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Adomian Decomposition Method for Solving Coupled KdV Equations

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ABSTRACT

In this article, we study a numerical solution of the coupled kdv equation with initial condition by the Adomian Decomposition Method.The solution is calculated in the form of a convergent power series with easily computable components. Numerical results obtained by this method have been compared with the exact solution to show that the Adomian Decomposition method is a powerful method for the solution of coupled kdv equation.

Indexing terms/Keywords

Adomian decomposition method; Coupled KdV equations.

Academic Discipline And Sub-Disciplines

Numerical analysis, Partial differential equations, Pure mathematics.

SUBJECT CLASSIFICATION

Numerical Solutions of partial differential equation.

TYPE (METHOD/APPROACH)

Adomian decomposition method (ADM),Fractional iteration method(FIM),Differetial transformation method (DTM),Variational iteration method(VIM)

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1- Introduction

Nonlinear phenomena that appear in many areas of scientific fields such as solid state physics, plasma physics, fluid dynamics, mathematical biology and chemical kinetics can be modeled by partial differential equation. A broad class of analytical solutions methods and numerical solutions methods were used in handle these problems. The Adomian decomposition method has been proved to be effective and reliable for handling differential equations, linear or nonlinear.

Various methods for seeking explicit travelling solutions to nonlinear partial differential equations are proposed such as Wadati et al. (1992), Wadati et al. (1975), Wadati (2001), Wadati (1972), Drazin et al. (1997). In the beginning of the 1980, a so-called Adomian decomposition method (ADM), which appeared in Adomian (1994), Adomian and Serrano (1998), Adomian et al. (1995), Deeba and Khuri (1996), Oldham (1974), Podlubny (1999), Wazwaz (2002), Wazwaz (2000), ElWakil et al. (in press), Abdou (2005), Kaya and El-Sayed (2003), Seng and Abbaoui (1996), and Lesnic (2006) has been to solve effectively. The nonlinear equations are solved easily and elegantly without transforming the equation by using the ADM. The technique has many advantages over the classical techniques, mainly, it avoids linearization and perturbation in order to find explicit solutions of a given nonlinear equations.

2- The Adomian decomposition

For the purpose of illustration of the methodology to the proposed method, using **ADM**, we begin by considering the differential equation

$$LU + RU + NU = g \tag{1}$$

with prescribed conditions, where U is the unknown function, L is the highest order derivative

which is assumed to be easily invertible, R is a linear differential operator of less order than L (operator L is linear also), NU represents the nonlinear term and g is the source term.

Assuming the inverse operator L exists and it can be taken as the definite integral with respect

to t from t_0 to t, i.e.

$$L_t^{-1} = \int_0^t (\bullet) dt \tag{2}$$

Applying the inverse operator L_t^{-1} to both sides of equation (1) and using the initial conditions we find

$$U = f - L_t^{-1}[RU + NU] \tag{3}$$

where the function $f(x)$ represents the term arising from integrating the source term g and from using the given initial or boundary conditions, all are assumed to be prescribed. The nonlinear operator [NU] can be decomposed by an infinite series of polynomials given by

$$NU = \sum_{n=0}^{\infty} A_n(u_0, u_1, ..., \mathrm{u}_n) \tag{4}$$

where $A_n(u_0, u_1, ..., u_n)$ the appropriate Adomian's polynomials are defined by Adomian. (1994) Adomian G, Serrano SE. (1998).

$$A_n = \frac{1}{n}[\frac{d^n}{d\lambda^n}(\sum_{k=0}^{\infty} \lambda^n u_k)]_{\lambda=0}, \quad n \succ 0 \tag{5}$$

This formula is easy to compute by using Mathematica software or by setting a computer code to get as many polynomials as we need in the calculation of the numerical as well as explicit solutions. The Adomian decomposition method assumes a series that the unknown function $\mathrm{u}(x,t)$ can be expressed by an infinite series of the form

$$u(x,t)=\sum_{n=0}^{\infty}u_n(\mathrm{x},\mathrm{t}) \qquad (6)$$

Identifying the zeros component $\mathrm{u}(x,0)$ the remaining components where $n \succ 1$ can be determined by using the recurrence relation

$$\mathrm{u}_0(x)=\mathrm{f}(\mathrm{x}) \qquad (7)$$

$$\mathrm{u}_{n+1}(x,t)=-\mathrm{L}_t^{-1}[R(u_n)-A_n], \quad n\geq 0 \qquad (8)$$

Then other polynomials can be generated in a similar way. The scheme (8) can easily determine the components $\mathrm{u}_n(x,\mathrm{t})$ It is in principle, possible to calculate more components in the decomposition series to enhance the approximation. One cannot compute an infinite number of terms; only a quite limited number of terms are determined of the series $\sum_{n=0}^{\infty}\mathrm{u}_n(x,\mathrm{t})$ and hence the solution $\mathrm{u}(x,\mathrm{t})$ is readily obtained. It is interesting to note that we obtained the solution by using the initial condition only.

3. Application

For simplicity, we are interested to deal with Adomian decomposition solution associated with the operator L_t^{-1} rather than the other operators in our example.

Nonlinear partial deferential equations are known to describe a wide variety of phenomena not only in physics, where applications extend over magneto fluid dynamics, water surface gravity waves, electromagnetic radiation reactions and ion acoustic waves in plasma, but also in biology, chemistry and several other fields. The main objective of the present paper is to use the Adomian decomposition method to the Coupled KdV equations which given by Lou (2006), Feng X (1996), Billingham (2004), Rogers and Ames (1988).

$$\mathrm{u}_t+u_{xxx}+2u\,u_x+2u_x\,v=0 \qquad (9)$$

$$v_t+v_{xxx}+2v\,v_x+2v_x\,u=0 \qquad (10)$$

With initial conditions

$$u(x,0)=e^{-kx} \qquad (11)$$

$$\mathrm{v}(x,0)=-e^{-kx} \qquad (12)$$

Where K is constant.

Equations (9, 10) can be written in an operator form as

$$\mathrm{Lu}=-[\mathrm{L}_{xxx}\,u+2N_1(u,u_x)+2K_1(u_x,v)] \qquad (13)$$

$$Lv=-[\mathrm{L}_{xxx}\,v+2N_2(v,v_x)+2K_2(v_x,u)] \qquad (14)$$

Where $L=\dfrac{d}{dt}$ and $L_{xxx}=\dfrac{d^3}{dx^3}$.

The Adomian Decomposition Method **(ADM)** assumes a series solution of the unknown functions

$$u(x,t)=\sum_{n=0}^{\infty}u_n(\mathrm{x},\mathrm{t}) \qquad (15)$$

$$v(x,t)=\sum_{n=0}^{\infty}v_n(x,t). \tag{16}$$

Substituting equations (15, 16) with initial conditions into equations (13, 14) yields

$$\sum_{n=0}^{\infty}u_n(x,t)=u(x,0)-L_t^{-1}(L_{xxx}u)-L_t^{-1}[2N_1(u,u_x)+2K_1(u_x,v)] \tag{17}$$

$$\sum_{n=0}^{\infty}v_n(x,t)=v(x,0)-L_t^{-1}(L_{xxx}v)-L_t^{-1}[2N_2(v,v_x)+2K_2(v_x,u)], \tag{18}$$

where the functions $N_1(u,u_x),\ K_1(u_x,v),\ N_2(v,v_x),\ K_2(v_x,u)$ are

$$N_1(u,u_x)=uu_x=\sum_{n=0}^{\infty}A_{1n}(u,u_x)=u_0u_{0x}+(u_1u_{0x}+u_0u_{1x})+...=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}u_mu_{(n-m)x} \tag{19}$$

$$K_1(u_x,v)=u_xv=\sum_{n=0}^{\infty}B_{1n}(u_x,v)=u_{0x}v_0+(u_{0x}v_1+u_{1x}v_0)+...=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}v_mu_{(n-m)x} \tag{20}$$

$$N_2(v,v_x)=vv_x=\sum_{n=0}^{\infty}A_{2n}(v,v_x)=v_0v_{0x}+(v_1v_{0x}+v_0v_{1x})+...=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}v_mv_{(n-m)x} \tag{21}$$

$$K_2(v_x,u)=v_xu=\sum_{n=0}^{\infty}B_{2n}(v_x,u)=v_{0x}u_0+(v_{0x}u_1+v_{1x}u_0)+...=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}u_mv_{(n-m)x} \tag{22}$$

Identifying the zeros components of u_0 *and* v_0 the remaining components $u_n(x,t)$ and $v_n(x,t), n\succ 1$ can be determined by using recursive relations given by

$$u(x,0)=e^{-kx}$$

$$v(x,0)=-e^{-kx},$$

$$u_{n+1}(x,t)=-L_t^{-1}(L_{xxx}u)-L_t^{-1}[2N_1(u,u_x)+2K_1(u_x,v)], \tag{23}$$

$$v_{n+1}(x,t)=-L_t^{-1}(L_{xxx}v)-L_t^{-1}[2N_2(v,v_x)+2K_2(v_x,u)] \tag{24}$$

The remaining components u_n *and* v_n can be completely determined such that each term that determined by using the previous terms, and the series solutions thus entirely evaluated.

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$$
\begin{aligned}
u_1 &= \frac{t}{e^x}, & v_1 &= -\frac{t}{e^x} \\
u_2 &= \frac{1}{2}\frac{t^2}{e^x}, & v_2 &= -\frac{1}{2}\frac{t^2}{e^x} \\
u_3 &= \frac{1}{6}\frac{t^3}{e^x}, & v_3 &= -\frac{1}{6}\frac{t^3}{e^x}
\end{aligned} \tag{25}
$$

....

$$u_n(x,t) = \frac{1}{n!}\frac{t^n}{e^x}, \quad v_n(x,t) = -\frac{1}{n!}\frac{t^n}{e^x} \quad .$$

The solution of $u(x,t)$ *and* $v(x,t)$ are

$$
\begin{aligned}
u(x,t) &= e^{-x+t} \\
v(x,t) &= -e^{-x+t}.
\end{aligned} \tag{26}
$$

Adomian solutions coincides with the exact solution

$$(u,v) = (e^{-x+t}, -e^{-x+t}) \tag{27}$$

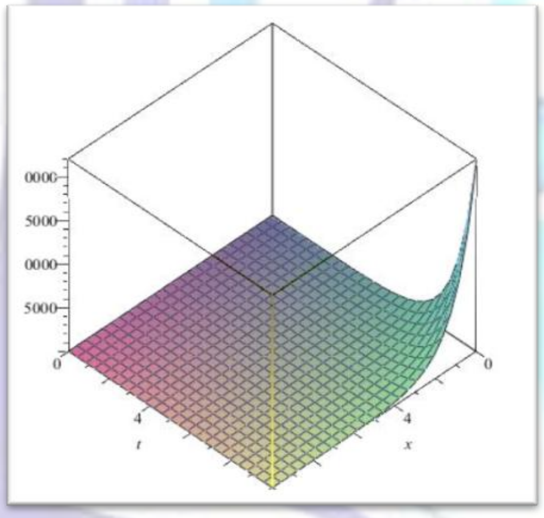


Fig.1 (the exact solution of u(x,t)in (27))

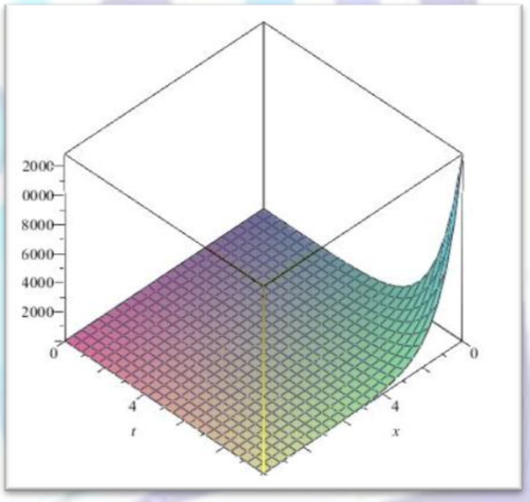


Fig.2 (the numerical solution of u(x,t) in (23))

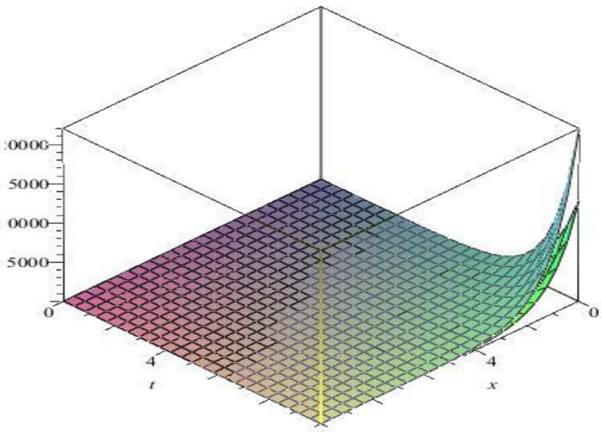
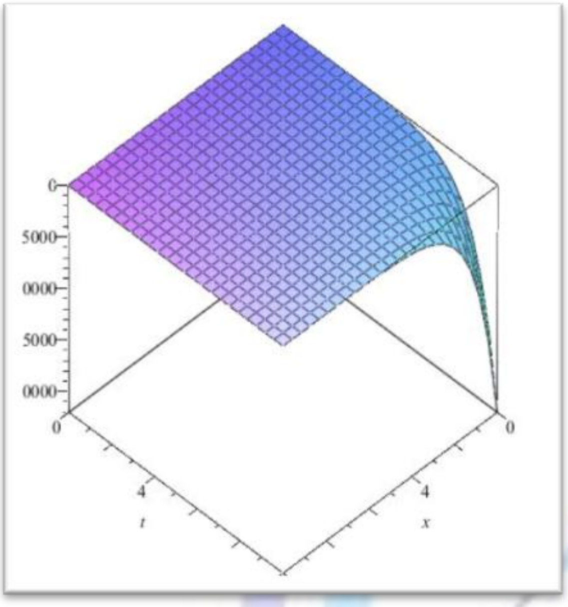


Fig. 3 (comparison between the exact solution and numerical solution of u (x,t))



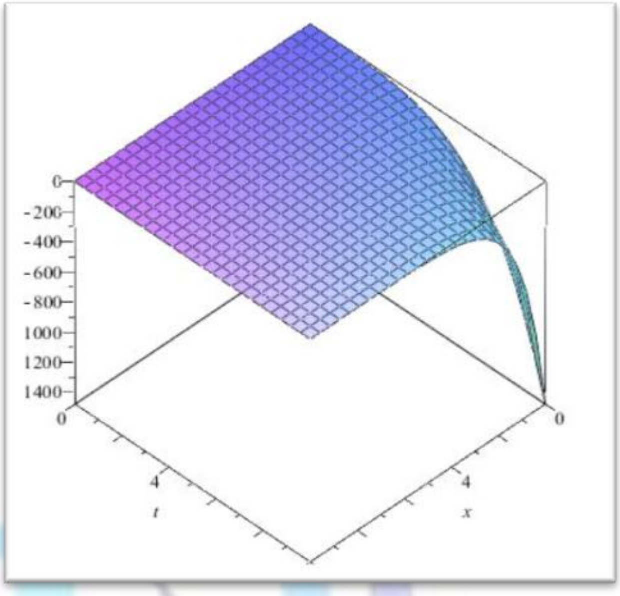


Fig. 4 (the exact solution of v(x,t)in (27)) Fig. 5 (the numerical solution of v(x,t)in(24))

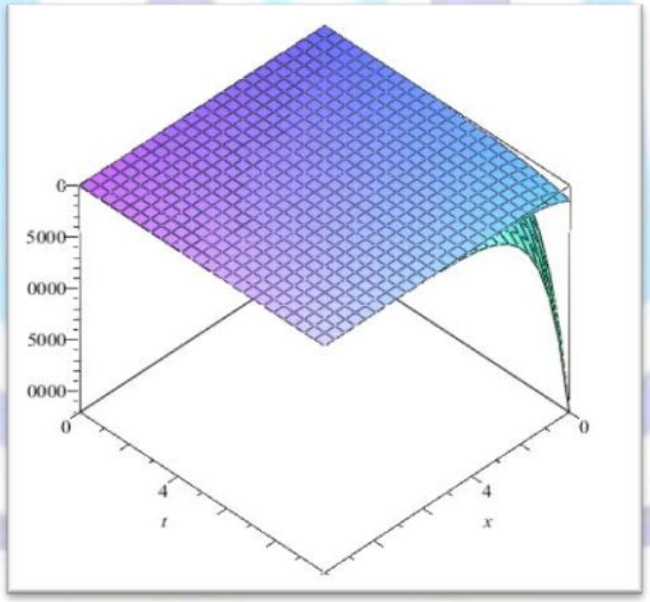


Fig. 6 (comparison between the exact and numerical solution of v(x,t))

CONCLUSIONS

In this work. The Adomian decomposition method has been successfully applied to find the solution of nonlinear Coupled KdV Equations is presented in Fig. (2,5). This method converts this equation to recurrences relation whose terms are computed using maple 15.

Moreover, the computations are simpler and faster than classical techniques.

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