduced by Coxeter in [13] and later named by Watkins[14].

Definition: For integers n and k with $2 \leq 2k < n$, the generalized Petersen graph $G_p(n,k)$ has the vertx set $V(G(n,k)) = u_0, u_1, \dots u_{n-1}, v_0, v_1, v_{n-1}$ and the edge set $E(G(n,k)) = [u_i , u_{i+j}], [u_i, v_i], [v_i\ v_{i+k}]$, where i is an integer and all subscripts are read modulo n.

As the name suggests $G_p(n, k)$ is the generalized Petersen graph. In particular for n = 5 and k = 2, we get the Petersen graph. Note that a generalized Petersen graph is a cubic graph.

In this section, we consider edge jumps on the generalized Petersen graph where $n \geq 5$, $n \in N$ and k = 1.

The new graph obtained after a single edge jump will bedenotedby G^j i.e., $G^j = G – e(G) + e(\overline{G})$.

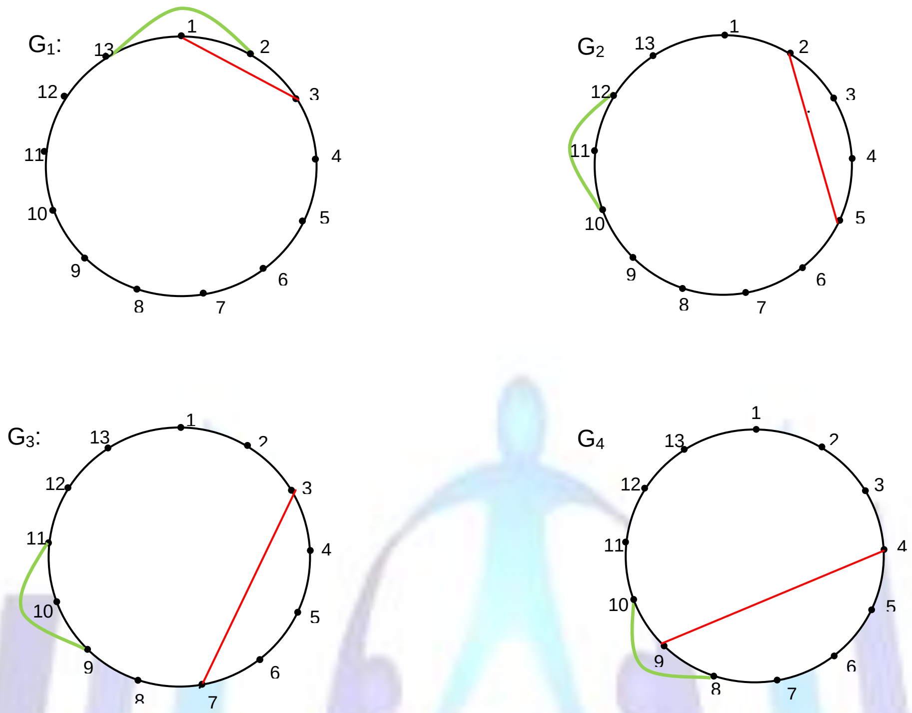
It was shown that complete graphs, cycles, trees and complement of line graphs are edge jump distance graphs. Now by slight modification of the graphs used in proofs of the theorems proved by Chartrand et al.[5] we show that $G_p(n,k)$, where k = 1 results in a edge jump distance graph(EJDG). We first generate two cycles as shown in [5] and then show that the generalized Petersen graph is a jump distance graph.

Theorem 2.1: The generalized Petersen graph $G_p(n,1)$ is a edge jump distance graph(EJDG) for $n \geq 4$.

ISSN 2347-1921

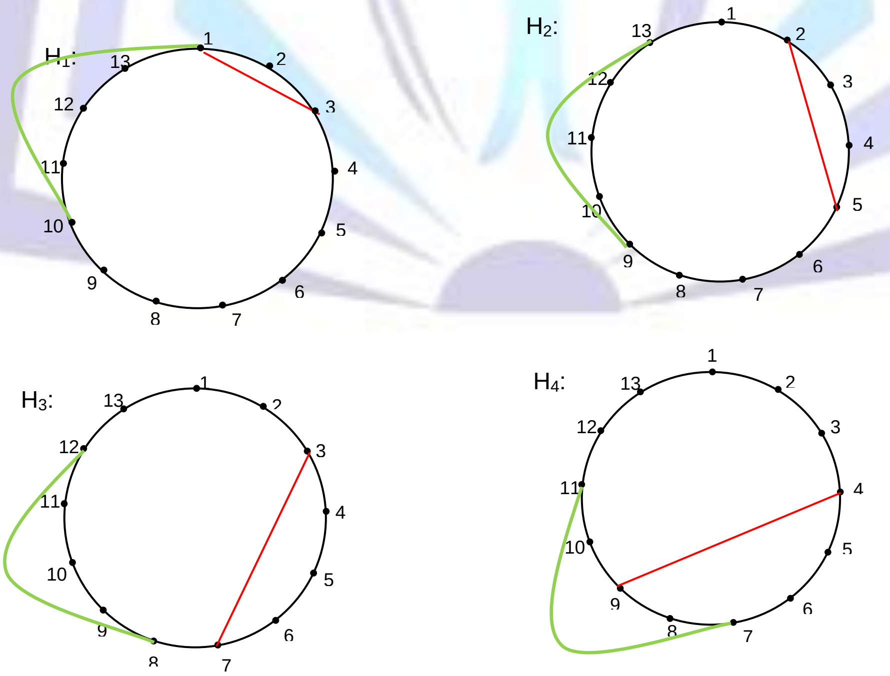
Proof: Let $n \geq 4$, be a positive integer and let $C : v_1, v_2, v_3, \ldots, v_{2n+5}$ be a 2n+5 cycle. For i = 1, 2, 3, . . . ,n. Let

$F_i = C = v_i v_{2i+1}$. Since F_i $(1 \leq i \leq n)$ contains a cycle of length i+ 2, it follows that the graphs $G_1, G_2, \ldots, G_n$ are pairwise non-isomorphic. For n = 4, the graphs G_1, G_2, G_3, G_4 are shown below.



In a similar way we generate one more cycle and then show that the edge jump betweeneach of these F_i and G_i is equal to one. The construction for F_i is as follows.

The edges highlighted in red show the formation of a cycle in both of G_i and H_i, where as the edges highlighted in green show the jump distance between each of $G_i$and H_i is one.



For i = 1, 2, 3, . . . , n, add an edge to the cycle starting from v_2 that is in the backward direction.

ISSN 2347-1921

$F_1 = C + v_2v_{2n+5}$, $F_2 = C + v_1v_{2n+4}$, $F_3 = C + v_{2n+3}v_{2n+1}$, $F_4 = C + v_{2n+2}V_{2n}$.

Add an edge from v_2 and make a cycle of length 3 such that the end vertex of cycle of F_1 is not equal to the starting vertex of cycle of F_2.

Similarly in G_i add a cycle of length 5 starting from v_1 in the backward manner. Now it follows that each F_i and G_i contains a cycle of length 3 and 5 such that they are pair wise non isomorphic.

Next, for i = 1, 2, n-1, . . . , let $G_i = F_i \cup F_{i+1}$ and $G_n = F_n \cup F_1$. Clearly the graphs G_i and G_k $(1 \leq i \leq k \leq n)$ differ by exactly one edge when k = i+1 or when k = n and i = 1 and differ by two edges otherwise.

Thus, since $d_j(G_i, G_{i+1}) = 1$ $(i \leq i \leq n-1)$ $d_j(G_n, G_1) = 1$, it follows that $d_j\{(G_1, G_2, G_3, \ldots G_n)\} = C_n$.

Also, for i = 1, 2,…. n-1, let $H_i = F_i \cup G_i$ and $H_n = F_n \cup G_n$. Clearly the graphs F_i and G_i differ by exactly two edges. Thus, $d_j(F_I, G_i) = 1$ $(1 \leq i \leq n)$ and $d_j(F_n, G_n) = 1$, it follows that $d_j\{(G_1, \ldots. G_n, F_1, \ldots. F_n)\} = 1 \cong G_p(n,1)$.

Example 1: For n = 4, we show that $G_p(4,1)$ is a edge jump distance graph.

Step 1: For the graphs G_1 to G_4 and H_1 to H_4 obtained after adding the edges (highlighted in red) which are used in the generation of the cycle, we add one more new edge to each of G_i and H_i in the manner below prescribed to show the edge jump distance between each of these G_i and H_i forms a generalized Petersen graph.

Edge jumps on graph G_1 to G_4 and H_1 to H_4.

In G_1, add the edge from v_1 to v_{13}, in H_1 join v_1 to v_{10}.

In G_2 join v_{12} to v_{10} and in H_2 join v_{13} to v_9.

In G_3 join v_{11} to v_9 and in H_3 join v_{12} to v_8.

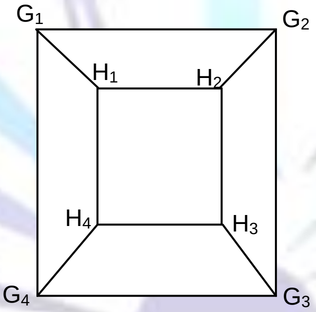
In G_4 join v_{10} to v_8 and in H_4 join v_{11} to v_7.

Step 2: Apply edge jump to the newly added vertices.

Step 3: We now perform the edge jump operation between each of G_i and H_i. That is the edge v_1v_{13} is jumped to the edge v_1v_{10} and thus the $d_j(G_1, H_1) = 1$.

In a similar way the jump operation is carried in the rest of the graphs to establish the j-distance relation.

Step 4: Thus, a relation is brought between the vertices of G and H in showing that the generalized Petersen graph is a edge jump distance graph.



Edge jump on $G_p(4,1)$

Lemma 2.2 : The star $K_{1,n}$ is a edge jump distance graph for $n \geq 2$, where $n \in N$.

Proof: Consider a path P of order 3n, where 1+n is the order of the star considered. Here we give a step by step procedure as proof to show that the star is a EJDG.

Step 1: Let $P = v_1, v_2, \ldots. v_{3n}$ be a labeled path and G_i be the graph formed by the following edge addition.

Step 2: For i = 1, 2, the graph G_i is formed as follows. $G_i = P + v_i v_{2i+1}$ and for the remaining i, that is, i = 3, 4, . . . ,n+1, G_i is formed as follows.

$G_i = P + v_{2i-1}v_{2i}$. Since $G_i (i \leq i \leq n+1)$ contains a cycle of length 3, it follows that the graphs $G_1, G_2, \ldots. G_{n+1}$ are pairwise isomorphic. Also , edge jump distance graph is obtained only between each of G_1 and G_{i+1} , for i = 1, 2,. . . , n+1. We can also note that the edge jump cannot be performed between G_2 and G_3, G_3 and G_4, …. G_n and G_{n+1}. Next for i = 1, 2, . . . ,

ISSN 2347-1921

n, let $H_i = G_1 \cup G_{i+1}$ and let $H_n = G_1 \cup G_{n+1}$. Thus, $d_j(H_1, H_{n+1}) = 1$ and $d_j(H_1, H_n) = 1$, it follows that $D_j\{(H_1, H_2, . . . , H_{n+1})\} = K_{1,n}$..

Example 1 : For n = 3, we show that the star $K_{1,3}$ is a edge jump distance graph.

Proof: To show this, one must note that the consecutive graphs must be non-isomorphic. That is, the edge jump distance between each of G_1 and G_{i+1} must be one. Hence the edges are added in this manner.

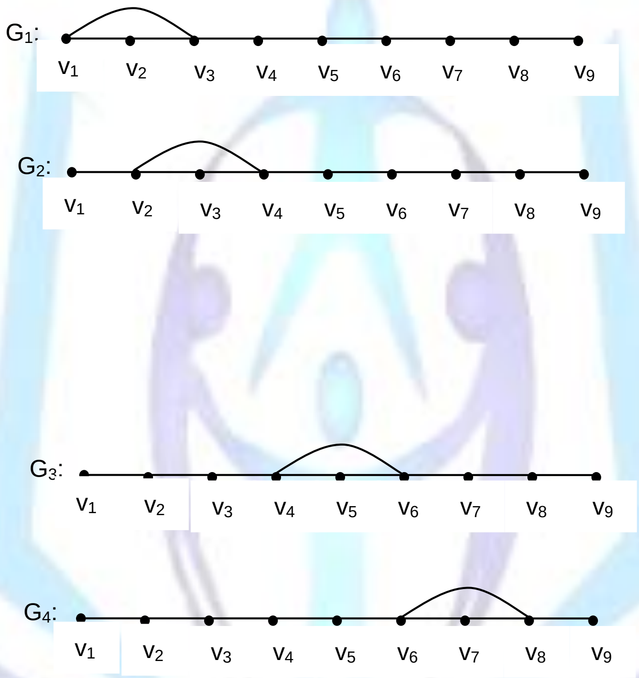
In G_1, join the edge between the vertices v_1 to v_3, which is shown in the below figure.

In G_2, join the edge between the vertices v_2 and v_4. In G_3 join v_4 to v_6 and in G_4 join v_6 to v_8.

Applying edge jump between the graphs, we find that G_1 is non-isomorphic to G_2, G_3 and G_4. This implies that we can use the edge jump definition on these graphs to show that they form a star.

Thus, $d_j(G_1, G_2) = d_j(G_1, G_3) = d_j(G_1, G_4) = 1$ and $d_j(G_2, G_3) = d_j(G_2, G_4) = d_j(G_3, G_4) = 2$.

Thus, the star $K_{1,3}$ forms a star.



The star $K_{1,3}$ is a EJDG

The next result in on almost self-centered graphs.

Lemma 2.3: An edge jump in $G_p(n, 1)$ for n > 5, results in an almost self-centered graph.

Proof: By the definition of edge jump, $G^j = G - e(G) + e(\overline{G})$ such that e(G) = uv and $e(\overline{G})$ = wx, where G^j is the new graph obtained after a single edge jump. As $G_p(n, 1)$ is a cubic graph any edge jump changes the degree of atmost 4 vertices. The end vertices of the edge selected for removal, lose one degree each and the edge replaced elsewhere gain one degree each thus resulting in a non-regular graph. The generalized Petersen graph $G_p(n,1)$ is a self-centered graph, that is radius being equal to diameter and rad = diam = n-2. If n is odd in $G_p(n,1)$, then each vertex consists of two eccentric vertices and if n is even, every vertex consists of a single eccentric vertex. When an edge jump is performed, the end vertices of the edge removed become eccentric vertices to each other retaining their earlier eccentricity. As the end vertices of the edge removed have neighbours which are at distance one from each other and that doesnot change after the removal of the edge.

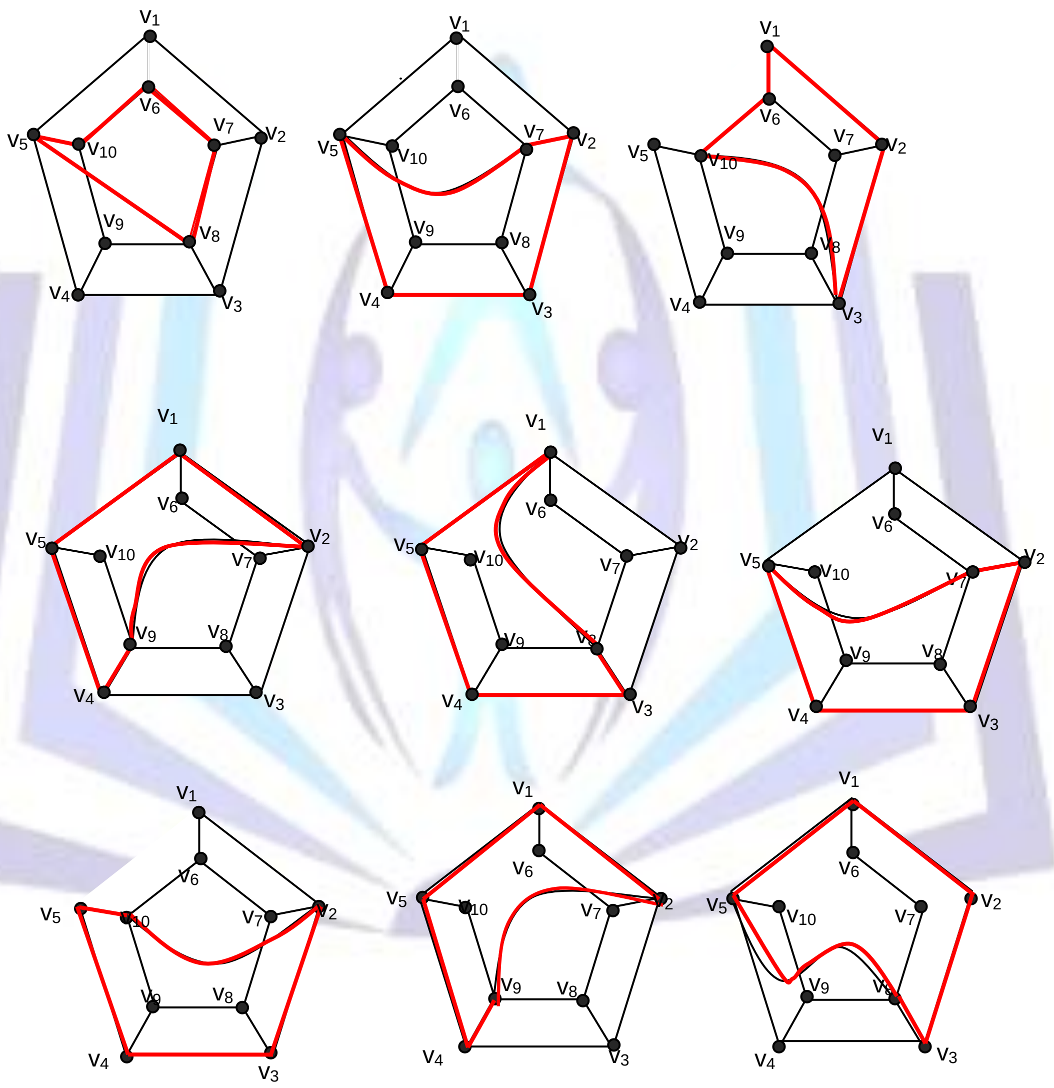
Hence an edge jump results in the reduction of eccentricity of the newly replaced edgeby one. Also, for some of the edge jumps the vertex adjacent to this newly added edge loses its eccentricity by one. Thus, in the graph we have atmost 3 vertices whose eccentricity is one less than its earlier eccentricity. This operation results in an induced cycle along the newly added edge from the complement. Thus,in the process of edge jump either twovertices or three vertices (the three vertices whose eccentricity reduces, will have the same eccentricity, that is one less than the earlier eccentricity) lose their eccentricity, while the rest of the vertices retain the values thus resulting in a almost self-centered graph.

ISSN 2347-1921

Lemma 2.4: An edge jump on $G_p(5, 1)$ results in a almost self-centered graph if the jump doesnot induce a cycle of length 5.

Proof: The definition of edge jump leads to change in degree of atmost four vertices which results in a non regular graph where as the generalized Petersen graph is a cubic graph. The girth of $G_p(n,k)$ graph is four. Thus we notice that the length of the induced cycle ranges from a minimum length 3 to a maximum of n-1. Among these, if the edge jump results in a induced cycle of length five, where one of the edges of the cycle includes this newly added edge from the complement,then it results in a almost self-centered graph, otherwise for rest of the jumps, it always results in a self-centered graph. Also we can notice that the new edge replaced forms a C_4 and this edge being common to both the cycles.When an edge jump is performed, the end vertices of the edge removed become eccentric vertices to each other retaining their original eccentricities. As the end vertices of the edge removed have neighbours which are at distance one from each other and that doesnot change after the removal of the edge.

The following edge jumps shown below result in an almost self-centered graph. The edge jumps and the induced cycle is shown in colored lines.



Recall the generalized Petersen graph and note that the average eccentricity depends on the value of n. Hence, for any n and for k = 1, the average eccentricity of generalized Petersen graph is given by $\left(\frac{n}{2}+1\right)$ or $\left\lfloor \frac{n}{2} \right\rfloor + 1$ for n being even or odd.

Theorem 2.5: The average eccentricity of the generalized Petersen graph $G_p(n,1)$ for any edge jump performed is less than n - 2, for n ≥ 6.

Proof: We shall be using the above Lemma (2.3) and Lemma (2.4) to prove this.We know that $G_p(n,1)$ is self-centered with radius = diameter = $\left(\frac{n}{2}+1\right)$ or $\left\lfloor\frac{n}{2}\right\rfloor+1$ (for n $\geq$5)(for n being even or odd). The average eccentricity of the generalized Petersen graph is $\left(\frac{n}{2}+1\right)$ or $\left\lfloor\frac{n}{2}\right\rfloor+1$, since it is a self-centered graph with radius equal to diameter and thus the average eccentricity would remain the same as radius (diameter). According to the above Lemma 2.3 and Lemma 2.4, an edge jump changes the degree of atmost 4 vertices resulting in a non - regular graph. Thereis a change in eccentricity of two or more vertices depending on the edge jump which induces a cycle of minimum length 3 to a maximum length of n-1.As the end vertices of the edge removed have neighbours which are at distance one from each other and that doesnot change after the removal of the edge.One can also note that depending on the edge jump, the eccentricity of one of the neighbours of newly replaced edge may or may not change. Thus we have atmost 2 to 3 vertices whose eccentricity is reduced by a maximum of one. Thus summing up the eccentricities of the vertices leads the sum to be always less than n - 2.

3: EDGE JUMPS ON PATHS, STARS, TREES AND CYCLES

As mentioned earlier edge jumps on graphs were introduced by Chartrand et al. in [5]. In this section we consider edge jumps on star, path and trees to transform them into respective class of graphs with the order of the graphs being retained.

LEMMA 3.1: Let S be a star with n vertices and let T be a arbitrary tree with n vertices and maximum degree Δ, then $d_j(t,S) = n - 1 - \Delta$.

Proof: Let u be the vertex of T with maximum degree Δ. Evidently, the subtree S_0 of T whose edge set is the set of all edges incident with u is a star with Δ+1 vertices and this S_0 is isomorphic to a subtree of S. We know that any subtree of S with atleast three vertices is a star, and the tree T cannot contain a subtree with more than Δ+1 vertices isomorphic to a subtree of S_0. As the maximum degree of S is n-1 and that of tree is Δ, it is necessary to perform atleast n-1-Δ edge jumps to obtain a graph isomorphic to S from T. Thus $d_j(T,S) = n - 1 - \Delta$.

LEMMA 3.2: Let P be a path with n vertices and T be an arbitrary tree with n vertices. Let 'd' be the diameter of the tree. Then $d_j(T, P) = n - 1 - d$.

Proof: Let P_0 be the diametrical path in T. This is a subtree of T which is isomorphic to a subtree of P and has d+1 vertices. As any subtree of P is a path, the tree cannot contain a subtree with more than d+1 vertices isomorphic to a subtree of P. As the diameter of P is n-1 and that of T is d, it is necessary to perform atleast n – 1-d edge jumps to obtain a P from T.

Corollary 3.3: Let S be a star with n vertices and let P be a path with n vertices. Then $d_j(P, S) = n - 3$.

Proof: Let P be a path with n vertices and S be a star with n vertices, the maximum degree of S is n-1 and that of P is 2. Hence leaving the last 2 edges, it is necessary to perform n – 1- 2 edge jumps to obtain a star S_0, isomorphic to S from P. Thus n – 3 edge jumps are required.

Remark 3.4 : An edge jump performed on any pendant edge of a complete binary tree of n levels results in a disconnected unicyclic graph i.e.,$G \cup K_1$.

Remark 3.5: An edge jump on level one binary tree (a path P_3) is not possible since there is no edge present in the complement that satisfies the definition of edge jump.

Remark 3.6: An edge jump on a path is possible if and only if the length of the path is greater than or equal to 3.

Theorem 3.7: The average eccentricity of C_n, after a single edge jump, lies between $\frac{n}{2}-\varepsilon \leq \frac{n}{2} \leq \frac{n}{2}+\varepsilon$ or $\left\lfloor\frac{n}{2}\right\rfloor-\varepsilon \leq \left\lfloor\frac{n}{2}\right\rfloor \leq \left\lfloor\frac{n}{2}\right\rfloor+\varepsilon$.

Proof: We know that the average eccentricity of the even cycle is given by $\left(\frac{n}{2}\right)$ and that of an odd cycle is given by $\left\lfloor\frac{n}{2}\right\rfloor$. Since the cycle is a minimum sized self-centered graph, we can consider any edge for the jump operation. Consider any edge of G for the edge jump operation. This leads to the change in eccentricity of the vertices which ranges from $\left(\frac{n}{2}\right)$ to $\left[\frac{n}{2}-\varepsilon\right]$ or $\left\lfloor\frac{n}{2}\right\rfloor$ to $\left\lfloor\frac{n}{2}\right\rfloor-\varepsilon$ such that $\varepsilon \neq n$. Clearly, the resulting graph is not a self-centered graph. Summing up the eccentricities we find that the minimum average eccentricity is not greater than $\frac{n}{2}-\varepsilon$ or $\left\lfloor\frac{n}{2}\right\rfloor-\varepsilon$ and the maximum average eccentricity is $\left(\frac{n}{2}\right)+\varepsilon$ or $\left\lfloor\frac{n}{2}\right\rfloor+\varepsilon$. Hence the proof.

4: ALGORITHM

In this section we develop algorithms to generate self centered graphs from cycles using the concept of edge jump operation. In literature construction of self-centered and almost self-centered graphs using the concept of edge operations was done by Huilgol et al. in [15]. Here we use edge jump operation in generating almost self-centered graph from cycles.

In the following algorithm we generate almost self-centered graphs from cycles using edge jump operation.

Algorithm 4.1

Step 1: Consider a cycle of length n.

Step 2: Find the eccentricity of the cycle as e = $\frac{n}{2}$ for even n and e = $\left\lfloor\frac{n}{2}\right\rfloor$ for odd n.

Step 3: Consider any edge from the complement for the edge jump operation. Using the definition of edge jump replace e(G) with e $(\overline{G})$.

Step 4: If the newly added edge forms an induced cycle of length (n-2) along with two pendant edges then the newly formed graph is an almost self – centered graph. Then goto Step – 7.

Else goto Step 5.

Step 5: If the newly added edge forms an induced cycle of length greater than n-2 or lesser than n-2 then perform Step – 4 until length = n-2.

Step 6: Repeat Steps 4 and 5 until we obtain an almost self- centered graph.

Step 7: Stop.

5. Edge jumps on Prisms

Recalling the definition of prisms, the following results are for edge jumps on prisms.

Theorem 5.1: An edge jump on a prism $C_n \times K_2$, when n is even is an almost self-centered graph.

Proof: Consider a cycle C_n of even length, i.e., n being even and consider the prism of this cycle ,C_n x K_2. By the definition of edge jump, consider any edgeof the cycle for the operation. The average eccentricity of the even cycle is $\left(\frac{n}{2}\right)$. By taking the prism of the cycle the eccentricity of the vertices of the graph will be $\left(\frac{n}{2}\right)+1$. Since the prism is self-centered, the average eccentricity of the graph will be $\left(\frac{n}{2}\right)+1$. Depending on the edge operation,if the edge jump induces a cycle of length three we find that eccentricity of the new edge decreases by at least one, where as the eccentricities of the remaining vertices remains the same. Also, for few of the edge jumps, i.e., if the edge jump induces a cycle of different length then the eccentricities of the neighbouring vertices reduce by one, thus forming an almost self-centered graph. Also, there might be a reduction of the eccentricity of at most one more vertex depending on the edge jump. Thus the remaining vertices retain their eccenticities resulting in a almost self-centered graph.

Remark 5.2: An edge jump on $C_5 \times K_2$ always results in a self-centered graph if the newly added edge doesnot induce a cycle of length four.

Remark 5.3: An edge jump on a $C_3 \times K_2$ (for any edge of the cycle) results in a self-centered graph.

We know that the eccentricity of C_3 is one. After taking the prism of C_3, the eccentricity of the prism becomes two. The prismof $C_3 \times K_2$is a cubic graph, where each vertex has atmost three neighbours at distance one, and the remaining two vertices at distance two.The vertices of the edge, used for edge jump operation loses one degree each and thus results in just two vertices at distance one and the rest of the vertices at distance two.This applies to all the edges of the cycle used for edge jump operation.Thus, for any edge jump considered results in a self-centered graph.

Theorem 5.4: If k < n is the average eccentricity of the prism of the path, $P_n \times K_2$, then the average eccentricity of the prism after a single edge jump lies between $n-2 \le k \le n+2$.

Proof: Let n be the length of the path, P_n. Let "d" and "r" denote the diameter and radius of the path. The eccentricity of the vertices of the path when n is odd ranges from (n-1) to $\left\lfloor \frac{n}{2} \right\rfloor$ and when n is even it ranges from (n-1) to $\left(\frac{n}{2} \right)$. Let "k" denote the average eccentricity of the path. We find that avearge eccentricity of the path is always greater than the radius of the path for n being either odd or even. Taking the prism of the path, let d' and r' denote the diameter and the radius of the prism. When the edge jump is performed on any one of the edges of the prism, we find that the diameter and the radius of the prism increase by one. The eccentricities of the prism, when n is odd ranges from n to $\left\lceil \frac{n}{2} \right\rceil$ and when n is even it ranges from n to $\frac{n}{2}+1$. On performing the edge jump operation on any edge of the prism we find that the eccentricities of the new graph G^j ranges from n to $\left\lfloor \frac{n}{2} \right\rfloor$ for n being odd and for n being even it ranges from n to $\left(\frac{n}{2} \right)$ for different edge jumps performed on the prism. Thus the eccentricities of the vertices of the prism vary depending on the length of the cycle induced in the graph due to edge jump. The length of the cycle induced along with the newly added edge varies from C_3 to a maximum of C_{n+1} . Thus summimg up the eccentricities of the newly obtained graph, we find that

the minimum average eccentricity is less than or equal to n - 2 and the maximum average eccentricity is greater than or equal to n + 2.

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