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SELECTION OF A LEADER FOR A COMPANY BY USING PRIORITIZATION METHODS OF ANALYTIC HIERARCHY PROCESS – AN ILLUSTRATION

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ABSTRACT:

This paper aims at selection of a leader for a company (whose founder is about to retire) with one of the popular decision making techniques. Analytic Hierarchy Process is an approach to decision making that involves structuring multiple criteria into hierarchy. There are several methods to calculate the priority vectors. Three prioritization methods are used to select a leader among three alternatives and four criteria. It is observed that Dick stands first in ranking and Tom stands second and Harry stands third in rankings. Consistency Ratio (C.R) also calculated for all pair-wise comparison matrices and analyzed.

KEY WORDS:

Analytic Hierarchy Process; Multi Criteria Decision Making; Pair-wise Comparison; Priority Vector.

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INTRODUCTION:

Analytic Hierarchy Process (AHP) has been introduced by Thomas.L.Saaty in 1970s. It is one of the most widely used decision making approaches. Decision making problems are known as Multi Criteria Decision Making problems, because a decision found to be superior with respect to one criterion may be found inferior with respect to another. Analytic Hierarchy Process is a tool to resolve hierarchy problems. The problem is decomposed and consisting of several criteria or sub criteria and a set of alternatives are evaluated with respect to each criterion. The evaluations of alternatives are usually done by prioritization methods. Here three prioritization methods namely (1). Geometric Mean Method (2). Additive Normalization Method (3). Stochastic Vector Method are used to calculate the priorities. Hierarchy problem is shown in the Fig.1.

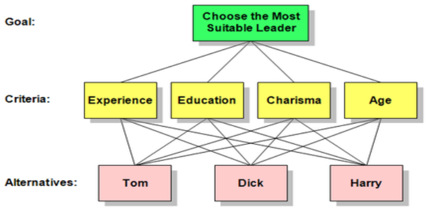

Fig.1 The AHP hierarchy

2. METHODOLOGY:

2.1. Definitions and notations for the pair wise comparison matrix:

Definition1. A comparison matrix A is said to be positive reciprocal if $a_{ij} = 1, a_{ij} > 0\, and\, a_{ij} = 1/a_{ji}$

Definition 2. A positive reciprocal matrix is perfectly consistent if $a_{ik}a_{kj} = a_{ij}$ for all i, j and k

Definition 3. A positive reciprocal matrix is approximately consistent if $a_{ik} \times a_{kj} \cong a_{ij}$ for all i, j and k, where '$\cong$' denotes approximately or close to.

Definition 4. A positive reciprocal matrix is said to be transitive if A>B and B>C then A>C.

Definition 5. The pair wise comparison matrix can pass the consistency test, if the consistency ratio C.R = $\frac{C.I}{R.I}$ < 0.1, where the consistency index (C.I) = $\frac{\lambda_{\max} - 1}{n - 1}$, R.I is the average random index based on matrix size, $\lambda_{\max}$ is the maximum eigenvalue of matrix A, and n is the order of matrix A (Saaty,1991).

Table 1: Saaty's Ratio scale for pair wise comparison of importance of weights of Criteria/alternatives

Intensity of importance	Definition	Explanation
1	Equal importance	Two elements contribute equally to the property
3	Moderate importance of one over another	Experience and judgment slightly favor one over the other
5	Essential or strong importance	Experience and judgment strongly favor one over another
7	Very strong importance	An element is strongly favored and its dominance is demonstrated in practice.
9	Extreme importance	The evidence favoring one element over another is one of the highest possible order of affirmation
2,4,6,8	Intermediate values between two adjacent judgments	Comprise is needed between two judgments
Reciprocals	When activity i compared to j is assigned one of the above numbers, the activity j compared to i is assigned its reciprocal	
Rational	Ratios arising from forcing consistency of judgments	

Table 2: Average Random Index (R.I) based on matrix size (adopted from Saaty,2000)

Size of the matrix	Random consistency index(R.I)
1	0
2	0
3	0.52
4	0.89
5	1.11
6	1.25
7	1.35
8	1.40
9	1.45
10	1.49

2.2. Prioritization methods:

Here in this paper, three prioritization methods namely (1). Geometric Mean Method (2). Additive Normalization Method (3). Stochastic Vector Method are used to calculate the priorities of three alternatives (Tom, Dick and Harry) with four criteria (Experience, Education, Charisma and Age) and the result is analyzed.

2.2.1. Geometric Mean Method (GMM):

This method is used to find the weights to the criteria or alternatives. The pair-wise comparison matrix of alternatives is shown in table3 where A_1, A_2.........A_n represent the alternatives which are to be ranked. Also a_{11}, a_{22},.....a_{nn} show the opinions of an expert. The Geometric Mean Method is explained below which is used to calculate the priority weight vectors.

Table 3: pair-wise comparisons

	A_1	A_2	....	A_n
A_1	a_{11}	a_{12}	....	a_{1n}
A_2	a_{21}	a_{22}	....	a_{2n}
. . . .	. . . .	. . . .	. . . .	. . . .
A_n	a_{n1}	a_{n2}	....	a_{nn}

Obtain the geometric row means of each row as

$$a_1 = (a_{11} * a_{12} * a_{13} * \dots \dots . . a_{1n})^{\frac{1}{n}}$$

$$a_2 = (a_{21} * a_{22} * a_{23} * \dots \dots . . a_{2n})^{\frac{1}{n}}$$

$$a_n = (a_{n1} * a_{n2} * a_{n3} * \dots\dots.. a_{nn})^{\frac{1}{n}}$$

The normalized vector of $(a_1, a_2 \dots\dots\dots\dots\dots..a_n)$ becomes the. solution vector.

2.2.2. Additive Normalization Method (ANM):

To obtain the priority vector *w* by this method it is enough to divide the elements of each column of matrix *A* by sum of that column (i.e. normalize the column), then add the elements in each resulting row and finally divide this sum by the number of elements in the row. This procedure is described by relations (1) and (2).

$$a_{ij}' = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}}, i, j = 1,2, \dots, n \qquad (1)$$

$$w_i = \left(\frac{1}{n}\right)\sum_{j=1}^{n} a_{ij}', i = 1,2, \dots, n \qquad (2)$$

2.2.3. Stochastic Vector Method (SVM) Algorithm:

Step-1: If the PCM is consistent i.e.$a_{ij} = a_{ik}a_{kj}$ for each element, then use GMM and go to Step-6

Step-2: If the PCM is not consistent i.e.$a_{ij} \neq a_{ik}a_{kj}$ for at least one i and j, then divide each row vector by its trace to get a stochastic row vector and let A^s be the stochastic matrix of such rows.

Step-3: Let X_0 be the initial guess stochastic fixed vector and the next vector is obtained by $X_1 = A^s X_0$

Step-4: While the error of $|X_0 - X_1|$ is less than the pre assigned value do $X_1 = A^s X_0$ and $X_0 = X_1$

Step-5: Write "The solution vector by SVM is X_1" Go to Step-7.

Step-6: Write "The solution vector by GMM isX_1"

Step-7: END

3. ILLUSTRATIVE EXAMPLE:

Consider an example which describes the use of the AHP in choosing a leader for a company whose founder is about to retire. There are several competing candidates and several competing criteria for choosing the most suitable one.

In order to select a leader for the company by using three prioritization methods and to analyze the result an illustration was taken from AHP literature, Saaty (2008), Chapter 5.

3.1. Geometric Mean Method (GMM):

Table3.1 (i)

C_1:Experience	Tom	Dick	Harry	Priority Vector
Tom	1	1/4	4	0.217
Dick	4	1	9	0.717
Harry	1/4	1/9	1	0.066

$$\lambda_{\max} = 3.0369, C.R = 0.04$$

Table3.1 (ii)

C_2:Education	Tom	Dick	Harry	Priority Vector
Tom	1	3	1/5	0.188
Dick	1/3	1	1/7	0.081
Harry	5	7	1	0.731

$$\lambda_{\max} = 3.0649, C.R = 0.06$$

Table3.1 (iii)

C_3:Charisma	Tom	Dick	Harry	Priority Vector
Tom	1	5	9	0.743
Dick	1/5	1	4	0.194
Harry	1/9	1/4	1	0.063

$$\lambda_{\max} = 3.0713, C.R = 0.07$$

Table3.1 (iv)

C_4:Age	Tom	Dick	Harry	Priority Vector
Tom	1	1/3	5	0.265
Dick	3	1	9	0.672
Harry	1/5	1/9	1	0.063

$$\lambda_{\max} = 3.0291, C.R = 0.03$$

Table3.1 (v): Criteria compared with respect to reaching the goal

Criteria	Experience	Education	Charisma	Age	Priority Vector
Experience	1	4	3	7	0.547
Education	1/4	1	1/3	3	0.127
Charisma	1/3	3	1	5	0.270
Age	1/7	1/3	1/5	1	0.057

$$\lambda_{\max} = 4.1184, C.R = 0.04$$

Table3.1 (vi): AHP solution by GMM

Criteria	Experience	Education	Charisma	Age	Priority Vector
Alternatives					
Tom	0.119	0.024	0.201	0.015	0.359
Dick	0.392	0.010	0.052	0.038	0.492
Harry	0.036	0.093	0.017	0.004	0.149
Criteria Weights	0.547	0.127	0.270	0.057	1.000

3.2. Additive Normalization Method (ANM):

Table3.2 (i)

C_1:Experience	Tom	Dick	Harry	Priority Vector
Tom	1	1/4	4	0.220
Dick	4	1	9	0.713
Harry	1/4	1/9	1	0.067

$$\lambda_{\max} = 3.0369, C.R = 0.04$$

Table3.2 (ii)

C_2:Education	Tom	Dick	Harry	Priority Vector
Tom	1	3	1/5	0.193
Dick	1/3	1	1/7	0.083
Harry	5	7	1	0.724

$$\lambda_{\max} = 3.0649, C.R = 0.06$$

Table3.2 (iii)

C_3:Charisma	Tom	Dick	Harry	Priority Vector
Tom	1	5	9	0.735
Dick	1/5	1	4	0.199
Harry	1/9	1/4	1	0.065

$$\lambda_{\max} = 3.0713, C.R = 0.07$$

Table3.2 (iv)

C_4:Age	Tom	Dick	Harry	Priority Vector
Tom	1	1/3	5	0.267
Dick	3	1	9	0.669
Harry	1/5	1/9	1	0.064

$$\lambda_{\max} = 3.0291, C.R = 0.03$$

Table3.2 (v):Criteria compared with respect to reaching the goal

Criteria	Experience	Education	Charisma	Age	Priority Vector
Experience	1	4	3	7	0.538
Education	1/4	1	1/3	3	0.132
Charisma	1/3	3	1	5	0.271
Age	1/7	1/3	1/5	1	0.059

$$\lambda_{\max} = 4.1184, C.R = 0.04$$

Table3.2 (vi): AHP solution by ANM

Criteria / Alternatives	Experience	Education	Charisma	Age	Priority Vector
Tom	0.118	0.025	0.199	0.016	0.358
Dick	0.384	0.011	0.054	0.040	0.489
Harry	0.036	0.096	0.018	0.003	0.153
Criteria Weights	0.538	0.132	0.271	0.059	1.000

3.3. Stochastic Vector Method (SVM):

Table3.3 (i)

C_1:Experience	Tom	Dick	Harry	Priority Vector
Tom	1	1/4	4	0.255
Dick	4	1	9	0.679
Harry	1/4	1/9	1	0.066

$$\lambda_{\max} = 3.0369, C.R = 0.04$$

Table3.3 (ii)

C_2:Education	Tom	Dick	Harry	Priority Vector
Tom	1	3	1/5	0.225
Dick	1/3	1	1/7	0.079
Harry	5	7	1	0.696

$$\lambda_{\max} = 3.0649, C.R = 0.06$$

Table3.3 (iii)

C_3:Charisma	Tom	Dick	Harry	Priority Vector
Tom	1	5	9	0.696
Dick	1/5	1	4	0.241
Harry	1/9	1/4	1	0.063

$$\lambda_{\max} = 3.0713, C.R = 0.07$$

Table3.3 (iv)

C_4:Age	Tom	Dick	Harry	Priority Vector
Tom	1	1/3	5	0.306
Dick	3	1	9	0.628
Harry	1/5	1/9	1	0.066

$$\lambda_{\max} = 3.0291, C.R = 0.03$$

Table3.3 (v): Criteria compared with respect to reaching the goal

Criteria	Experience	Education	Charisma	Age	Priority Vector
Experience	1	4	3	7	0.490
Education	1/4	1	1/3	3	0.150
Charisma	1/3	3	1	5	0.305
Age	1/7	1/3	1/5	1	0.055

$$\lambda_{\max} = 4.1184, C.R = 0.04$$

Table3.3 (vi): AHP solution by SVM

Criteria / Alternatives	Experience	Education	Charisma	Age	Priority Vector
Tom	0.125	0.034	0.212	0.017	0.388
Dick	0.333	0.012	0.074	0.034	0.453
Harry	0.032	0.104	0.019	0.003	0.158
Criteria Weights	0.490	0.150	0.305	0.055	1.000

Table 4: Priority vectors of GMM,ANM,SVM

GMM	ANM	SVM
0.359	0.358	0.388
0.492	0.489	0.453
0.149	0.153	0.158

Table 4 shows the priority vectors using the three methods as calculated above. Covariance of the priority vectors by using three methods are calculated.

C12= Covariance (GMM,ANM)= 0.019525

C23= Covariance (ANM,SVM)= 0.017198

C31= Covariance (SVM,GMM)= 0.01757

4. CONCLUSIONS AND IMPLCATIONS:

In this paper we have applied three prioritization methods to select a leader for a company whose founder is about to retire. It can be concluded that alternative Dick is ranked first and Alternative Tom is ranked next. Alternative Harry is ranked last. It can be observed that the ranking of the alternatives by three prioritization methods yield the same result though the weights of the elements differ in all the three methods. Though the ranking of the elements of priority vectors do not differ,the elements differ and the variance can be seen with covariance calculated for all the three vectors. Experience is the most important criterion with respect to reaching the goal, followed by Charisma, Education, and Age. Consistency Ratio (C.R) or Inconsistency Factor for all Pair-wise Comparison Matrices has also been calculated and mentioned, it is acceptable (i.e. < 0.1) for all the Pair-wise Comparison Matrices.

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