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Tri-generative Stochastic Model of Population Growth

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ABSTRACT

This paper investigates the population growth of a certain species in which every generation reproduces twice. First we probe the cases of 100% regeneration. We find that the population stabilizes for all the case although the stable values are different and exhibit interesting patterns. Then we study the survival period of a species by randomizing the reproduction probabilities within a window at same predefined ages. Now the population meets three different possibilities when left over for long periods – Dies, Stabilizes, Shoots up. We carry out a detail study for these outcomes in the parameter space defined by the reproduction probabilities.

Indexing terms/Keywords

Population Growth; Probability Space; Random Probability

Academic Discipline And Sub-Disciplines

Mathematics, Stochastic Modeling

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INTRODUCTION

This paper investigates the population growth of a certain population in which every generation reproduces twice. The interest in conducting such a research was generated from the work done in the thesis [1] which proposes a stochastic model to describe the dynamics of human population in a certain environment. The pioneering work of Thomas Robert Malthus in 1798 shaped the early views on human population growth. Malthus argued that any population, if left unrestricted, would grow exponentially. Therefore, human population would continue to grow until it would become too large to be supported by the food grown on available agricultural land. Malthusian model was amended by Verhulst in 1838 by additional limiting term to mark the environmental constraints acting on the growth rate. There were oppositions to Malthusian paradigm from the eighteenth century onward. Marquis de Condorcet, Ester Boserupt, Julian Simon, and Marie-Jean-Antoine-Nicolas Caritat and many others argued that population growth can stimulate technological innovation.

We use the information gained from earlier research on human population dynamics done by previous authors [2, 3, 4] and construct a phenomenological model to estimate future tendencies. Analytical modelling or insight derived from system analysis, combined with simulation, often provides the most informative approach for understanding and interpreting population data. Statistical theory has established that the best predictive power is achieved by relatively simple models with a small or intermediate number of parameters, rather than by complex models, because limited data do not permit accurate estimation of many parameters. Ideal tool for describing human population dynamics would be a model [5, 6], simple enough to be intelligible and complicated enough to be potentially realistic and empirically testable to be credible.

We study the survival period of a species by varying the regeneration probabilities at predefined ages. First we study the cases when whole of population regenerates at the age of 18, 21 and 24. Then we look at more realistic cases when only a fraction regenerates with a certain probability. While in the first case the population always stabilized to a certain value over a period of times, in the latter case the population meets three different outcomes when left over for long periods – Dies, Stabilizes or Shoots up. We carry out a detail study for these possibilities in the parameter space defined by the random reproduction probabilities.

Current paper deals with a rather simplistic picture in form of tri-generative growth. In future we will follow up with more complicated and sophisticated models by extending our regeneration windows to approach a continuous regeneration. However the tri-generative model itself exhibits very interesting results which certainly demand an analytical description.

RANGES FOR BIRTH AND DEATH RATES

We first explain the various ranges we have used to define the birth and death rate for the population

Birth Rate

We start with an initial population and then let the populations reproduce at the age of 18, 21 and 24 with 100% probability. In the first section we study how the population stabilizes in these cases. In the following section we will introduce random probabilities of reproduction at the same predefined ages. Our code uses a uniform random number generator to select this probability within a given window. We provide a probability window to the birth rate at the age of 18 and 21 while we keep the birth rate at 24 to be fixed at 45%.

Death Rate

In our model, each individual in the population dies at the age of 50. In this paper our goal is to study the population dynamics as a function of the birth rates. We plan to introduce variability even in the death rate in follow up papers and analyze how the population dynamics change under those conditions.

The Measurable Parameters

We focus on two parameters to study the influence of different birth rates and initial population – 1. The stable population reached by a species for the cases when it does become stable 2. The bounded regions in the parameter space where population either becomes stable or die or shoots up.

POPULATION SURVIVAL WITH FIXED PROBABILITIES FOR REGENERATION

We first simulate the growth dynamics of an initial population of 50. Figure 1 below shows how the population changes with time:

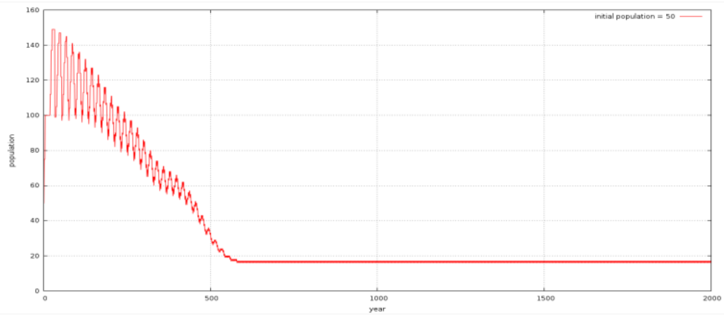

Fig 1: Population change with initial value 50

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The most important fact we notice here is that the population stabilizes at an average of 16.6. As we will show the larger initial populations, the stable population reached is almost always larger than the initial population. In the initial population we started we equal number of males and females. As each generation is equally likely to be a male or a female, the ratio maintains through out the process. Also as we were anticipating, the average age of the population once it stabilizes is 24.5, which is just half the maximum age possible for any individual. We run our simulation for 2000 years while the stable value is reached much earlier in around 500 years.

We further run simulation with different initial population. We illustrate the cases with initial populations 500 and 5000. For these cases with fixed generative rates, the peak population which is reached is always 3 times the initial population. This is evident from Figure 1, 2 & 3. However the final stable population is a function of initial population. For all the cases we considered, except for the case discussed above, the stable value reached is larger than the initial population.

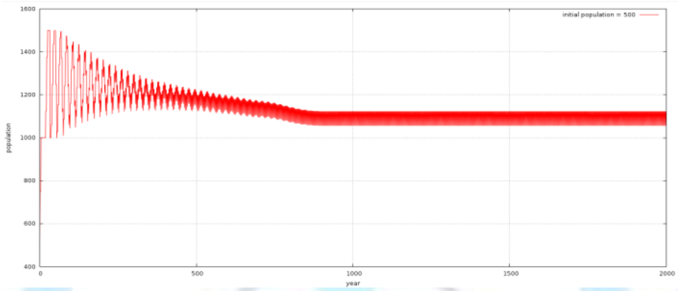

Fig 2: Population change with initial value 500

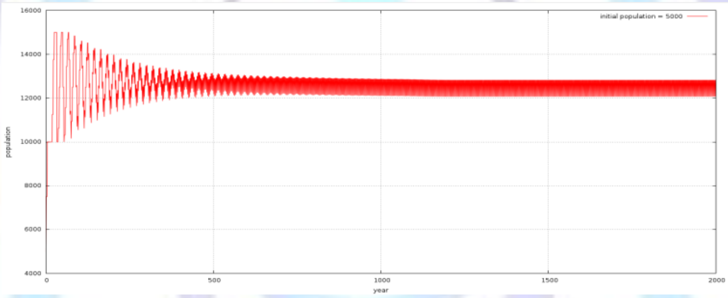

Fig 3: Population change with initial value 5000

POPULATION SURVIVAL WITH STOCHASTIC PROBABILITIES FOR REGENERATION

In the previous section we discussed the growth of a given population when the regeneration at age of 18 and 21 takes place with 100% probability and 45% probability at the age of 24. Now we investigate more realistic scenarios when the generation rates at the given two ages are not fixed. That means that not all the population at the age of 18 or 21 would regenerate but only a fraction of them. We define a window for this fraction and let the code pick a random value within that window during every run. This uncertainty makes the model more realistic and also gives more interesting results.

An important outcome of this uncertainty is that a population does not always reach a stable value. Instead depending on the different regenerative rates at 18 and 21, the population may stabilize, die or shoot off. We define our parameter space using the regenerative rate windows at the two ages, 18 & 21, and find the regions in which the population may end with different outcomes. To make our simulations more robust, we average our results over 1000 runs. Averaging washes away any outcomes which may be very unlikely.

We first define our windows in the following manner. When at the age of 18, 75-80% of the population regenerates while at the age of 21, 85-90% of the population regenerates, we use the notation (75-80, 85-90). We investigate the different windows for different initial populations. Following are the figures for initial population of 100 and windows (75-80, 75-80), (75-80, 80-85), (75-80. 85-90) & (75-80, 90-95):

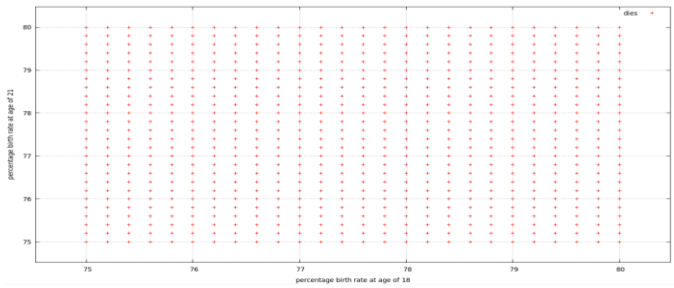

Fig 4: Initial Population 100 with probabiliy window (75-80, 75-80)

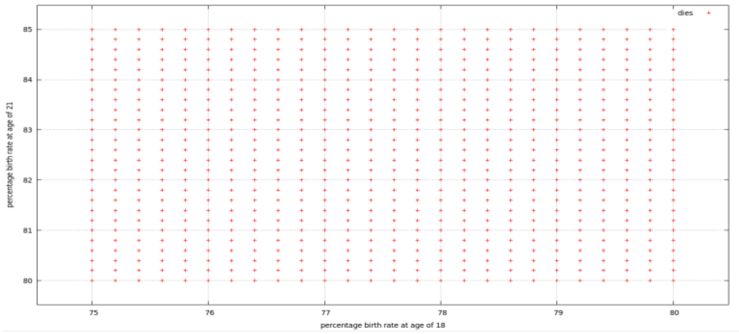

Fig 5: Initial Population 100 with probabiliy window (75-80, 80-85)

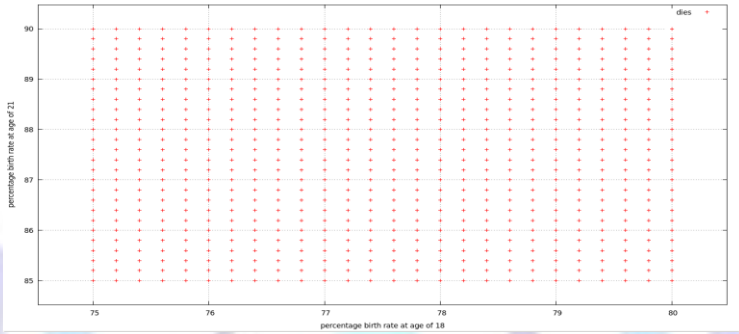

Fig 6: Initial Population 100 with probabiliy window (75-80, 85-90)

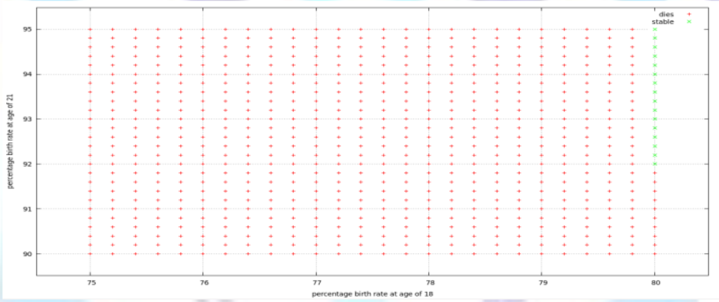

Fig 7: Initial Population 100 with probabiliy window (75-80, 90-95)

The red dots in the figures mean that the population doesn't survive and goes extinct after a certain period, while the green dots imply that the population stabilizes after some time. Evidently the populations don't survive for low probability windows. For the three cases, (75-80, 75-80), (75-80, 80-85), (75-80. 85-90), populations always go extinct. However for the case (75-80. 90-95), the population graph shows few green dots too. A portion of the last column has green dots and population actually stabilizes over the range (80, 92) to (80, 95). We move a slightly left or below to this portion and the population dies after some period. This is a sort of phase transition in the probability parameter space.

This sudden change in the outcome with a minute change in probabilities reflects a sort of phase transition. This is depicted in the graph below which shows how the population evolves in two different cases.

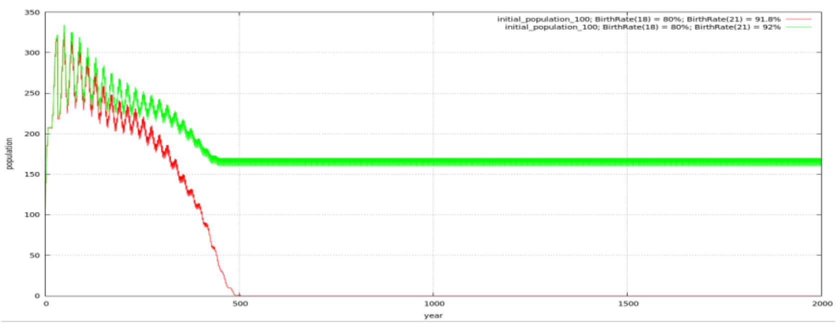

Fig 8: Initial Population 100 with probabilities (80, 91.8) & (80, 92)

The red population which has a regeneration probability of 91.8 at 21 dies within around 500 years. However the green population which has a slightly higher probability of 92 at 21 survives and stabilizes. Consequently the probability

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becomes a criticial variable near the boundary of red and green dots in Fig 7. The critical values are definitely model dependent and carry a substantial amount of information about the model.

Clearly there is a significant dependence on the initial population also. A larger initial population always enhances the chance of survival. To illustrate this, we run the simulations for window (75-80, 85-90) as above but for an initial population of 500 & 1000. Following are the graphs for these initial populations:

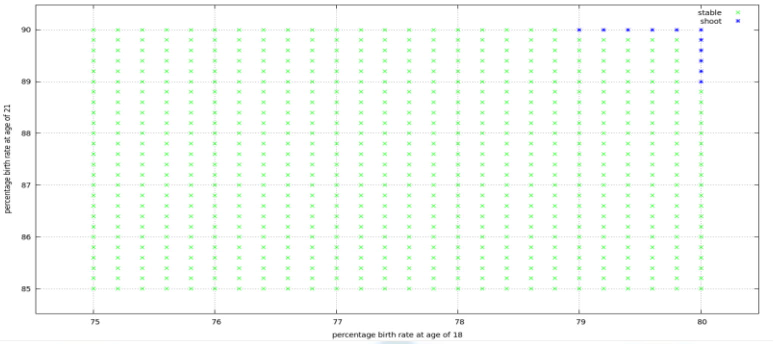

Fig 9: Initial Population 500 with probabiliy window (75-80, 85-90)

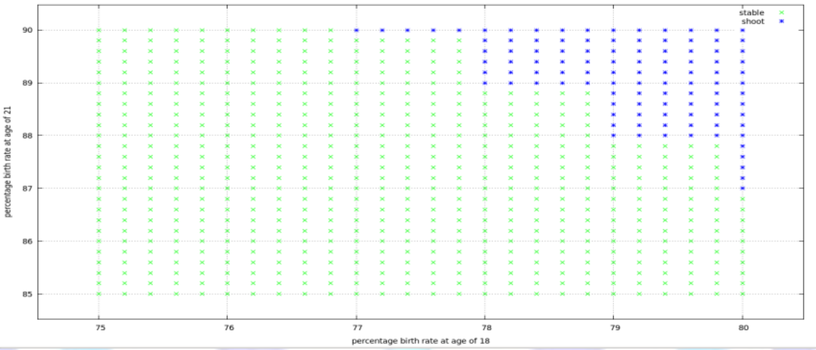

Fig 10: Initial Population 1000 with probabiliy window (75-80, 85-90)

Now comparing the figure 6, 9 & 10, we see how the initial population increase the chances of the population surviving. While with initial value of 100 the population dies, it survives for initial value of 500 and 1000. In fact in figure 9 we see how the last column has blue dots which mean that the population shoots up. As we go to initial value of 1000, the blue patch moves further inwards and population is more likely to shoot up.

We further study the asymmetry between the two ages of regeneration. The two windows (75-80, 85-90) and (85-90, 75-80) are just mirror images of each other. One is constructed by flipping the probability windows at two different ages. We run simulations with flipped windows and study how the survival probability of a population changes. We first consider the two windows mentioned above, (75-80, 85-90) and (85-90, 75-80), for initial population of 500. The following fig. 11 displays the graph for the window (85-90, 75-80) while figure 8 shows the graph for the flipped window. It is clearly visible that both graphs are exactly the same in these two different cases. We also checked the flipped graphs for same window for an initial population of 100 and they also look the same.

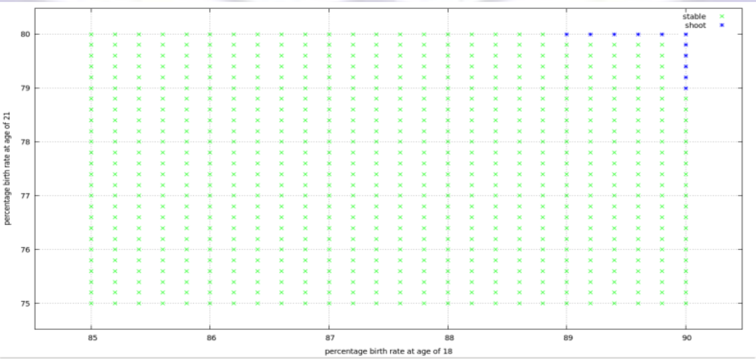

Fig 11: Initial Population 500 with probabiliy window (85-90, 75-80)

However this symmetry is not retained for larger initial populations. To prove this, we run simulation for the flipped window (85-90, 75-80) with an initial population of 1000. The graph below shows the outcome:

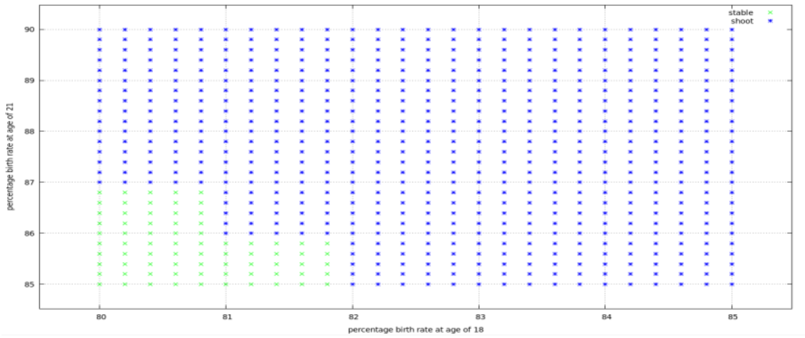

Fig 12: Initial Population 1000 with probabiliy window (85-90, 75-80)

If we compare the outcome in Figure 12 with one in Figure 10, it is evident that there is an asymmetry between the two flipped cases. Apparently if we allow for a higher regeneration at the age of 18 the population is more likely to grow exponentially. It poses an important question as to why this asymmetry appears only for larger initial population.

CONCLUSIONS

In this model we discovered quite a few interesting results. The model proves that the evolution of any population depends on its initial value. Further we found that a population may very well go extinct if the birth rate goes below a minimum threshold. This threshold value depends on the initial population as well as the dynamics. A very interesting fact we notice is that the birth rate has a critical value for a given system and a slight change around this value can disrupt the outcome. A theoretical explaination of such a behavior will be helpful in dealing with ecological systems.

Another observation made in the model is the asymmetry among flipped windows. This explains that birth rates at different ages have different effects on the population. A higher birth rate an lower ages lead to higher population growth. We plan to upgrade this model by incorporating or tuning on other variable like death rate, sex ratio etc.

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