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The Resultant formula of Masses m_1/m_2,…,m_n in Space oxyz at a Point

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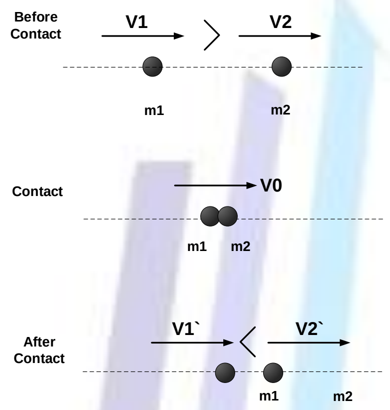
Abstract

In this paper, the formula of the contact resultant for masses (m-1,m-2,…,m-n) in space(oxyz) is calculated and proved. Regarding the importance of masses movement in space and their contact with each other, it is felt that in order to design and optimize dynamic systems (dynamic mechanics), a reasonable relation should be established between their subsets. This paper attempts to prove such a relation in the simplest possible way.

Key words

Contact, material points, internal stress



Regarding the established formulas, please now find below some examples in each of which different assumptions have been assumed.

Ex.-1) Collision of 2 masses m_1 and $m_2$at speeds V_1 and V_2 on the oxy plane is considered.

Ex.-2) Collision of 3 masses m_1,$m_2$and m3 at speeds V_1,V_2 and V_3 on the oxy plane is considered.

Ex.-3) Collision of 2 masses $m_1$and m_2 at speeds V_1 and V_2 on 3D space oxyz is considered.

In order to solve Ex. 3 the masses technical specifications should studied on 3 planes oxy ,oxz and oyz .

In order to solve Ex.3 and 4, it will suffice to have the masses technical specifications on 2 planes oxz and oxy by which to obtain the masses technical specification on plane oyz.

Now, in order to determine the masses technical specifications on plane oyz, the following trigonometric relation can be applied.

$$\frac{V_z}{Vx} = \frac{V_z}{V_y} = \frac{V_y}{V_x} \Rightarrow \tan\alpha = \tan\beta . \tan\gamma \ \Rightarrow tab\ \beta = \frac{\tan\alpha}{\tan\gamma}$$

The above can be named the dynamic trigonometry formula from then on, the above formula can be considered as the (dynamic trigonometry) formula the subset of which have been proved.

Dynamic trigonometry

Having relations z' = $\frac{V_2}{V_x}$and y' = $\frac{V_y}{V_x}$ on coordinates planes oxz and oxy now, the following relations can be written on 3 coordinates planes oxy, oyz and oxz as following:

1. z' = $\frac{V_2}{V_x}$ = tan ∝ on oxz
2. y' = $\frac{V_y}{V_x}$ = tan γ on oxy

By dividing Eq.1 by Eq.2 , Eq.3 can be obtained on coordinates plane oyz:

3. $\frac{z'}{y'} = \frac{V_z/V_x}{V_y/V_x} = \frac{V_z}{V_y} = \tan\beta\ on\ oyz$ ⇨ tan β = $\frac{V_z}{V_y}$

Using Eqs.3 , 2 and 1, Eq. 4 can be obtained

4. tan∝ = tan γ . tan β .

Eq.4 can be designated as the (dynamic trigonometric) primary formula basis and foundation. Next formula will thus be the subsets of Eq.4.

Exercise:

Having the following specifications, 2 masses are moving in space oxyz when they collide at a point there. Calculate the post collisionspecifications.

(The 2 masses are material points moving forwards (x,y,z) positive or negative axes).

Numbers for the problem statement have been assumed for practice.

Masses precoalition technical specifications on plane oxy:

oxy = Angle of vector V in space relative to plane oxy

oxz = Angle of vector V in space relative to plane oxz

oyz = Angle of vector V in space relative to plane oyz

During contact, the contact forces between 2 spheres are equal to an inverse of each other. The set linear momentum will not thus change.

Therefore it is included from the linear momentum law that:

$$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$$

Let any force, except for the very large internal ones due to contact which impact the spheres during contact, be relatively small,and so that the impact from the contact is considered negligible compared with the impact from any internal contact forces.

Inthis paper, the contact resultant for masses (m_1, m_2… m_n) at a point in space (oxyz) is presented in a formula whose proof is included as follows:

M=(m_1,m_2,… … …,m_n):equivalent mass after masses contact

V=(V_1,V_2,… … …,V_n) : equivalent velocity after masses contract

∑ m.v_x =masses(m_1,m_2,…,… …,m_n) on the abscissa

The algebraic sum of the masses velocity components on the (x)axis

∑ m.V_y= (m_1,m_2,… … …,m_n) on the ordinate.

The algebraic sum of the masses velocity components on the (y) axis

∑ m.V_z= (m_1,m_2,… … …,m_n) on the heights axis

The algebraic sum of the masses velocity component on the (z) axis

In this paper, 2 masses m_1 and m_2 with the velocity of v_1 and v_2, respectively, are calculated at contact. If 2 points o_1 and o_2 are considered in the origin of the oxy coordinates with masses m_1 and m_2, respectivelyat the above mentioned point, we will have:

The velocity image on the (y) and (x) axis

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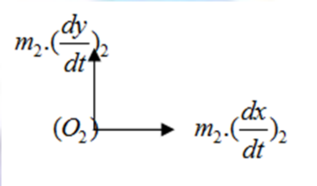
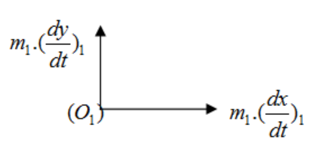
$$O_1 \left| \begin{aligned} V_{x_1} &= m_1.(\frac{dx}{dt})_1 \\ V_{y_1} &= m_1(\frac{dy}{dt})_1 \end{aligned} \right. \Rightarrow \left| \begin{aligned} & y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{V_{y_1}}{V_{x_1}} \\ & \frac{V_{y_1}}{V_{x_1}} = \frac{m_1.(\frac{dy}{dt})_1}{m_1.(\frac{dx}{dt})_1} \end{aligned} \right.$$

$$m\,.V = \sqrt{(\sum m.V_x)^2 + (\sum m.V_y)^2 + (\sum m.V_z)^2}$$

$$O_2 \left| \begin{aligned} V_{x_2} &= m_2.(\frac{dx}{dt})_2 \\ V_{y_2} &= m_2.(\frac{dy}{dt})_2 \end{aligned} \right. \Rightarrow \frac{V_{y_2}}{V_{x_2}} = \frac{m_2.(\frac{dy}{dt})_2}{m_2.(\frac{dx}{dt})_2}$$





For the calculation of the 2 points (O_1) and (O_2) resultant, it will suffice to proceed as follows:
Points (O_1,O2,… … …,O_n) are material points with no internal stress.

$$\Sigma m.Vx = m_1.(\frac{dx}{dt})_1 + m_2.(\frac{dx}{dt})_2$$

$$\Sigma m.Vy = m_1.(\frac{dy}{dt})_1 + m_2.(\frac{dy}{dt})_2$$

N.B.: Sign(+) or (-) shows the velocities opposition or codirectionality. We have not thus added the extra(±) sign.
Let mass and velocity be shown as m and v, respectively. We will thus have:
On the (oxy) coordinates

$$m.V = \sqrt{\overline{\Sigma m.Vx}^2 + \overline{\Sigma m.Vy}^2}$$

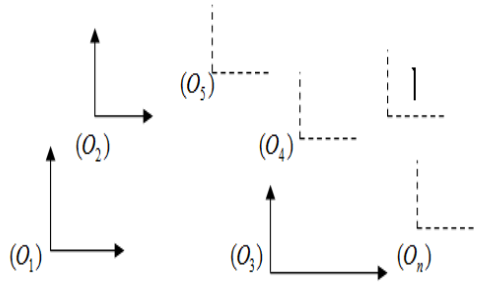
$$m.V = \sqrt{\left[m_1.(\frac{dx}{dt})_1 + m_2.(\frac{dx}{dt})_2\right]^2 + \left[m_1.(\frac{dy}{dt})_1 + m_2.(\frac{dy}{dt})_2\right]^2}$$

Upon obtaining the formula, its specific case will be studied.
Specific case: If 2 points (O_1) and (O_2) have the static state after contact, their resultant will be zero.

$$m.V = 0 \Rightarrow \sqrt{\overline{\Sigma m.Vx}^2 + \overline{\Sigma m.Vy}^2} = 0 \;\Rightarrow \overline{\Sigma m.Vx}^2 + \overline{\Sigma m.Vy}^2 = 0$$

$$\left[m_1.(\frac{dx}{dt})_1 + m_2.(\frac{dx}{dt})_2\right]^2 + \left[m_1.(\frac{dy}{dt})_1 + m_2.(\frac{dy}{dt})_2\right]^2 = 0$$

In the origin of coordinates (oxy), instead of 2 points (O_1) and (O_2), if there are however many points:



$$\Sigma m.Vx = m_1.(\frac{dx}{dt})_1 + m_2.(\frac{dx}{dt})_2 + m_3.(\frac{dx}{dt})_3 + ... + m_n.(\frac{dx}{dt})_n \quad \Sigma m.Vy = m_1.(\frac{dy}{dt})_1 + m_2.(\frac{dy}{dt})_2 + m_3.(\frac{dy}{dt})_3 + ... + m_n.(\frac{dy}{dt})_n$$

$$m.V = \sqrt{\overset{2}{\overline{\Sigma m.Vx}} + \overset{2}{\overline{\Sigma m.Vy}}}$$

By obtaining the values of ∑m.V_x and ∑m.V_y, they can be put in

$$m.V = \sqrt{\overset{2}{\overline{\Sigma m.Vx}} + \overset{2}{\overline{\Sigma m.Vy}}}$$

to have the total resultant. If we consider that the object is in an equilibrium state after contact at O_1,O_2,O_3,…., O_n, we will have:

$$m.V = \sqrt{\overset{2}{\overline{\Sigma m.Vx}} + \overset{2}{\overline{\Sigma m.Vy}}} = 0$$

$$\overset{2}{\overline{\Sigma m.Vx}} + \overset{2}{\overline{\Sigma m.Vy}} = 0$$

Now, points O_1, O_2, O_3,…, O_n are studied and calculated in 3 dimensions of (oxyz).

$$m.V = \sqrt{\overset{2}{\overline{\Sigma m.Vx}} + \overset{2}{\overline{\Sigma m.Vy}} + \overset{2}{\overline{\Sigma m.Vz}}}$$

$$\Sigma m.Vx = m_1.(\frac{dx}{dt})_1 + m_2.(\frac{dx}{dt})_2 + m_3.(\frac{dx}{dt})_3 + ... + m_n.(\frac{dx}{dt})_n \quad \Sigma m.Vx = m_1.(\frac{dx}{dt})_1 + m_2.(\frac{dx}{dt})_2 + m_3.(\frac{dx}{dt})_3 + ... + m_n.(\frac{dx}{dt})_n$$

$$\Sigma m.Vy = m_1.(\frac{dy}{dt})_1 + m_2.(\frac{dy}{dt})_2 + m_3.(\frac{dy}{dt})_3 + ... + m_n.(\frac{dy}{dt})_n \quad \Sigma m.Vz = m_1.(\frac{dz}{dt})_1 + m_2.(\frac{dz}{dt})_2 + m_3.(\frac{dz}{dt})_3 + ... + m_n.(\frac{dz}{dt})_n$$

By obtaining the values of ∑m.V_z, ∑m.V_y and ∑m.V_x they can be put in

$$m.V = \sqrt{\overset{2}{\overline{\Sigma m.Vx}} + \overset{2}{\overline{\Sigma m.Vy}} + \overset{2}{\overline{\Sigma m.Vz}}}$$

to have the total resultant. If we consider that the object is in the static and equilibrium state after contact at points O_1, O_2, O_3,… … …, O_n, we will have

$$m.V = \sqrt{\overset{2}{\overline{\Sigma m.Vx}} + \overset{2}{\overline{\Sigma m.Vy}} + \overset{2}{\overline{\Sigma m.Vz}}}^{=0}$$

$$\overset{2}{\overline{\Sigma m.Vx}} + \overset{2}{\overline{\Sigma m.Vy}} + \overset{2}{\overline{\Sigma m.Vz}} = 0$$

Example : 1

$$\begin{cases} m_1 = 4\,kg \\ V_1 = \sqrt{2} m/sec \\ \tan \gamma_1 = 1 \end{cases}$$

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$$\begin{cases} m_2 = 2\,kg \\ V_2 = \sqrt{5} m/sec \\ tan\,\gamma_2 = 2 \end{cases}$$

No.	Suppose the issue before the collision			**(oxy)**					
	m **kg**	**V$_{m/sec}$**	$\frac{m.Vy}{m.Vx} = tan\,\gamma$	**Sinγ =** $\frac{tan\,\gamma}{\sqrt{1+tan^2\gamma}}$	**Cos γ =** $\frac{1}{\sqrt{1+tan^2\gamma}}$	**Vx = V.Cosγ**	**m.Vx**	**Vy= V.Sin γ**	**m.Vy**
1	4	$\sqrt{2}$	1	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	+4	1	+4
2	2	$\sqrt{5}$	2	$\frac{2\sqrt{5}}{5}$	$\frac{\sqrt{5}}{5}$	1	+2	2	+4
consequence	m.V=10		$\frac{8}{6} = \frac{4}{3}$	$^4/_5$	$^3/_5$		∑(m.Vx)=6		∑(m.Vy)=8

$$\tan\gamma = \frac{\sum(m.V_y)}{\sum(m.V_x)} = \frac{8}{6} = \frac{4}{3}\ , m.V = \sqrt{\sum(m.V_x)^2 + \sum(m.V_y)^2}$$

$$m.V = \sqrt{(6)^2 + (8)^2} = \sqrt{36+64} = \sqrt{100} = 10 \Rightarrow \tan\gamma = {}^4/_3$$

$$\sin\gamma = \frac{\tan\gamma}{\sqrt{1+\tan^2\gamma}} = \frac{^4/_3}{\sqrt{1+\left(\frac{4}{3}\right)^2}} = \frac{^4/_3}{\sqrt{\frac{9+16}{3}}} = \frac{4}{\sqrt{25}} = {}^4/_5 \Rightarrow m.V = 10$$

$$\cos\gamma = \frac{1}{\sqrt{1+\tan^2\gamma}} = \frac{1}{\sqrt{1+\left(\frac{4}{3}\right)^2}} = \frac{1}{\sqrt{1+\frac{16}{9}}} = \frac{1}{\sqrt{\frac{9+16}{9}}} = {}^3/_5$$

Example : 2

$$\begin{cases} m_1 = 4\,kg \\ V_1 = \sqrt{2} m/sec \\ tan\,\gamma_1 = 1 \end{cases}$$

$$\begin{cases} m_2 = 2\,kg \\ V_2 = \sqrt{5} m/sec \\ \tan\gamma_2 = 2 \end{cases}$$

$$\begin{cases} m_3 = 2\,kg \\ V_3 = \sqrt{2} m/sec \\ tan\,\gamma_3 = -1 \end{cases}$$

No.	Suppose the issue before the collision			(oxy)					
	m$_{kg}$	V$_{m/sec}$	$\frac{m.Vy}{m.Vx} = tan\,\gamma$	Sin γ = $\frac{tan\,\gamma}{\sqrt{1+tan^2\gamma}}$	Cos γ = $\frac{1}{\sqrt{1+tan^2\gamma}}$	Vx = V.Cos γ	m.Vx	Vy=V. Sin γ	m.Vy
1	4	$\sqrt{2}$	1	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	+4	1	+4
2	2	$\sqrt{5}$	2	$\frac{2\sqrt{5}}{5}$	$\frac{\sqrt{5}}{5}$	1	+2	2	+4
3	2	$\sqrt{2}$	-1	$\frac{-\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	+2	-1	-2
consequence	m.V=10		$\frac{\sum(m.Vy)}{\sum(m.Vx)} = \frac{3}{4}$	$^3/_5$	$^4/_5$		∑(m.Vx)=8		∑(m.Vy)=6

$$\tan\gamma = \frac{\sum(m.V_y)}{\sum(m.V_x)} = \frac{6}{8} \Rightarrow \tan\gamma = \frac{3}{4},\ m.V = \sqrt{\sum(m.V_x)^2 + \sum(m.V_y)^2}$$

$$m.V = \sqrt{(8)^2 + (6)^2} = \sqrt{64+36} = 10 \Rightarrow \mathbf{m.V = 10}$$

$$\sin\gamma = \frac{\tan\gamma}{\sqrt{1+\tan^2\gamma}} = \frac{3/4}{\sqrt{1+\frac{9}{16}}} = \frac{3/4}{\sqrt{\frac{9+16}{3}}} = \frac{3/4}{5/4} = 3/5 \Rightarrow Sin\,\gamma = \frac{3}{5}$$

$$\cos\gamma = \frac{1}{\sqrt{1+\tan^2\gamma}} = \frac{1}{\sqrt{1+\frac{9}{16}}} = \frac{4}{5} \Rightarrow Cos\,\gamma\ 4/5$$

Example : 3

$$\begin{cases} m_1 = 2\ kg \\ V_1 = 1m/sec \\ tan\,\gamma_1 = \sqrt{3} \end{cases}$$

$$\begin{cases} m_2 = 1\ kg \\ V_2 = 2m/sec \\ tan\,\gamma_2 = \frac{\sqrt{3}}{3} \end{cases}$$

No.	Suppose the issue before the collision			(oxy)						
	m kg	V m/sec	$\frac{m.Vy}{m.Vx} = tan\,\gamma$	Sin γ = $\frac{tan\ \gamma}{\sqrt{1+tan^2\gamma}}$	Cos γ = $\frac{1}{\sqrt{1+tan^2\gamma}}$	Vx = V.Cosγ	m.Vx	Vy=V.S in γ	m.Vy	m.V
1	2	1	$\sqrt{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\sqrt{3}$	2
2	1	2	$\frac{\sqrt{3}}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	1	$\sqrt{3}$	1	2
consequence			$\frac{\sum(m.Vy)}{\sum(m.Vx)} = 1$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$			$\sum$(m.Vx) =(1+$\sqrt{3}$)	$\sum$(m.Vy) = ($\sqrt{3}$ + 1)	($\sqrt{3}$ + 1) ($\sqrt{2}$)

$$\tan\gamma = \frac{\sum(m.V_y)}{\sum(m.V_x)} = \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = 1 \Rightarrow \tan\gamma = 1 \Rightarrow Sin\,\gamma = \frac{\sqrt{2}}{2} \Rightarrow Cos\gamma = \frac{\sqrt{2}}{2}$$

$$m.V = \sqrt{\sum(m.V_x)^2 + \sum(m.V_y)^2} = \sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}+1)^2} = \sqrt{(\sqrt{3}+1)^2 + (1+1)} \Rightarrow \mathbf{m.V=(\sqrt{3+1})\sqrt{2}}$$

$$oxz\ ) \Rightarrow \begin{cases} \tan\gamma = 1 \Rightarrow \sin\gamma = \frac{\sqrt{2}}{2}, \cos\gamma = \frac{\sqrt{2}}{2} \\ m.V = (\sqrt{3}+1)(\sqrt{2} \\ \sum(m.Vy) = (\sqrt{3}+1) \\ \sum(m.Vx) = (\sqrt{3}+1) \end{cases}$$

Example : 4

$$\begin{cases} m_1 = 2 \\ V^1 = \frac{\sqrt{2}}{2} \\ tan\,\alpha_1 = 1 \end{cases}$$

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$$\begin{cases} m_2 = 1 \\ V^2 = 2\sqrt{3} \\ \tan\gamma_2 = \sqrt{3} \end{cases}$$

No.	Suppose the issue before the collision			(oxz)						
	m kg	V m/sec	$\frac{m.Vz}{m.Vx} = \tan\propto$	Sin ∝ = $\frac{\tan\propto}{\sqrt{1+\tan^2\propto}}$	Cos ∝ = $\frac{1}{\sqrt{1+\tan^2\propto}}$	Vx = V.Cos∝	Vz = V.Sin∝	m.Vx	m.Vz	m.V
1	2	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\sqrt{2}$
2	1	$2\sqrt{3}$	$\sqrt{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	3	$\sqrt{3}$	3	$2\sqrt{3}$
consequence	m.V= $\sqrt{20+2\sqrt{3}}$		$\frac{4}{1+\sqrt{3}}$	$\frac{4\sqrt{66}}{33}$	$\frac{\sqrt{33}}{33}$			∑(m.Vx) =(1+$\sqrt{3}$)	∑(m.Vz) = 4	$\sqrt{20+2\sqrt{3}}$

$$m.V=\sqrt{\sum(m.V_x)^2+\sum(m.V_z)^2} = \sqrt{\left(\sqrt{3}+1\right)^2+(4)^2} = \sqrt{1+3+2\sqrt{3}+16}=\sqrt{20+2\sqrt{3}}$$

$$\boldsymbol{m.V = \sqrt{20+2\sqrt{3}}}$$

$$\tan\propto = \frac{\sum(m.Vz)}{\sum(m.Vx)} = \frac{4}{1+\sqrt{3}} \Rightarrow \tan\propto = \frac{4}{1+\sqrt{3}} \Rightarrow Sin\ \propto = \frac{4\sqrt{66}}{33}\ \ , Cos\ \propto = \frac{\sqrt{33}}{33}$$

$$\text{oxz })\Rightarrow \begin{cases} m.V = \sqrt{20+2\sqrt{3}} \\ \sum(m.Vz) = 4 \\ \sum(m.Vx) = \left(\sqrt{3}+1\right) \\ \tan\propto = \frac{4}{(1+\sqrt{3})} \end{cases}$$

Example : 5

No.	Suppose the issue before the collision			**(oyz)**						
	m kg	V m/sec	$\frac{m.Vz}{m.Vy} = \tan\beta$	Sin β = $\frac{\tan\beta}{\sqrt{1+\tan^2\beta}}$	Cos β = $\frac{1}{\sqrt{1+\tan^2\beta}}$	Vy = V.Cos β	Vz = V.Sin β	m.Vy	m.Vz	m.V
1	2	1	$\frac{1}{\sqrt{3}}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	1	2
2	1	$\sqrt{10}$	3	$\frac{3\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	1	3	1	3	$\sqrt{10}$
consequence	m.V= $\sqrt{20+2\sqrt{3}}$		$\frac{\sum(m.Vz)}{\sum(m.Vy)} = \frac{4}{\sqrt{3}+1}$	$\frac{4\sqrt{4+2+\sqrt{3}}}{\sqrt{20+2+\sqrt{3}}}$	$\frac{\sqrt{4+2+\sqrt{3}}}{\sqrt{20+2+\sqrt{3}}}$			∑(m.Vy)=(1+$\sqrt{3}$)	∑(m.Vz)= 4	$\sqrt{20+2\sqrt{3}}$

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$$\left.\begin{matrix}\dfrac{V_x}{V_y}\\ \dfrac{V_z}{V_y}\\ \dfrac{V_y}{V_x}\end{matrix}\right\} \Rightarrow \tan \propto = \tan \beta . \tan \gamma \Rightarrow \tan \beta = \frac{\tan \propto}{\tan \gamma} = \frac{{}^{4}/_{\sqrt{3}} + 1}{1}$$

$$= \frac{4}{\sqrt{3}+1} \tan \beta = \frac{4}{\sqrt{3}+1}$$

$$m.V = \sqrt{\sum (m.V_y)^2 + \sum (m.V_z)^2} = \sqrt{(\sqrt{3}+1)^2 + (4)^2} = \sqrt{3+1+2\sqrt{3}+16} = \sqrt{20+2\sqrt{3}}$$

$$m.V = \sqrt{20+2\sqrt{3}}$$

$$\text{oxz)} \Rightarrow \begin{cases} m.V = \sqrt{20+2\sqrt{3}} \\ \sum(m.Vz) = 4 \\ \sum(m.Vy) = (\sqrt{3}+1) \\ \tan \beta = \frac{4}{1+\sqrt{3}} \end{cases}$$

$$\text{No. 1)} \begin{Bmatrix} Vz = V.\operatorname{Sin}\beta \Rightarrow \frac{1}{2} = V.\frac{1}{2} \Rightarrow V = 1m/\sec \\ Vy = V.\operatorname{Cos}\beta \Rightarrow \frac{\sqrt{3}}{2} = V.\frac{\sqrt{3}}{2} \Rightarrow V = 1\ m/sec \end{Bmatrix} \Rightarrow V = 1m/\sec$$

$$\text{No. 2)} \begin{cases} Vz = V.\operatorname{Sin}\beta \Rightarrow 3 = V.\frac{3\sqrt{10}}{10} \Rightarrow V = \frac{10}{\sqrt{10}} = \frac{10\sqrt{10}}{10} = \sqrt{10} \\ Vy = V.\operatorname{Cos}\beta \Rightarrow 1 = V.\frac{\sqrt{10}}{10} \Rightarrow V = \frac{10}{\sqrt{10}} \Rightarrow V = \sqrt{10}m/sec \\ V = \frac{10\sqrt{10}}{10} \Rightarrow \sqrt{10} \Rightarrow V = \sqrt{10}m/\sec \end{cases}$$

$$\text{oxyz)} \Rightarrow m.V = \sqrt{\sum(m.V_x)^2 + \sum(m.V_y)^2 + \sum(m.V_z)^2} = \sqrt{(1+\sqrt{3})^2 + (1+\sqrt{3})^2 + (4)^2} = \sqrt{1+(\sqrt{3})^2(2)+16}$$

$$= \sqrt{(1+3+2\sqrt{3}) + (2) + 16} = \sqrt{2(4+2+\sqrt{3}+16} = \sqrt{24+4\sqrt{3}} = \sqrt{4*6+4\sqrt{3}} = 2\sqrt{6+\sqrt{3}}$$

$$\text{oxyz)} \begin{cases} m.V = 2\sqrt{6+\sqrt{3}kgm/sec} \\ \operatorname{Cos}\theta_{xy} = \dfrac{\sqrt{\sum(m.V_x)^2 + \sum(m.V_y)^2}}{m.V} = \dfrac{\sqrt{2(1+\sqrt{3})^2}}{2\sqrt{6+\sqrt{3}}} \\ \operatorname{Cos}\theta_{xz} = \dfrac{\sqrt{\sum(m.V_x)^2 + \sum(m.V_z)^2}}{m.V} = \dfrac{\sqrt{(1+\sqrt{3})^2 + (4)^2}}{2\sqrt{6+\sqrt{3}}} \\ \operatorname{Cos}\theta_{yz} = \dfrac{\sqrt{\sum(m.V_y)^2 + \sum(m.V_z)^2}}{m.V} = \dfrac{\sqrt{(1+\sqrt{3})^2 + (4)^2}}{2\sqrt{6+\sqrt{3}}} \end{cases}$$

Conclusion

As observed, the resultant velocity of the set of the particles which simultaneously contact each other in a 2D space was obtained using the energy conservation law. This was proceeded to be extended to the 3D state as well. The formula obtained finally will provide the possibility to predict the resultant velocity of the set of the particles which simultaneously contact each other in a 3D dimension.

Resource

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