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SOME RESULTS ON KENMOTSU MANIFOLDS ADMITTING QUARTER SYMMETRIC NON METRIC CONNECTION

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ABSTRACT

In this paper we have studied the some curvature properties of a quarter-symmetric non metric connection of Kenmostu manifolds. We also studied the some properties of projective ricci tensor, pseudo projective curvature tensor and m-projective curvature tensor.

Keywords

Kenmostu manifold; quarter-symmetric non metric; m-projective; pseudo projective.

Academic Discipline And Sub-Disciplines

Mathematics, Differentiable Manifold

SUBJECT CLASSIFICATION

53A20, 53B05

1. INTRODUCTION

In 1972, K. Kenmostu studied a class of contact Riemannian manifold and call it Kenmostu manifold[5]. He studied that if Kenmostu manifold satisfies the condition $R(X,Y).R=0$, then it is a negative curvature -1, where *R* is the Riemannian curvature tensor of type (1, 3) and $R(X,Y)$ denotes the derivation of the tensor algebra at each point of the tangent space. Further Kenmostu manifold is studied by Jun, De and Pathak[3], Yano and Imai[6], Singh and Pandey[10], Tripathi[7] and many others.

In 1975, Golab[12] introduced the quarter-symmetric linear connection on a differentiable manifold. Let T be the torsion tensor defined as

$$T(X,Y)=\overline{\nabla}_X Y-\overline{\nabla}_Y X-[X,Y]$$

A linear connection $\overline{\nabla}$ in an n-dimensional differentiable manifold is said to be quarter-symmetric connection if its torsion tensor is defined by

$$T(X,Y)=\eta(Y)\phi X-\eta(X)\phi Y$$

Where η is 1-form, ϕ is tensor of type (1, 1) and *X* and *Y* is vector fields.

A quarter-symmetric linear connection $\overline{\nabla}$ is satisfies the condition

$$\overline{\nabla}_X g\neq 0 \text{ for all } X\in TM$$

then $\overline{\nabla}$ is said to be quarter-symmetric non metric connection otherwise it is metric connection where *TM* is the lie algebra of vector field of manifold. The quarter-symmetric non metric connection is studied by Prakash and Pandey[2], Prakash and Narain[1], Singh and Pandey[10], Yano and Imai[6], Biswas and De[11], Rastogi[14], Shukla and Jaiswal[8], Mishra and Pandey[9], Mkhopadhyay, Ray and Barua[13], Biwas and Sengupta[4] and many others.

This paper is organized as follows:

After the introduction, in section2 we have the brief introduction of Kenmostu manifold admitting the quarter-symmetric non metric connection. We studied the projective ricci tensor of quarter-symmetric non metric Kenmostu manifold in section3. We discuss the pseudo projective tensor and M-projective curvature in section4 and section5 respectively. At last in section6 we prove some results base on the some curvature tensor (concircular curvature and conharmonic curvature).

2. PRELIMINILIARIES

Let *M* be an odd *n*-dimensional almost contact metric structure (ϕ,ξ,η,g), where ϕ is a tensor field of $(1,1)$ type, η is 1-form and *g* is the Riemannian metric satisfying the conditions:

$$\overline{\overline{X}}=-X+\eta(X)\xi \tag{2.1}$$

$$\phi X=\overline{X} \tag{2.2}$$

$$\eta(\phi X)=0 \tag{2.3}$$

$$\eta(\xi)=1 \tag{2.4}$$

$$\phi\xi=0 \tag{2.5}$$

$$rank(\phi)=n-1 \tag{2.6}$$

$$g(\phi X,\phi Y)=g(X,Y)-\eta(X)\eta(Y) \tag{2.7}$$

For any vector fields *X* and *Y* on *M*.

The fundamental 2-form Φ in an almost contact metric manifold is defined by

$$\Phi(X,Y)=g(\phi X,Y) \tag{2.8}$$

It can be easily shown that

$$\Phi(\phi X,\phi Y)=\Phi(X,Y) \tag{2.9}$$

An almost contact metric manifold is said to be a Kenmotsu manifold if

$$(\nabla_X \phi)Y = -\eta(Y)\phi X - g(X, \phi Y)\xi \tag{2.10}$$

$$\nabla_X \xi = X - \eta(X)\xi \tag{2.11}$$

Where ∇ is the Levi-Civita connection of *g.*

Also in a Kenmotsu manifold the following relations hold:

$$(\nabla_X \eta)Y = g(X,Y) - \eta(X)\eta(Y) \tag{2.12}$$

$$R(X,Y,\xi) = \eta(X)Y - \eta(Y)X \tag{2.13}$$

$$S(X,\xi) = -(n-1)\eta(X) \tag{2.14}$$

$$S(\phi Y, \phi Z) = S(Y,Z) - (n-1)\eta(X)\eta(Y) \tag{2.15}$$

$$\eta(R(X,Y)Z) = \eta(Y)g(X,Z) - \eta(X)g(Y,Z) \tag{2.16}$$

Where *R* is curvature tensor and *S* is ricci tensor of the Kenmotsu manifold *M.*

Hence, the quarter-symmetric non metric connection of Kenmotsu manifold is defined by[10]

$$\overline{\nabla}_X Y = \nabla_X Y - \eta(Y)\phi X + g(X,Y)\xi \tag{2.17}$$

Such that

$$(\overline{\nabla}_X g)(Y,Z) = -\eta(Y)g(X,Z) - \eta(Z)g(X,Y) \tag{2.18}$$

It also satisfies[10]:

$$(\overline{\nabla}_X \eta)Y = -\eta(X)\eta(Y) \tag{2.19}$$

$$(\overline{\nabla}_X \phi)Y = \eta(Y)\phi X - \eta(X)Y + \eta(X)\eta(Y)\xi - g(X,\phi Y)\xi \tag{2.20}$$

$$\overline{R}(X,Y,Z) = R(X,Y,Z) + g(Y,Z)X - g(X,Z)Y + \eta(Y)\eta(Z)\phi X - \eta(X)\eta(Z)\phi Y \tag{2.21}$$

$$\overline{S}(Y,Z) = S(Y,Z) + (n-1)g(Y,Z) \tag{2.22}$$

$$\overline{r} = r + n(n-1) \tag{2.23}$$

$$\overline{R}(X,Y,Z) + \overline{R}(Y,Z,X) + \overline{R}(Z,X,Y) = 0 \tag{2.24}$$

$$\overline{S}(Y,Z) - \overline{S}(Z,Y) = 0 \tag{2.25}$$

Where $\overline{R}$ and $\overline{S}$ are curvature tensor and ricci tensor of Kenmotsu manifold admitting the quarter-symmetric non metric connection respectively.

3. PROJECTIVE RICCI TENSOR

Definition3.1: Let M be an odd *n*-dimensional Kenmotsu manifold with the quarter-symmetric non metric connection $\overline{\nabla}$. The projective ricci tensor tensor of M with respect to quarter-symmetric non metric connection $\overline{\nabla}$ is defined by

$$\overline{\hat{P}}(X,Y) = \frac{n}{(n-1)}\overline{S}(X,Y) - \frac{\overline{r}}{(n-1)}g(X,Y) \tag{3.1}$$

By using (2.22) and (2.23), we get

$$\overline{\hat{P}}(X,Y) = \hat{P}(X,Y) \tag{3.2}$$

Hence we state:

Theorem1: Projective ricci tensor of Kenmostu manifold with the Riemannian connection ∇ is identical to the projective ricci tensor of Kenmostu manifold admitting the quarter-symmetric non metric connection $\overline{\nabla}$.

From (3.1), we have

$$\overline{\hat{P}}(X,Y)+\overline{\hat{P}}(Y,X)=0 \tag{3.3}$$

From (3.3) we can state:

Theorem2: Projective ricci tensor of Kenmostu manifold admitting the quarter-symmetric non metric connection is symmetric.

Again if $\overline{\hat{P}}=0$, then we have

$$\overline{S}(X,Y)=\frac{\overline{r}}{(n-1)}g(X,Y) \tag{3.4}$$

Therefore we can state:

Theorem3: If projective ricci tensor is flat, then the Kenmostu manifold *M* admitting the quarter-symmetric non metric connection become the Einstein manifold.

4. PSEUDO PROJECTIVE CURVATURE TENSOR

Definition4.1: Let *M* be an odd *n*-dimensional Kenmostu manifold with the quarter-symmetric non metric connection $\overline{\nabla}$, then the pseudo projective curvature tensor of *M* with respect to quarter-symmetric non metric connection $\overline{\nabla}$ is defined by

$$\overline{\tilde{P}}(X,Y)Z = a\overline{R}(X,Y)Z+b\left[\overline{S}(Y,Z)X-\overline{S}(X,Z)Y\right]-\frac{\overline{r}}{n}\left\{\frac{a}{(n-1)}+b\right\}\left[g(Y,Z)X-g(X,Z)Y\right] \tag{4.1}$$

Where *a* and *b* are constant, such that $a,b\neq 0$

By using (4.1), (2.21), (2.22) and (2.23), we have

$$\overline{\tilde{P}}(X,Y)Z = a\left[R(X,Y)Z+\eta(Y)\eta(Z)\phi X-\eta(X)\eta(Z)\phi Y\right]+b\left[S(Y,Z)X-S(X,Z)Y\right]-\frac{r}{(n-1)}\left[\frac{a}{(n-1)}+b\right]\left\{g(Y,Z)X-g(X,Z)Y\right\} \tag{4.2}$$

Using (2.24) and (2.25) in (4.2), we have

$$\overline{\tilde{P}}(X,Y)Z+\overline{\tilde{P}}(Y,Z)X+\overline{\tilde{P}}(Z,X)Y=0 \tag{4.3}$$

Hence we can state:

Theorem 4: In Kenmostu manifold, admitting the quarter-symmetric non metric connection, the pseudo projective curvature tensor is cyclic.

In (4.1) taking inner product with *U* and contracting with *X* and *U,* we get

$$\tilde{S}(Y,Z)= a\overline{S}(Y,Z)+(n-1)b\overline{S}(Y,Z)-\frac{\overline{r}}{n}\left\{\frac{a}{(n-1)}+b\right\}(n-1)g(Y,Z) \tag{4.4}$$

If $\overline{S}=0$ and $\overline{r}=0$, then $\tilde{S}=0$

Now we can state:

Theorem5: In Kenmostu manifold with quarter-symmetric non metric connection, the ricci tensor of pseudo projective curvature is vanishes, if ricci tensor and scalar curvature of quarter-symmetric non metric Kenmostu manifold is vanishes.

If pseudo projective curvature is flat, from (4.1) we have

$$a\overline{R}(X,Y,Z,U)+b\left[\overline{S}(Y,Z)g(X,U)-\overline{S}(X,Z)g(Y,U)\right]= \frac{\overline{r}}{n}\left\{\frac{a}{(n-1)}+b\right\}\left[g(Y,Z)g(X,U)-g(X,Z)g(Y,U)\right] \tag{4.5}$$

In (4.5), we put $X=U=e_i$ and taking summation both sides we obtain

$$\overline{S}(Y,Z)=\frac{\overline{r}}{n}g(Y,Z) \tag{4.6}$$

Hence from (4.6), we can state:

Theorem6: If the pseudo projective curvature tensor of quarter-symmetric non metric Kenmostu manifold is flat then Kenmostu manifold become Einstein manifold.

5. m-PROJECTIVE CURVATURE TENSOR

Definition5.1: Let *M* be an odd *n*-dimensional Kenmostu manifold with the quarter-symmetric non metric connection $\overline{\nabla}$, then the m-projective curvature tensor of *M* with respect to quarter-symmetric non metric connection $\overline{\nabla}$ is defined by

$$\overline{W}^*(X,Y)Z= \overline{R}(X,Y)Z-\frac{1}{2(n-1)}\left[\overline{S}(Y,Z)X-\overline{S}(X,Z)Y+g(Y,Z)\overline{Q}X-g(X,Z)\overline{Q}Y\right] \tag{5.1}$$

From (2.24), (2.25) and (5.1), we obtain

$$\overline{W}^*(X,Y)Z+\overline{W}^*(Y,Z)X+\overline{W}^*(Z,X)Y=0 \tag{5.2}$$

Hence we can state:

Theorem7: In Kenmostu manifold admitting the quarter-symmetric non metric connection, m-projective curvature tensor is cyclic.

If $\overline{S}=0$, then from (5.1) we get

$$\overline{W}^*(X,Y)Z=\overline{R}(X,Y)Z \tag{5.3}$$

Colloary8: If ricci tensor of Kenmostu manifold with the quarter-symmetric non metric connection is vanishes then m-projective curvature tensor and curvature tensor of quarter-symmetric non metric connection are identical.

If $W^*=0$ and taking inner product with *U* in (5.1) we obtain

$$g\left(\overline{R}(X,Y)Z,U\right)= \frac{1}{(n-1)}\left[\overline{S}(Y,Z)g(X,U)-\overline{S}(X,Z)g(Y,U)+\overline{S}(X,U)g(Y,Z)-\overline{S}(Y,U)g(X,Z)\right] \tag{5.4}$$

From (5.4), we have

$$\overline{S}(Y,Z)=\frac{\overline{r}}{n}g(Y,Z) \tag{5.5}$$

Therefore we can state:

Theorem9: Let *M* be the Kenmostu manifold admitting the quarter-symmetric non metric connection. If m-projective curvature of *M* is flat then *M* becomes Einstein manifold.

Again we take the inner product with *U* in (5.1) and put $X=U=e_i$, we get

$$S^*(Y,Z) = \bar{S}(Y,Z) - \frac{1}{2(n-1)}\left[(n-2)\bar{S}(Y,Z) + \bar{r}g(Y,Z)\right] \tag{5.6}$$

Theorem10: If *M* be a Kenmostu manifold admitting a quarter-symmetric non metric connection whose ricci tensor and scalar curvature both vanishes then the ricci tensor with respect to m-projective curvature also vanishes.

6. SOME CURVATURE PROPERTY

Definition6.1: Let *M* be an odd *n*-dimensional Kenmostu manifold admitting the quarter-symmetric non metric connection $\bar{\nabla}$. The concircular curvature tensor of *M* with respect to quarter-symmetric non metric connection is defined by

$$\bar{Z}(X,Y)U = \bar{R}(X,Y)U - \frac{\bar{r}}{n(n-1)}\left[g(Y,U)X - g(X,U)Y\right] \tag{6.1}$$

From (6.1), we can easily find

$$\bar{Z}(X,Y)U + \bar{Z}(Y,U)X + \bar{Z}(U,X)Y = 0 \tag{6.2}$$

Hence we can say,

Theorem11: The concircular curvature tensor of Kenmostu manifold *M* with respect to quarter-symmetric non metric connection is cyclic.

Now if concircular curvature tensor is flat and taking inner product with respect to vector field *V* in (6.1), we obtain

$$\bar{S}(Y,Z) = \frac{\bar{r}}{n}g(Y,Z)$$

Therefore we can state:

Theorem12: Let *M* be the Kenmostu manifold admitting the quarter-symmetric non metric connection. If the concircular curvature tensor is flat then *M* gives the Einstein manifold.

Definition6.2: Let *M* be an odd *n*-dimensional Kenmostu manifold admitting the quarter-symmetric non metric connection $\bar{\nabla}$. The conharmonic curvature tensor of *M* with respect to quarter-symmetric non metric connection is defined by

$$\bar{V}(X,Y)Z = \bar{R}(X,Y)Z - \frac{1}{(n-2)}\left[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\right] \tag{6.3}$$

From (6.3), we have

$$\bar{V}(X,Y)Z + \bar{V}(Y,Z)X + \bar{V}(Z,X)Y = 0 \tag{6.4}$$

Again we can state:

Theorem13: The conharmonic curvature tensor of Kenmostu manifold *M* with respect to quarter-symmetric non metric connection is cyclic.

If $\bar{S} = 0$, then from (6.3) we have

$$\bar{V}(X,Y)Z = \bar{R}(X,Y)Z \tag{6.5}$$

From (5.3) and (6.5), we obtain

$$\bar{V}(X,Y)Z = \bar{W}^*(X,Y)Z \tag{6.6}$$

Now from (6.6), we state:

Theorem14: If ricci tensor of Kenmostu manifold with the quarter-symmetric non metric connection is vanishes then m-projective curvature tensor and concircular curvature tensor of quarter-symmetric non metric connection are identical.

If conharmonic curvature tensor is flat and taking inner product with respect to vector field *U* in (6.3), we obtain

$$\bar{r} = 0 \tag{6.7}$$

Hence finally we can state:

Theorem15: Let *M* be the Kenmostu manifold admitting the quarter-symmetric non metric connection. If the conharmonic curvature tensor of *M* is flat then scalar curvature of Kenmostu manifold admitting the quarter-symmetric non metric connection is vanishes.

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