MODELLING NIGERIA POPULATION GROWTH RATE

[1]Olayiwola, O.M., [2]Lawal. G. O. [3]Amalare, A. A [4]Agboluaje, S. A,   [4]Fantola, J. O,

[1]Department of Statistics, Federal University of Agriculture, Abeokuta, Nigeria

[2]Department of Statistics, Yaba College of Technology, Lagos State, Nigeria

[3]Department of Mathematics and Statistics, Lagos State Polytechnic, Ikorodu, Nigeria

[4]Department of Mathematics and Statistics, The Polytechnic, Ibadan, Nigeria

Correspondence Author: laniyimathew@yahoo.com

ABSTRACT

Thomas Robert Malthus Theory of population highlighted the potential dangers of over population. He stated that while the populations of the world would increase in geometric proportions, the food resources available for them would increase in arithmetic proportions. This study was carried out to find the trend, fit a model and forecast for the population growth rate of Nigeria.The data were based on the population growth rate of Nigeria from 1982 to 2012 obtained from World Bank Data (data.worldbank.org). Both time and autocorrelation plots were used to assess the Stationarity of the data. Dickey-Fuller test was used to test for the unit root. Ljung box test was used to check for the fit of the fitted model. Time plot showed that the random fluctuations of the data are not constant over time. There was an initial decrease in the trend of the growth rate from 1983 to 1985 and an increase in 1986 which was constant till 1989 and then slight fluctuations from 1990 to 2004 and a general increase in trend from 2005 to 2012. There was a slow decay in the correlogram of the ACF and this implied that the process is non stationary. The series was stationary after second differencing, Dickey-Fuller = -4.7162, Lag order = 0, p-value = 0.01 at α= 0.05. The p-value (0.01) and concluded that there is no unit root i.e the series is stationary having d=2. Correlogram and partial correlogram for the second-order differenced data showed that the ACF at lag 1 and lag 5 exceed the significant bounds and the partial correlogram tailed off at lag 2.The identified order for the ARIMA(p,d,q) model was ARIMA(2,2,1). The estimate of AR1 co-efficient (ϕ_1)=1.5803 is observed to be statistically significant but the estimated value does not conforms strictly to the bounds of the stationary parameter hence (ϕ_1) was excluded from the model. (ϕ_2) =-0.9273 is observed to be statistically significant and conformed strictly to the bounds of the stationary parameter , hence was maintained in the model.

The estimate of MA1 co-efficient (θ_1) = - 0.1337 was observed to be statistically significant conformed strictly to the bounds of the parameter invertibility. For ARIMA (2, 2, 0) the estimate of AR1 co-efficient (ϕ_1) =1.5430 was observed to be statistically significant and not conformed strictly to the bounds of the parameter stationary, hence excluded from the model.

The estimate of AR 2 co-efficient (ϕ_2) =-0.9000 is observed to be statistically significant and conformed strictly to the bounds of the parameter stationary, hence retained in the model.

The ARIMA (2, 2, 0) is considered the best model. It has the smallest AIC. The Ljung test showed that residuals are random and implies that the model is fit enough for the data. The forecast Arima function gives us a forecast of the Population Growth Rate in the next thirty eight (38) years, as well as 80% and 95% prediction intervals for those predictions i.e up to 2050

Indexing terms/Keywords: Population Growth Rate; Modelling; Parameter invertibility; ARIMA

Academic Discipline And Sub-Disciplines

Statistics, Mathematics, Economiocs

SUBJECT CLASSIFICATION

 Mathematics Subject Classification

TYPE (METHOD/APPROACH)

INTRODUCTION

Nigeria is a country located in West Africa whose official name is the Federal Republic of Nigeria. It is known as the "Giant of Africa" because it is considered the most populous country in Africa and seventh most populous country in the world (Library of Congress-Federal Research Division, 2008).

Nigeria is a high fertility country and there is evidence that its large population inhibit government's efforts in meeting the basic needs of the people. With a population that already exceeds 130 million people and growing roughly 3 percent annually (United Nations, 2004).Population growth rate is normally influenced by three main factors namely- birth, death and migration. The birth rate, low death rate and migration are the sources of high population growth in Nigeria. It has experienced a population explosion for at least the last 50 years due to high fertility rates. Growth was fastest in the 1980s, after child mortality had dropped, and has slowed slightly. According to the 2012 revision of the world population prospects the total population was 159,708,000 in 2010, compared to only 37,860,000 in 1950.Over the years the birth rate and death rate of Nigeria had fluctuated and has affected its population growth. Nigeria population currently is 168.8 million (2012).

The population growth rate is the rate at which the number of individuals in a population increases in a given time period as a fraction of the initial population and is often expressed as a percentage of the number of individuals in the population at the beginning of that period. (2.8% annual change, 2012). A positive growth rate ratio indicates that the population is increasing, while a negative growth ratio indicates the population is decreasing. A growth ratio of zero indicates that there were same number of people at the two times.

According to the United Nations Population Fund (UNFPA), formerly the United Nations Fund for Population Activities, estimates 31.10 in 2011. Global human population growth is around 75 million annually, or 1.1% per year. The global population has grown from 1 billion in 1800 to7 billion in 2012. It is expected to keep growing to reach 10 billion by the end of the century.

Objectives of Study

This research is aimed at finding the trend of the population growth rate within 1982-2012, fitting a model for the population growth rate and forecasting for the population total in the nearest future.

Data

The data collection was done from a secondary source. The data was obtained from the health profile of the World Bank Data (data.worldbank.org). It captures a time series data of 31 years i.e 1982 to 2012.

Literature Review

Nigeria's population total in 2006 was 140,431,790 according to the National population commission projections and UN projections. It was 168,833,776 in 2012. It was assumed the annual growth rate of 2.8% annual change in 2012. The 1952/53 population census in spite of some technical difficulties was considered the best census conducted in Nigeria before 1991 census. (Phillips, 1997).The sex distribution is balanced, the population almost equally divided between the males and females with 71,345,488 males and 60,086,302 females. Thus about 50.8% of the populations are males and 49.2% are females. (Phillips,1997).

The Thomas Robert Malthus Theory of population highlighted the potential dangers of over population. In his famous book 'An Essay on the Principles of Population' Malthus stated that while the populations of the world would increase in geometric proportions, the food resources available for them would increase in arithmetic proportions. Malthus theory was based on the assumption that the power of population is much greater than the power of the earth to provide subsistence to man. In his own words population would grow at such a high rate that it would outstrip food supply.

According to Malthus, disease, food shortage and death due to starvation, were nature's way to control population growth. He proposed that human beings adopt measures like abortion, delay in marriage and celibacy to check population growth. By the end of the 19th century, when living standards improved and birth rates dropped in the western countries. Concerns of overpopulation became irrelevant. However in underdeveloped countries which have agrarian economies. Malthus theory often finds credibility.

Although economists before Malthus made some of the observations which Malthus made, it was his work that had great influence on the major classical economists that followed him. Perhaps the most important legacy of Malthus has been the treatment of population growth. These and other classical theory emphasizes the importance of both fertility and mortality in population growth.

Modern time series forecasting methods are essentially rooted in the idea that the past tells us something about the future. How we are to extrapolate future events based on this information, constitute the main subject matter of time series analysis.

Typically, the approach to forecasting time series is to first specify a model, although this need not be so. This model is a statistical formulation of the dynamic relationships between that which we observe (i.e. the so called information set), and those variables we believe are related to that which we observe. It should thus be stated immediately that this discussion will be restricted in scope to those models which can formulated parametrically.

The "classical" approach to time series forecasting derives from regression analysis. The standard regression model involves specifying a linear parametric relationship between a set of explanatory variables (or exogenous variables) and the dependent (or endogenous variable). The parameters of the model can be estimated in a variety of ways, going back as far as Gauss in 1794 with the "Least Squares" method, but the approach always culminates in striving for some form of statistical orthogonality between the explanatory variables and the residuals of the regression.

Both Wiener (1949) and Kolmogorov (1941) were pioneers in the field of linear prediction, and while their approaches differed (Wiener worked in the frequency domain popular amongst engineers, while Kolmogorov worked in the time domain), it is clear that their solutions to the same basic geometrical problem were equivalent (Priestley (1981) ch.10). Wiener's work, in particular, was especially relevant to modern time series forecasting in that he was among the first to

rigorously formulate the problem of "signal extraction." That is, given observations on a time series corrupted by additive noise, what is the optimal estimator (in the mean-squared error (MSE) sense) of the latent or underlying signal (or state variable).

Given the historical context of massive systems of equations models popular among macro econometric forecasters of 1950's (Klein-Goldberger model (1955) or Adelmans (1959) for details) it quickly became apparent that forecasting models derived from a signal. These methods became popular due to their simplicity and ease of application for the general practitioner. However, these methods are considered ad hoc since they fail to incorporate theoretical considers into their decompositions of cyclical components and they are formulated without recourse to a well specified statistical model. Consequently, they also do not allow for prediction intervals since there is no accounting for predictive variance. In recent years, there has been an increasing emphasis by national statistical offices to include uncertainty in their official population projections so that the user community has a more realistic sense for what future might hold. For most national statistical offices this has involved the inclusion of several plausible (deterministic) projection variants based on assumptions regarding future fertility, mortality and migration in a cohort-component population projection framework. We focus on the issues and practicalities of including uncertainty from a probabilistic view point. Population forecasts are based on past patterns, where a long time series of data are very valuable for assessing our uncertainty for the future.

With respect to the dimensionality of population projections, the simplest models rely on the extrapolations of population size, population growth rates or crude rates related to particular components of demographic change (fertility, mortality and migration). The adding of age and sex leads to the cohort-component framework of population accounting developed by Leslie (1945).

According to the various project works that were considered, it can be concluded that the Least Square method and the Autoregressive Integrated Moving Average Model were mostly adopted for the forecasting of the future values.

Methodology

Box-Jenkins model identification

Stationarity and Seasonality

The first step in developing a Box–Jenkins model is to determine if the time series is stationary and if there is any significant seasonality that needs to be modeled.

Detecting Stationarity

Stationarity can be assessed from a run sequence plot. The run sequence plot should show constant location and scale. It can also be detected from an autocorrelation plot. Specifically, non-stationarity is often indicated by an autocorrelation plot with very slow decay.

Finally, unit root tests provide a more formal approach to determining the degree of differencing such as Kwiatkowski-Phillips-Schmidt-Shin (KPSS) and Phillips-Perron Unit Root Tests are carried out employing the unit root testing procedures of Hamilton (1994). The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for the null hypothesis of a level stationary against an alternative of unit root together with the Philips-Peron test for the null hypothesis of a unit root against the alternative of a stationary series.

The decision rule is that for the KPSS test if the p-value of its test statistic is greater than the critical value of say 0.05, then reject the null hypothesis of having a level stationary series and therefore conclude the alternate hypothesis that it has a unit root. The Philips-Peron Test on the other hand test for the null hypothesis of unit root against an alternative hypothesis of stationarity by rejecting the null hypothesis if its p-value is less than the critical value chosen.

The Dickey-Fuller test: just like the Phillips Perron test, the null hypothesis is: H_0: The series has a unit root (the process is not stationary) against H_1: The series has no unit root (the process is stationary). Decision rule: reject H_0 if p-value is less than α.

Differencing to achieve Stationarity

Box and Jenkins recommend the differencing approach to achieve stationarity. However, fitting a curve and subtracting the fitted values from the original data can also be used in the context of Box–Jenkins models.

Seasonal Differencing

At the model identification stage, the goal is to detect seasonality, if it exists, and to identify the order for the seasonal autoregressive and seasonal moving average terms. For many series, the period is known and a single seasonality term is sufficient. For example, for monthly data one would typically include either a seasonal AR 12 term or a seasonal MA 12 term. For Box–Jenkins models, one does not explicitly remove seasonality before fitting the model. Instead, one includes the order of the seasonal terms in the model specification to the ARIMA estimation software. However, it may be helpful to apply a seasonal difference to the data and regenerate the autocorrelation and partial autocorrelation plots. This may help in the model identification of the non-seasonal component of the model. In some cases, the seasonal differencing may remove most or all of the seasonality effect.

Identify p and q

Once stationarity and seasonality have been addressed, the next step is to identify the order (i.e., the p and q) of the autoregressive and moving average terms. These are determined by examining the values of the autocorrelations and the partial autocorrelations with their corresponding plots as explained below.

Autocorrelation and Partial Autocorrelation plots

The primary tools for doing this are the autocorrelation plot and the partial autocorrelation plot. The sample autocorrelation plot and the sample partial autocorrelation plot are compared to the theoretical behavior of these plots when the order is known.

Order of Autoregressive Process (p)

Specifically, for an AR (1) process, the sample autocorrelation function should have an exponentially decreasing appearance. However, higher-order AR processes are often a mixture of exponentially decreasing and damped sinusoidal components.

For higher-order autoregressive processes, the sample autocorrelation needs to be supplemented with a partial autocorrelation plot. The partial autocorrelation of an AR(p) process becomes zero at lag p + 1 and greater, so we examine the sample partial autocorrelation function to see if there is evidence of a departure from zero. This is usually determined by placing a 95% confidence interval on the sample partial autocorrelation plot (most software programs that generate sample autocorrelation plots will also plot this confidence interval). If the software program does not generate the confidence band, it is approximately, with N denoting the sample size.

Order of Moving-Average Process (q)

The autocorrelation function of a MA(q) process becomes zero at lag q + 1 and greater, so we examine the sample autocorrelation function to see where it essentially becomes zero. We do this by placing the 95% confidence interval for the sample autocorrelation function on the sample autocorrelation plot. Most software that can generate the autocorrelation plot can also generate this confidence interval.

The sample partial autocorrelation function is generally not helpful for identifying the order of the moving average process.

Parameter Estimation

After identifying the order of the tentative model, the parameters of the model are estimated using the maximum likelihood estimation to determine the AR and MA parameters, as well as all other parameters reported in the study.

Three other penalty function statistics namely the Akaike information criteria (AIC), the Schwarz Bayesian information criteria (BIC) as well as the corrected Akaike information criteria (AICc) are explained in penalizing fitted models based on the principle of parsimony. These statistics were one of the various checks used to verify the adequacy of the chosen models. Comparatively, models with the smallest AIC and BIC are deemed to have residuals which resembles a white noise process. Twice the number of estimated parameters minus two times the log likelihood gives the AIC value of a model i.e $AIC = -2\log L + 2k$ The BIC is computed as $-2In(L) + In(n)k$, where L is the likelihood, n denotes the number of residuals and k is the number of free parameters.

Each parameter estimate reports standard error for that particular parameter. Using the parameter estimate and its standard error, a test for statistical significance (t-value) are then conducted. For statistically significant parameters, the absolute values of the *t*-ratios are expected to be greater than 1.96 or 2 in order for the parameters to be maintained in the model whereas parameters which are not significant are trimmed or removed from the model.

Furthermore, the estimated AR and MA parameters must also conform to certain boundary condition that is they must lie between -1 and 1. If the AR and MA parameters do not lie within those bounds of stationarity then the parameters of the model are re-estimated or if possible a different candidate model is alternatively considered for estimation. All these checks when strictly adhered to would lead to obtaining reliable results from the model.

Diagnostic Checking

The diagnostic stage of the Box-Jenkins ARIMA process is to examine whether the fitted model follows a white noise process. This can be done by studying the autocorrelation values (r_k) one at a time, and to develop a standard error formula to test whether a particular r_k value is significantly different from zero. Theoretically, it is envisage that all autocorrelation coefficients for a series of random numbers must be zero. However, because of the presence of finite samples, each sample autocorrelations might not be exactly zero. The ACF coefficients of white noise data is said to have a sampling distribution that can be approximated by a normal curve with mean zero and standard error of $\frac{1}{\sqrt{n}}$, where n gives the number of data points in the observed series.

For a white noise process, 95% of all sample autocorrelation values must lie within a range specified by the mean plus or minus 1.96 standard errors. In this case, since the mean of the process is zero and the standard error is $\frac{1}{\sqrt{n}}$, one should expect about 95% of all sample autocorrelation values r_k to be within the range of $\pm 1.96\sqrt{n}$ or $(-1.96\sqrt{n} < r_k < 1.96\sqrt{n})$. If this condition does not hold, then the model fitted do not follow a white noise process, or the residuals are not white noise. The correlogram of the ACF would therefore show lines at the critical values $\pm 1.96\sqrt{n}$ of for easily verification.

The Ljung-Box test is a modified version of the portmanteau test statistic developed by Ljung and Box (1978) is also used. The modified Ljung- Box Q statistic tests whether the model's residuals have a mean of zero, constant variance and serially uncorrelated values r_k (a white noise check). The test statistic is given by;

$$Q = n(n+2)\sum_{k=1}^{h}\frac{r_k^2}{(n-k)}$$

Where n denote the number of data points in the series, r_k^2 is the square of the autocorrelation at lag k, and h is the maximum lag being considered. The hypothesis to be tested is formulated in the form;

H_0: The set of autocorrelations for residual is white noise (model fit data quite well)

H_1: The set of autocorrelations for residual is different from white noise

The test statistic (Q) is compared with a chi-square distribution written as $\chi^2_{\alpha(h-p-q)}$, where α is taken to be 5% (0.05), h is the maximum lag being considered, and p and q are the order of the AR and MA processes respectively. The decision is to accept the null hypothesis (H_0) if $Q < \chi^2_{\alpha(h-p-q)}$ and to reject the alternative hypothesis if $Q > \chi^2_{\alpha(h-p-q)}$. In other words, the residuals are not white if the test statistic Q lies in the extreme 5% of the right-hand tail of the chi-square distribution.

Forecast

In the forecasting stagewe use the FORECAST statement to forecast future values of the time series and the confidence interval.

Discussion

Time series analysis was used for the data analysis, the time plot of the sets of observation for the population growth rate
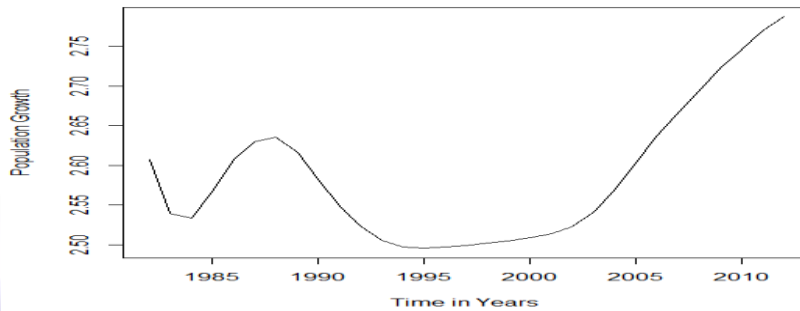
above is shown below;

Fig 1: time plot of the sets of observation for the population growth rate

It can be seen from fig 1 that the random fluctuations of the data are not constant over time. There was an initial decrease in the trend of the growth rate from 1983 to 1985 and an increase in 1986 which was constant till 1989 and then slight fluctuations from 1990 to 2004 and a general increase in trend from 2005 to 2012.
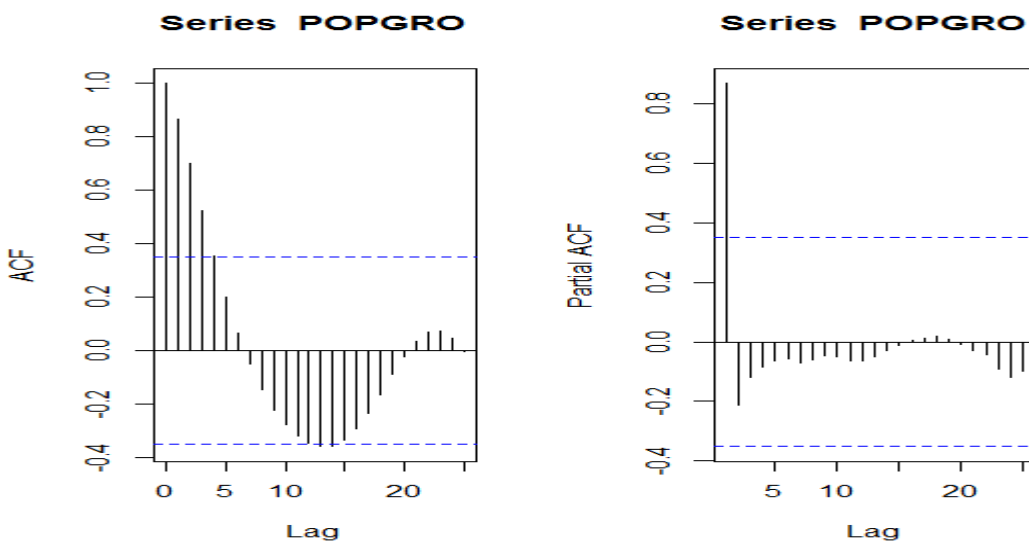
The slow decay in the correlogram of the ACF above implies that the process is non stationary.

To confirm the result from the correlogram, a formal test for stationarity was performed using the Dickey-Fuller test for unit root and the following was observed; Dickey-Fuller = -0.173, Lag order = 0, p-value = 0.9898

H_0: there is unit root (not stationary) against H_1: there is no unit root (stationary), at $\alpha = 0.05$

Since the p-value (0.9898) is greater than α, we cannot reject H_0 and conclusion that there is unit root i.e the series is not stationary and there is need for differencing.

Differenced Population Growth Rate data

After differencing, Augmented Dickey-Fuller test was carried out with the following results:

Dickey-Fuller = -3.1225, Lag order = 0, p-value = 0.1415, at $\alpha = 0.05$

Since the p-value (0.1415) is greater than α, we cannot reject H_0 and conclusion that there is unit root i.e the series is not stationary. This took us to further differencing and obtained the results, Dickey-Fuller = -4.7162, Lag order = 0, p-value = 0.01 at $\alpha = 0.05$. The p-value (0.01) is less than α, we reject H_0. And conclusion that there is no unit root i.e the series is stationary.

Since the data has been confirmed to be stationary at the second-order difference, we can go further to fit a model, having d=2.
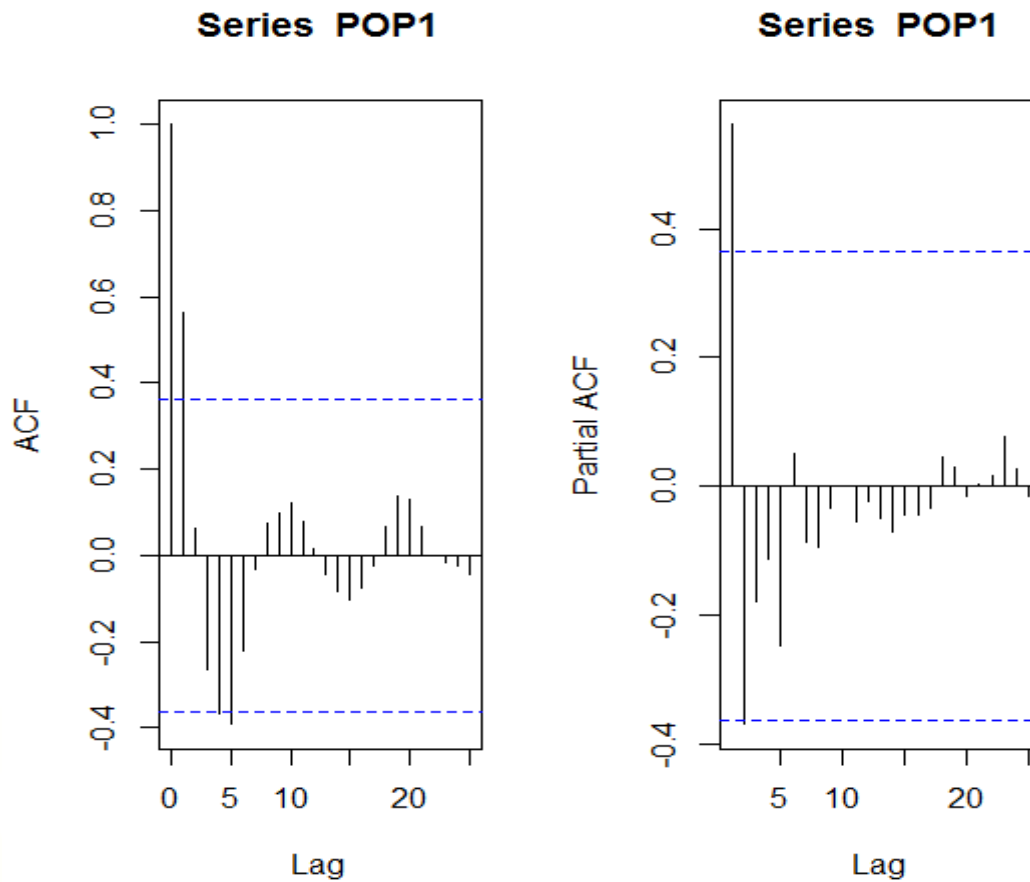

Fig 2: Correlogram and partial correlogram for the second-order differenced data.

From the figure above, the correlogram shows that the ACF at lag 1 and lag 5 exceed the significant bounds and the partial correlogram tailed off at lag 2. Since the correlogram is zero after lag 1, and the partial correlogram tails off to zero after lag 1 and PACF tails off at lag 2,then the identified order for the ARIMA(p,d,q) model is thus ARIMA(2,2,1). The selected models are then compared with an over fitted model ARIMA (2, 2, 0). Hence we have the following table;

ARIMA (2, 2, 1)

AIC=-201.48			
Co-efficient	Estimate	Standard error	t-value
AR1	1.5803	0.1130	13.9849
AR2	-0.9273	0.0769	-12.0585
MA1	-0.1337	0.2267	-0.5898

Table 2: Parameter Estimation

From table 2, the estimate of AR1 co-efficient (ϕ_1) =1.5803 is observed to be statistically significant since its test statistic (13.9849) >2 but the estimated value does not conforms strictly to the bounds of the stationary parameter since1.5803 does not lie between -1 and 1. Hence (ϕ_1) =1.5803 must not be maintained in the model.

Also (ϕ_2) =-0.9273 is observed to be statistically significant since the absolute value of its test statistic (12.0585) > 2 but the estimated value conforms strictly to the bounds of the stationary parameter since -0.9273 lies between -1 and 1. Hence (ϕ_2) =-0.9273 must be maintained in the model.

The estimate of MA1 co-efficient (θ_1) = - 0.1337 is observed to be statistically insignificant since the absolute value of its test statistic (0.5898) < 2. The estimated value also conforms strictly to the bounds of the parameter invertibility, since -0.1337 lies between -1 and 1. Hence

θ_1 =-0.1337 must not be maintained in the model.

ARIMA (2, 2, 0)

AIC=-203.15			
Co-efficient	Estimate	Standard error	t-value
AR1	1.5430	0.1088	14.182
AR2	-0.9000	0.0881	-10.2157

From the table above, the estimate of AR1 co-efficient (ϕ_1) =1.5430 was observed to be statistically significant since its test statistic (14.182) > 2. The estimated value also conformed strictly to the bounds of the parameter stationary, since 1.5430 does not lie between -1 and 1. Hence (ϕ_1) = 1.5430 must not be maintained in the model.

The estimate of AR 2 co-efficient (ϕ_2) =-0.9000 is observed to be statistically significant since the absolute value of its test statistic (10.2157) > 2. The estimated value also conformed strictly to the bounds of the parameter stationary, since -0.9000 lies between -1 and 1. Hence (ϕ_2) = -0.9000 must be maintained in the model.
The ARIMA (2, 2, 0) is considered the best model since it has the smallest AIC.

DIAGNOSTIC CHECK

| Test type	Chi-squared	Degree of freedom	p-value
Ljung -Box	32.8638	23	0.08347

The Ljung test as presented in the table above tests the hypothesis below:
H_0: The residuals are random or white noise agaist H_1: The residuals are not random at $\alpha = 0.05$
The result from the table shows that p-value $> \alpha$ H_0 cannot be rejected, hence we conclude that the residuals are random. This implies that the model is fit enough for the data, hence the model can be used to forecast.
FORECAST
The forecast Arima function gives us a forecast of the Population Growth Rate in the next thirty eight (38) years, as well as 80% and 95% prediction intervals for those predictions i.e up to 2050.

Forecasts from ETS(A,Ad,N)
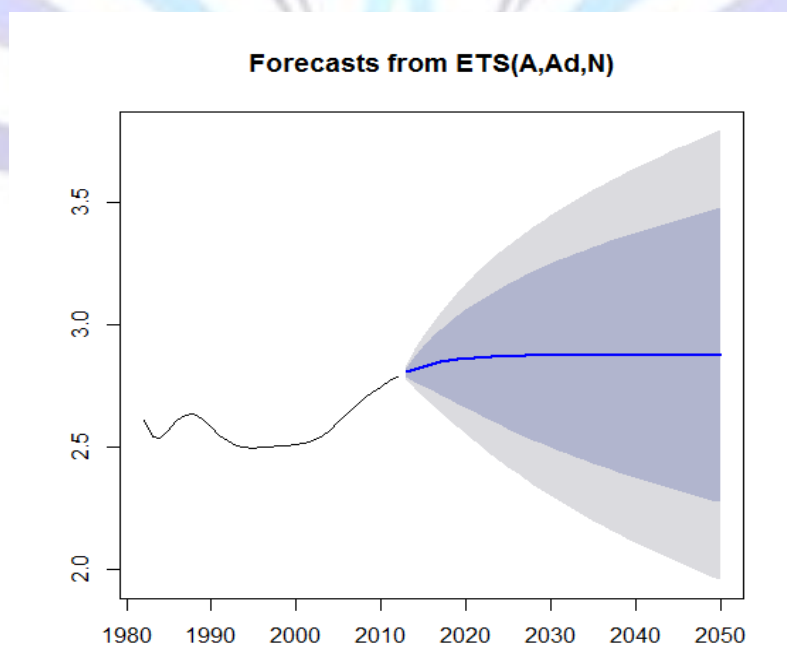
The forecasts are shown as a blue line, with the 80% prediction intervals as a dark blue gray shaded area, and the 95% prediction intervals as a light blue grey shaded area.

CONCLUSION

The Population growth rate has an upward increasing trend and hence it must be monitored. Therefore the Population of Nigeria is increasing rapidly and may explode to billions in the nearest 100 years. This study covered the population growth rate of Nigeria. The following were observed from the analysis; with the recent analysis performed Nigeria population may surpass that of the US by 2050.

The Population growth rate was stationary at the second order difference. The appropriate model for the Population growth rate was ARIMA (2, 2, 0) .

The model was the best fit model out of all other selected models because of it has the smallest AIC and the diagnostic test performed on the model using Box-pierce test shows the model is fit enough for the data and hence it was used to forecast.

Conclusion

The time series components found in the model was Trend. Stationarity of the data was attained with differencing the time series plot. The model was then found fit for the study. Therefore forecast was found to have a gradual constant increase in trend of the Population growth rate up to year 2050.

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