ISSN 2347-1921

TRIPLED COINCIDENCE POINTS FOR WEAKLY COMPATIBLE AND ITS VARIANTS IN FUZZY METRIC SPACES

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ABSTRACT: In this paper, we will introduce the concept of weakly commuting and variants of weakly commuting mappings (R-weakly commuting, R-weakly commuting of type (Af), type (Ag), type (P) mappings) for triplet in fuzzy metric spaces. Secondly, we introduce the notion of weakly compatibility and its variants weakly f-compatible maps and weakly g-compatible maps. At the end, we prove common fixed point theorems for a pair of weakly compatible map and their variants, which generalize the results of various authors present in fixed point theory literature. Our result is validated with a suitable example.

Indexing terms/Keywords

Tripled fixed point; fuzzy metric space; weakly compatible mappings.

Academic Discipline And Sub-Disciplines

Mathematics.

SUBJECT CLASSIFICATION

2010 AMS Classification. 47H10, 54H25.

TYPE (METHOD/APPROACH)

Fixed point theory and various mappings.

INTRODUCTION

Fixed point theory has been remained an important area of research for mathematicians. From Banach contraction principle to upto now much have been invented, applied, generalized in this particular direction. After a long research in fixed point theorems and their applications focus is now on coupled and tripled fixed point theory. In 2006, Bhaskar and Lakshmikantham [1] discussed the mixed monotone mappings and gave some coupled fixed point theorems which can be used to discuss the existence and uniqueness of solution for a periodic boundary value problem.

The concept of *tripled fixed point* has been introduced by Berinde and Borcut in 2011. In their manuscript, some new tripled point theorems are obtained using the mixed *g*-monotone mapping. Their results generalize and extend the Bhaskar and Lakshmikantham's research for nonlinear mappings. Moreover, these results could be used to study the existence of solutions of a periodic boundary value problem involving $y = f(t, y, y)$.

It is well known that a fuzzy metric space is an important generalization of the metric space. Many authors have considered this problem and have introduced it in different ways. For instance, George and Veeramani [11] modified the concept of a fuzzy metric space introduced by Kramosil and Michalek [18] and defined the Hausdorff topology of a fuzzy metric space. There exists considerable literature about fixed point properties for mappings defined on fuzzy metric spaces, which have been studied by many authors (see [5,7,8,13-19]). Zhu and Xiao [21] and Hu [13,14] gave a coupled fixed point theorem for contractions in fuzzy metric spaces, and Fang [7, 8] proved some common fixed point theorems under φ-contractions for compatible and weakly compatible mappings on probabilistic metric spaces.

In this paper, we give a new tripled fixed point theorem under weaker conditions than in [17] and give an example, which shows that the result is a genuine generalization of the corresponding result in [17].

2.PRELIMINARIES

Before proceeding towards our main result we will give some preliminaries:

Henceforth, X will denote a non-empty set and $X^3 = X \times X \times X$. Subscripts will be used to indicate the arguments of a function. For instance, $F(x, y, z)$ will be denoted by $Fxyz$ and $M(x, y, t)$ will be denoted by $Mxy(t)$. Furthermore, for brevity, $g(x)$ will be denoted by gx, metric space will be denoted by MS.

Definition 2.1 [17] Let (X, d) be a MS. A mapping $f: X \to X$ is said to be *Lipschitzian* if there exists $k \geq 0$ such that $d(fx, fy) \leq kdxy$ for all $x, y \in X$. The smallest k (denoted by kf) for which this inequality holds is said to be the *Lipschitz constant for f*. A Lipschitzian mapping $f: X \to X$ is a *contraction* if $kf < 1$.

Definition 2.2 [12] A triangular norm (also called a t-norm) is a map $*: [0, 1]^2 \to [0, 1]$ that is associative,commutative, nondecreasing in both arguments and has 1 as identity. For each a ∈ [0, 1], the sequence $\{* na\}_{n=1}^{\infty}$ is defined inductively by $*^1 a = a$ and $*^n a = (*^{n-1} a) * a$. A t-norm $*$ is said to be of H-type if the sequence $\{* na\}_{n=1}^{\infty}$ is equicontinuous at a = 1, i.e., for all $\varepsilon \in (0, 1)$, there exists $\eta \in (0, 1)$ such that if $a \in (1 - \eta, 1]$, then $*^m a > 1 - \varepsilon$ for all $m \in$ N.

The most important and well-known continuous t-norm of H-type is $* = \min$, that verifies $\min(a, b) \geq ab$ for all $a, b \in [0, 1]$. The following result presents a wide range of t-norms of H-type.

Lemma 2. 3 Let $\delta \in (0, 1]$ be a real number and let $*$ be a t-norm. Define $*\delta$ as $x *\delta\, y = x * y$, if $\max(x, y) \leq 1 - \delta$, and $x *\delta\, y = \min(x, y)$, if $\max(x, y) > 1 - \delta$. Then $*\delta$ is a t-norm of H-type.

Definition 2. 4 [18] A triplet $(X,M, *)$ is called a fuzzy metric space (in the sense of Kramosil and Michalek; briefly, a FMS) if X is an arbitrary non-empty set, $*$ is a continuous t-norm and $M : X\times X\times[0,\infty) \to [0, 1]$ is a fuzzy set satisfying the following conditions, for each $x, y, z \in X$, and $t, s > 0$:

(i) $M_{xy}(0) = 0$;

(ii) $M_{xy}(t) = 1$ if and only if $x = y$;

(iii) $M_{xy}(t) = M_{yx}(t)$;

(iv) $M_{xy}(\cdot) : [0,\infty) \to [0, 1]$ is left continuous;

(v) $M_{xy}(t) * M_{yz}(s) \leq M_{xz}(t + s)$.

In this case, we also say that (X, M) is a FMS under $*$. In the sequel, we will only consider FMS verifying:

(vi) $\lim_{t\to\infty} M_{xy}(t) = 1$ for all $x, y \in X$.

Lemma 2. 5

[17] $Mxy(\cdot)$ is a non-decreasing function on $[0,\infty)$.

FMS is said to be continuous at a point $x_0 \in X$ if, for any sequence $\{x_n\}$ in X converging to x_0, the sequence $\{gx_n\}$ converges to gx_0. If g is continuous at each $x \in X$, then g is said to be continuous on X. As usual, if $x_0 \in X$, we will denote $g{-}1(x_0) = \{x \in X: gx = x_0\}$.

Now, we will define fixed point, coincidence and weakly compatibility and its variants for the case of triplet.

Definition 2. 6. An element (x, y, z) $\in$ X × X × X is called a tripled fixed point of F: X × X × X → X if F (x, y, z) = x, F (y, z, x) = y, and F (z, x, y) = z.

Definition 2.7. An element (x, y,z) $\in$ X× X× X is called a tripled coincidence point of mappings F: X × X× X → X and g: X→X if

F(x,y,z) = g (x), F (y,z,x) = g (y) and F (z,x,y) = g (z).

In 1994, Mishra [20] introduced the concept of compatible mappings in fuzzy metric spaces akin to the concept of compatible mapping in metric spaces, see [8]. In 1994, Pant [21] introduced the concept of R-weakly commuting maps in metric spaces. Later on, Vasuki [28] initiated the concept of non compatible of mapping in fuzzy metric spaces and introduced the notion of R-weakly commuting mappings in fuzzy metric spaces and proved some common fixed point theorems for R-weakly commuting maps in the fuzzy metric space. Further, Pathak et al. [22] generalized the concept of R-weakly commuting maps of type (Ag) and type (Af) as follows.

Definition 2.8 A pair of self-mappings (f,g) of a fuzzy metric space (X,M,$*$) is said to be

i. weakly commuting if M(fgx,gfx,t) ≥ M(fx,gx,t).

ii. R-weakly commuting if there exists some R > 0 such that

M(fgx,gfx,t) ≥ M(fx,gx,t/R).

iii. R-weakly commuting mappings of type (Af) if there exists some R > 0 such that

M(fgx,ggx,t) ≥ M(fx,gx,t/R).

iv. R-weakly commuting mappings of type (Ag) if there exists some R > 0 such that

M(gfx,ffx,t) ≥ M(fx,gx,t/R).

In 2006, Imdad and Javid Ali [15] introduced the definition of R-weakly commuting mappings of type (P) as follow. A pair of self-mappings (f,g) of a fuzzy metric space (X,M,$*$) is said to be R-weakly commuting mappings of type (P) if there exists some R > 0 such that

M(ffx,ggx,t) ≥ M(fx,gx,t/R), for all x ∈ X and t > 0.

Now we introduce the following notions for tripled mappings.

Definition 2.9 The mappings $f : X \times X\times X \to X$ and $g : X \to X$ are said to be weakly commuting if

M(f(gx,gy,gz),gf(x,y,z),t) ≥ M(f(x,y,z),gx,t),

M(f(gy,gz,gx),gf(y,z,x),t) ≥ M(f(y,z,x),gy,t)

M(f(gz,gx,gy),gf(z,x,y),t) ≥ M(f(z,x,y),gz,t)

for all x,y,z in X and t > 0.

Definition 2.10 The mappings $f : X \times X\times X \to X$ and $g : X \to X$ are said to be

(i) R-weakly commuting if there exists some R > 0 such that

M(f(gx,gy,gz),gf(x,y,z),t) ≥ M(f(x,y,z),gx,t/R),

M(f(gy,gz,gx),gf(y,z,x),t) ≥ M(f(y,z,x),gy,t/R)

M(f(gz,gx,gy),gf(z,x,y),t) ≥ M(f(z,x,y),gz,t/R).

for all x,y,z in X and t > 0.

(ii) R-weakly commuting maps of type (*Af*) if there exists some R > 0 such that *M(f(gx,gy,gz),ggx,t) ≥ M(f(x,y,z),gx,t/R),*

M(f(gy,gz,gx),ggy,t) ≥ M(f(y,z,x),gy,t/R)

M(f(gz,gx,gy),ggz,t) ≥ M(f(z,x,y),gz,t/R).

for all x,y,z in X and t > 0.

(iii) R-weakly commuting maps of type (*Ag*) if there exists some R > 0 such that

M(gf(x,y,z),f(f(x,y,z),f(y,z,x),f(z,x,y),t) ≥ M(f(x,y,z),gx,t/R),

M(gf(y,z,x),f(f(y,z,x),f(z,x,y),f(x,y,z),t) ≥ M(f(y,z,x),gy,t/R)

M(gf(z,x,y),f(f(z,x,y),f(x,y,z),f(y,z,x),t) ≥ M(f(z,x,y),gz,t/R).

for all x,y,z in X and t > 0.

(iv) R-weakly commuting maps of type (*P*) if there exists some R > 0 such that

M(f(f(x,y,z),f(y,z,x),f(z,x,y), ggx,t) ≥ M(f(x,y,z),gx,t/R), *M(f(f(y,z,x),f(z,x,y),f(x,y,z), ggy,t) ≥ M(f(y,z,x),gy,t/R)*

M(f(f(z,x,y),f(x,y,z),f(y,z,x), ggz,t) ≥ M(f(z,x,y),gz,t/R).

for all x,y,z in X and t > 0.

Definition 2.11 The mappings F and g where $F: X \times X \times X \to X$ and $g: X \to X$ are said to be weakly compatible if $F(x,y,z) = g(x)$, $F(y,z,x) = g(y)$ and $F(z,x,y) = g(z)$ implies that

$gF(x, y, z) = F(gx, gy,gz)$, $gF(y, z, x) = F(gy, gz, gx)$ and $gF(z, x,y) = F(gz, gx,gy)$ for all $x, y, z \in X$, that is, the mappings commute at coincidence point.

Remark 2.12 It is easy to prove that if F and g are compatible, then they are weakly compatible,

but the converse need not be true. See the example in the next section.

Definition 2.13 The mappings $f : X \times X \times X \to X$ and $g : X \to X$ are said to be weakly f-compatible if either

$\lim_{n\to\infty} gf(x_n, y_n, z_n) = f(x,y,z)$, $\lim_{n\to\infty} gf(y_n, z_n, x_n) = f(y,z,x)$, $\lim_{n\to\infty} gf(z_n, x_n, y_n) = f(z,x,y)$,

or

$\lim_{n\to\infty} ggx_n = f(x,y,z)$, $\lim_{n\to\infty} ggy_n = f(y,z,x)$, $\lim_{n\to\infty} ggz_n = f(z,x,y)$.

whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $\lim_{n\to\infty} f(x_n,y_n,z_n) = \lim_{n\to\infty} g(x_n) = x$,

$\lim_{n\to\infty} f(y_n,z_n,x_n) = \lim_{n\to\infty} g(y_n) = y$, $\lim_{n\to\infty} f(z_n,x_n,y_n) = \lim_{n\to\infty} g(z_n) = z$.

and

$\lim_{n\to\infty} f(gx_n,gy_n,gz_n) = \lim_{n\to\infty} f(f(x_n, y_n, z_n), f(y_n, z_n, x_n), f(z_n, x_n, y_n)) = f(x,y,z)$,

$\lim_{n\to\infty} f(gy_n,gz_n,gx_n) = \lim_{n\to\infty} f(f(y_n, z_n, x_n), f(z_n, x_n, y_n), f(x_n, y_n, z_n)) = f(y,z,x)$,

$\lim_{n\to\infty} f(gz_n,gx_n,gy_n) = \lim_{n\to\infty} f(f(z_n, x_n, y_n), f(x_n, y_n, z_n), f(y_n, z_n, x_n)) = f(z,x,y)$.

for some $x,y,z \in X$.

Definition 2.14 The mappings $f : X \times X \times X \to X$ and $g : X \to X$ are said to be weakly g-compatible if either

$\lim_{n\to\infty} f(gx_n,gy_n,gz_n) = gx$, $\lim_{n\to\infty} f(gy_n,gz_n,gx_n) = gy$ and $\lim_{n\to\infty} f(gz_n,gx_n,gy_n) = gz$

or

$\lim_{n\to\infty} f(f(x_n, y_n, z_n), f(y_n, z_n, x_n), f(z_n, x_n, y_n)) = gx$,

$\lim_{n\to\infty} f(f(y_n, z_n, x_n), f(z_n, x_n, y_n), f(x_n, y_n, z_n)) = gy$,

$\lim_{n\to\infty} f(f(z_n, x_n, y_n), f(x_n, y_n, z_n), f(y_n, z_n, x_n)) = gz$.

whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $\lim_{n\to\infty} f(x_n,y_n,z_n) = \lim_{n\to\infty} g(x_n) = x$,

$\lim_{n\to\infty} f(y_n,z_n,x_n) = \lim_{n\to\infty} g(y_n) = y$, $\lim_{n\to\infty} f(z_n,x_n,y_n) = \lim_{n\to\infty} g(z_n) = z$.

and

$\lim_{n\to\infty} gf(x_n,y_n,z_n) = \lim_{n\to\infty} gg(x_n) = gx$,

$\lim_{n\to\infty} gf(y_n,z_n,x_n) = \lim_{n\to\infty} gg(y_n) = gy$, $\lim_{n\to\infty} gf(z_n,x_n,y_n) = \lim_{n\to\infty} gg(z_n) = gz$.

for some $x,y,z \in X$.

3. MAIN RESULTS

Recently, we have proved the following results for tripled fixed point in fuzzy metric spaces:

Theorem 3.1 [17] Let $*$ be a t-norm of H-type such that $s * t \geq st$ for all $s, t \in [0, 1]$. Let $k \in (0, 1)$ and $a, b, c \in [0, 1]$ be real numbers such that $a + b + c \leq 1$, let $(X,M, *)$ be a complete FMS and let $F : X^3 \to X$ and $g : X \to X$ be two mappings such that $F(X^3) \subseteq g(X)$ and g is continuous and F and g are compatible.

Suppose that for all $x, y, z, u, v,w \in X$ and all $t > 0$,

$M_{FxyzFuvw}(kt) \geq M_{gxgu}(t)^a * M_{gygv}(t)^b * M_{gzgw}(t)^c$.

Then there exists a unique $x \in X$ such that $x = gx = F_{xxx}$. In particular, F and g have, at least, one tripled coincidence point. Furthermore, (x, x, x) is the unique tripled coincidence point of F and g if we assume that $g{-}1(x_0) = \{x_0\}$ only in the case that $F \equiv x_0$ is constant on X^3.

In this result, in order to avoid the indetermination 0^0, we assume that $M_{gxgu}(t)^0 = 1$ for all $t > 0$ and all $x, y \in X$.

Now we are ready to prove our results for weakly compatible mappings.

Theorem 3.2 Let $*$ be a t-norm of H-type such that $s * t \geq st$ for all $s, t \in [0, 1]$. Let $k \in (0, 1)$ and $a, b, c \in [0, 1]$ be real numbers such that $a + b + c \leq 1$, let $(X, M, *)$ be a FMS and let $F: X^3 \to X$ and $g: X \to X$ be two weakly compatible mappings such that $F(X^3) \subseteq g(X)$ and $F(X^3)$ or $g(X)$ is complete.

Suppose that for all $x, y, z, u, v, w \in X$ and all $t > 0$,

$M_{FxyzFuvw}(kt) \geq M_{gxgu}(t)^a * M_{gygv}(t)^b * M_{gzgw}(t)^c$. (1)

Then there exists a unique $x \in X$ such that $x = gx = F_{xxx}$. In particular, F and g have, at least, one tripled coincidence point.

Proof . Throughout this proof, n and p will denote non-negative integers and $t \in [0, \infty)$.

Step 1. Definition of the sequences $\{x_n\}$, $\{y_n\}$ *and* $\{z_n\}$. Let $x_0, y_0, z_0 \in X$ be three arbitrary points of X. Since $F(X^3) \subseteq g(X)$, we can choose $x_1, y_1, z_1 \in X$ such that $gx_1 = F_{x_0y_0z_0}$, $gy_1 = F_{y_0z_0x_0}$ and $gz_1 = F_{z_0x_0y_0}$. Again, from $F(X^3) \subseteq g(X)$, we can choose $x_2, y_2, z_2 \in X$ such that $gx_2 = F_{x_1y_1z_1}$, $gy_2 = F_{y_1z_1x_1}$ and $gz_2 = F_{z_1x_1y_1}$. Continuing this process, we can construct sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ such that, for $n \geq 0$, $gx_{n+1} = F_{x_ny_nz_n}$, $gy_{n+1} = F_{y_nz_nx_n}$ and $gz_{n+1} = F_{z_nx_ny_n}$.

Step 2. $\{gx_n\}$, $\{gy_n\}$ *and* $\{gz_n\}$ *are Cauchy sequences*. Define, for $n \geq 0$ and all $t \geq 0$,

$\delta_n(t) = M_{gx_n gx_{n+1}}(t) * M_{gy_n gy_{n+1}}(t) * M_{gz_n gz_{n+1}}(t)$.

Since δ_n is a non-decreasing function and $t - kt \leq t \leq t/k$, we have that

$\delta_n(t - kt) \leq \delta_n(t) \leq \delta_n(t/k)$, for all $t > 0$ and $n \geq 0$. (2)

From inequality (1) we deduce, for all $n \in \mathbb{N}$ and all $t \geq 0$

$$M_{gx_n gx_{n+1}}(t) = M_{F_{x_{n-1}y_{n-1}z_{n-1}}F_{x_ny_nz_n}}(t) \geq M_{gx_{n-1}gx_n}\left(\frac{t}{k}\right)^a * M_{gy_{n-1}gy_n}\left(\frac{t}{k}\right)^b * M_{gz_{n-1}gz_n}\left(\frac{t}{k}\right)^c \quad (3)$$

$$M_{gy_n gy_{n+1}}(t) = M_{F_{y_{n-1}z_{n-1}x_{n-1}}F_{y_nz_nx_n}}(t) \geq M_{gy_{n-1}gy_n}\left(\frac{t}{k}\right)^a * M_{gz_{n-1}gz_n}\left(\frac{t}{k}\right)^b * M_{gx_{n-1}gx_n}\left(\frac{t}{k}\right)^c \quad (4)$$

$$M_{gz_n gz_{n+1}}(t) = M_{F_{z_{n-1}x_{n-1}y_{n-1}}F_{z_nx_ny_n}}(t) \geq M_{gz_{n-1}gz_n}\left(\frac{t}{k}\right)^a * M_{gx_{n-1}gx_n}\left(\frac{t}{k}\right)^b * M_{gy_{n-1}gy_n}\left(\frac{t}{k}\right)^c \quad (5)$$

According to (3), (4), (5) we have that

$$M_{gx_n gx_{n+1}}(t) \geq M_{gx_{n-1}gx_n}\left(\frac{t}{k}\right)^a * M_{gy_{n-1}gy_n}\left(\frac{t}{k}\right)^b * M_{gz_{n-1}gz_n}\left(\frac{t}{k}\right)^c \geq$$

$$\geq M_{gx_{n-1}gx_n}\left(\frac{t}{k}\right) * M_{gy_{n-1}gy_n}\left(\frac{t}{k}\right) * M_{gz_{n-1}gz_n}\left(\frac{t}{k}\right) = \delta_{n-1}(t/k);$$

$$M_{gy_n gy_{n+1}}(t) \geq M_{gy_{n-1}gy_n}\left(\frac{t}{k}\right)^a * M_{gz_{n-1}gz_n}\left(\frac{t}{k}\right)^b * M_{gx_{n-1}gx_n}\left(\frac{t}{k}\right)^c \geq$$

$$\geq M_{gy_{n-1}gy_n}\left(\frac{t}{k}\right) * M_{gz_{n-1}gz_n}\left(\frac{t}{k}\right) * M_{gx_{n-1}gx_n}\left(\frac{t}{k}\right) = \delta_{n-1}(t/k);$$

$$M_{gz_n gz_{n+1}}(t) \geq M_{gz_{n-1}gz_n}\left(\frac{t}{k}\right)^a * M_{gx_{n-1}gx_n}\left(\frac{t}{k}\right)^b * M_{gy_{n-1}gy_n}\left(\frac{t}{k}\right)^c \geq$$

$$\geq M_{gz_{n-1}gz_n}\left(\frac{t}{k}\right) * M_{gx_{n-1}gx_n}\left(\frac{t}{k}\right) * M_{gy_{n-1}gy_n}\left(\frac{t}{k}\right) = \delta_{n-1}(t/k).$$

This proves that, for all $t > 0$ and all $n \geq 0$,

$$M_{gx_n gx_{n+1}}(t), M_{gy_n gy_{n+1}}(t), M_{gz_n gz_{n+1}}(t) \geq \delta_{n-1}(t/k) \geq \delta_{n-1}(t). \quad (6)$$

Swapping t by $t - kt$, we deduce, for all $t > 0$ and $n \geq 0$, that

$$M_{gx_n gx_{n+1}}(t-kt), M_{gy_n gy_{n+1}}(t-kt), M_{gz_n gz_{n+1}}(t-kt) \geq \delta_{n-1}(t-kt). \quad (7)$$

Taking into account that $*$ is commutative and $* \geq \cdot$, and (3), (4), (5), we observe that

ISSN 2347-1921

$\delta_n(t) = M_{gx_n gx_{n+1}}(t) * M_{gy_n gy_{n+1}}(t) * M_{gz_n gz_{n+1}}(t)$

$\geq (M_{gx_{n-1}gx_n}(\frac{t}{k})^a * M_{gy_{n-1}gy_n}(\frac{t}{k})^b * M_{gz_{n-1}gz_n}(\frac{t}{k})^c) *$

$*(M_{gx_{n-1}gx_n}(\frac{t}{k})^c * M_{gy_{n-1}gy_n}(\frac{t}{k})^a * M_{gz_{n-1}gz_n}(\frac{t}{k})^b) *$

$*(M_{gx_{n-1}gx_n}(\frac{t}{k})^b * M_{gy_{n-1}gy_n}(\frac{t}{k})^c * M_{gz_{n-1}gz_n}(\frac{t}{k})^a) * =$

$= (M_{gx_{n-1}gx_n}(\frac{t}{k})^a * M_{gx_{n-1}gx_n}(\frac{t}{k})^c * M_{gx_{n-1}gx_n}(\frac{t}{k})^b) *$

$*(M_{gy_{n-1}gy_n}(\frac{t}{k})^b * M_{gy_{n-1}gy_n}(\frac{t}{k})^a * M_{gy_{n-1}gy_n}(\frac{t}{k})^c) *$

$*(M_{gz_{n-1}gz_n}(\frac{t}{k})^c * M_{gz_{n-1}gz_n}(\frac{t}{k})^b * M_{gz_{n-1}gz_n}(\frac{t}{k})^a) * \geq$

$\geq (M_{gx_{n-1}gx_n}(\frac{t}{k})^a . M_{gx_{n-1}gx_n}(\frac{t}{k})^c . M_{gx_{n-1}gx_n}(\frac{t}{k})^b) *$

$*(M_{gy_{n-1}gy_n}(\frac{t}{k})^b . M_{gy_{n-1}gy_n}(\frac{t}{k})^a . M_{gy_{n-1}gy_n}(\frac{t}{k})^c) *$

$*(M_{gz_{n-1}gz_n}(\frac{t}{k})^c . M_{gz_{n-1}gz_n}(\frac{t}{k})^b . M_{gz_{n-1}gz_n}(\frac{t}{k})^a) =$

$= M_{gx_{n-1}gx_n}(\frac{t}{k})^{a+b+c} * M_{gy_{n-1}gy_n}(\frac{t}{k})^{a+b+c} * M_{gz_{n-1}gz_n}(\frac{t}{k})^{a+b+c} \geq$

$\geq M_{gx_{n-1}gx_n}(\frac{t}{k}) * M_{gy_{n-1}gy_n}(\frac{t}{k}) * M_{gz_{n-1}gz_n}(\frac{t}{k}) = \delta_{n-1}(t/k)$.

If we join this property to (2),

$\delta_n(t) \geq \delta_{n-1}(t/k) \geq \delta_{n-1}(t) \geq \delta_{n-1}(t - kt)$, for all $t > 0$ and $n \geq 1$. (8)

Repeatedly applying the first inequality, we deduce that $\delta_n(t) \geq \delta_{n-1}(t/k) \geq \delta_{n-2}(t/k^2) \geq . . \geq \delta_0\, t/k^n)$

for all $t > 0$ and $n \geq 1$. This means that for all $t > 0$,

$\lim_{n\to\infty}\delta_n(t) \geq \lim_{n\to\infty}\delta_0(t/k^n) = 1$ implies $\lim_{n\to\infty}\delta_n(t) = 1$. (9)

Properties (6) and (8) imply that

$M_{gx_n gx_{n+1}}(t), M_{gy_n gy_{n+1}}(t), M_{gz_n gz_{n+1}}(t) \geq \delta_n(t) \geq \delta_{n-1}(t-kt)$. (10)

Next, we claim that

$M_{gx_n gx_{n+p}}(t), M_{gy_n gy_{n+p}}(t), M_{gz_n gz_{n+p}}(t) \geq *^p\, \delta_{n-1}(t-kt)$, for all $t > 0$, $n,p \geq 1$. (11)

We prove it by induction methodology in $p \geq 1$. If $p = 1$, (11) is true for all $n \geq 1$ and all $t > 0$ by (10).

Suppose that (11) is true for all $n \geq 1$ and all $t > 0$ for some p, and we are going to prove it for $p+1$.Applying (1), the induction hypothesis and that $* \geq \cdot$,

$M_{gx_{n+1}gx_{n+p+1}}(kt) = M_{F_{x_n y_n z_n} F_{x_{n+p} y_{n+p} z_{n+p}}}(kt) \geq$

$\geq M_{gx_n gx_{n+p}}(t)^a * M_{gy_n gy_{n+p}}(t)^b * M_{gz_n gz_{n+p}}(t)^c \geq$

$\geq (*^p\, \delta_{n-1}(t-kt))^a * (*^p\, \delta_{n-1}(t-kt))^b * (*^p\, \delta_{n-1}(t-kt))^c \geq$

$\geq (*^p\, \delta_{n-1}(t-kt))^a \cdot (*^p\, \delta_{n-1}(t-kt))^b \cdot (*^p\, \delta_{n-1}(t-kt))^c$

$= (*^p\, \delta_{n-1}(t-kt))^{a+b+c} \geq *^p\, \delta_{n-1}(t-kt)$.

Arguing in the same way, we come to

$M_{gx_{n+1}gx_{n+p+1}}(kt), M_{gy_{n+1}gy_{n+p+1}}(kt), M_{gz_{n+1}gz_{n+p+1}}(kt) \geq *^p\, \delta_{n-1}(t-kt)$.

Applying the axiom (v) of a FMS, (7) and the induction hypothesis, $M_{gx_n gx_{n+p+1}}(t) = M_{gx_n gx_{n+p+1}}(t - kt + kt) \geq$

$$\geq M_{gx_p gx_{n+1}}(t - kt) * M_{gx_{n+1} gx_{n+p+1}}(kt) \geq$$

$$\geq \delta_{n-1}(t-kt) * (*^{p} \delta_{n-1}(t-kt)) = *^{p+1} \delta_{n-1}(t-kt).$$

The same reasoning is also valid for $M_{gy_n gy_{n+p+1}}(t), M_{gz_n gz_{n+p+1}}(t)$. Therefore, (11) is true. This permits us to show that $\{gx_n\}$ is Cauchy. Suppose that $t > 0$ and $\varepsilon \in (0, 1)$ are given. By the hypothesis, as $*$ is a t-norm of H-type, there exists $0 < \eta < 1$ such that $*pa > 1-\varepsilon$ for all $a \in (1-\eta, 1]$ and for all $p \geq 1$. By (9), $\lim_{n\to\infty} \delta n(t) = 1$, so there exists $n_0 \in N$ such that $\delta n(t - kt) > 1 - \eta$ for all $n \geq n_0$. Hence from (11), we get

$M_{gx_n gx_{n+p}}(t), M_{gy_n gy_{n+p}}(t), M_{gz_n gz_{n+p}}(t) > 1 - \varepsilon$ for all $n \geq n_0$ and $p \geq 1$. Therefore, $\{gx_n\}$ is a Cauchy sequence. Similarly, $\{gy_n\}$ and $\{gz_n\}$ are also Cauchy sequences.

Step 3. We claim that g and F have a tripled coincidence point.

Without loss of generality, we can assume that $g(X)$ is complete, then there exist x, y, z and $u, v, w \in g(X)$, such that

$\lim_{n\to\infty} gx_n = g(u) = x$, $\lim_{n\to\infty} gy_n = g(v) = y$ and $\lim_{n\to\infty} gz_n = g(w) = z$.

$\lim_{n\to\infty} gx_{n+1} = \lim_{n\to\infty} F_{x_n y_n z_n} = g(u) = x.$

$\lim_{n\to\infty} gy_{n+1} = \lim_{n\to\infty} F_{y_n z_n x_n} = g(v) = y$

$\lim_{n\to\infty} gz_{n+1} = \lim_{n\to\infty} F_{z_n x_n y_n} = g(w) = z.$

From (1) we get

$$M_{F_{x_n y_n z_n} F_{uvw}}(kt) \geq (M_{gx_n gu}(t)^a * M_{gy_n gv}(t)^b * M_{gz_n gw}(t)^c) \geq$$

$$\geq (M_{gx_n gu}(t) * M_{gy_n gv}(t) * M_{gz_n gw}(t))$$

Since M is continuous, taking limit as $n\to\infty$, we have

$M_{gu F_{uvw}}(kt) = 1.$

which implies that $F(u, v, w) = g(u) = x$.

Similarly, we can show that $F(v, w, u) = g(v) = y$ and $F(w, u, v) = g(w) = z$. Since F and g are weakly compatible, we get that $gF_{uvw} = F_{g(u)g(v)g(w)}$, $gF_{vwu} = F_{g(v)g(w)g(u)}$ and $gF_{wuv} = F_{g(w)g(u)g(v)}$ g which implies that

$F_{xyz} = gx$. In a similar way, we can show that $F_{yzx} = gy$ and $F_{zxy} = gz$, so (x, y, z) is a tripled coincidence point of the mappings F and g.

Thus, $F_{xyz} = gx$, $F_{yzx} = gy$ and $F_{zxy} = gz$. (12)

Step 4. *We claim that* $x = F_{zxy}$, $y = F_{xyz}$ *and* $z = F_{yzx}$. We note that by condition (1),

$$M_{gx gy_{n+1}}(kt) = M_{F_{xyz} F_{y_n z_n x_n}}(kt) \geq M_{gx gy_n}(t)^a * M_{gy gz_n}(t)^b * M_{gz gx_n}(t)^c; \quad (13)$$

$$M_{gy gz_{n+1}}(kt) = M_{F_{yzx} F_{z_n x_n y_n}}(kt) \geq M_{gy gz_n}(t)^a * M_{gz gx_n}(t)^b * M_{gx gy_n}(t)^c; \quad (14)$$

$$M_{gz gx_{n+1}}(kt) = M_{F_{zxy} F_{x_n y_n z_n}}(kt) \geq M_{gz gx_n}(t)^a * M_{gx gy_n}(t)^b * M_{gy gz_n}(t)^c; \quad (15)$$

Let $\beta_n(t) = M_{gx gy_n}(t) * M_{gy gz_n}(t) * M_{gz gx_n}(t)$ for all $t > 0$ and $n \geq 0$. By (13), (14) and (15),

$$\beta_{n+1}(kt) = M_{gx gy_{n+1}}(kt) * M_{gy gz_{n+1}}(kt) * M_{gz gx_{n+1}}(kt)$$

$$\geq M_{gx gy_n}(t)^a * M_{gy gz_n}(t)^b * M_{gz gx_n}(t)^c *$$

$$* M_{gy gz_n}(t)^a * M_{gz gx_n}(t)^b * M_{gx gy_n}(t)^c *$$

$$* M_{gz gx_n}(t)^a * M_{gx gy_n}(t)^b * M_{gy gz_n}(t)^c =$$

$$= M_{gxgy_n}(t)^a * M_{gxgy_n}(t)^c * M_{gxgy_n}(t)^b *$$
$$* M_{gxgy_n}(t)^b * M_{gygz_n}(t)^a * M_{gygz_n}(t)^c *$$
$$* M_{gzgx_n}(t)^c * M_{gzgx_n}(t)^b * M_{gzgx_n}(t)^a \geq$$
$$\geq M_{gxgy_n}(t)^a . M_{gxgy_n}(t)^c . M_{gxgy_n}(t)^b *$$
$$* M_{gygz_n}(t)^b . M_{gygz_n}(t)^a . M_{gygz_n}(t)^c *$$
$$* M_{gzgx_n}(t)^c . M_{gzgx_n}(t)^b . M_{gzgx_n}(t)a =$$
$$= M_{gxgy_n}(t)^{a+b+c} * M_{gygz_n}(t)^{a+b+c} * M_{gzgx_n}(t)^{a+b+c} \geq$$
$$\geq M_{gxgy_n}(t) * M_{gygz_n}(t) * M_{gzgx_n}(t) = \beta_n(t).$$

This proves that $\beta_{n+1}(kt) \geq \beta_n(t)$ for all $n \geq 0$ and all $t > 0$. Repeating this process,

$\beta_n(t) \geq \beta_{n-1}(t/k) \geq \beta_{n-2}(t/k^2) \geq . . . \geq \beta_0(t/k^n)$, for all $t > 0$ and $n \geq 1$. (16)

Now, by (16), (13), (14) and (15),

$$M_{gxgy_{n+1}}(kt) \geq M_{gxgy_n}(t)^a * M_{gygz_n}(t)^b * M_{gzgx_n}(t)^c \geq \beta_n(t) \geq \beta_0(t/k^n); \quad (17)$$

$$M_{gygz_{n+1}}(kt) \geq M_{gygz_n}(t)^a * M_{gzgx_n}(t)^b * M_{gxgy_n}(t)^c \geq \beta_n(t) \geq \beta_0(t/k^n); \quad (18)$$

$$M_{gzgx_{n+1}}(kt) \geq M_{gzgx_n}(t)^a * M_{gxgy_n}(t)^b * M_{gygz_n}(t)^c \geq \beta_n(t) \geq \beta_0(t/k^n); \quad (19)$$

Therefore, $M_{gxgy_{n+1}}(kt), M_{gygz_{n+1}}(kt), M_{gzgx_{n+1}}(kt) \geq \beta_0(t/k^n)$ for all $t > 0$ and $n \geq 1$. Since

$\lim_{n\to\infty} \beta_0(t/k^n) = 1$ for all $t > 0$, we have, taking limit in (17), (18) and (19), that $\lim_{n\to\infty} gx_n = gz$, $\lim_{n\to\infty} gy_n = gx$ and $\lim_{n\to\infty} gz_n = gy$. This shows, using (12), that

$F_{xyz} = gx = \lim_{n\to\infty} gy_n = y$, $F_{yzx} = gy = \lim_{n\to\infty} gz_n = z$, $F_{zxy} = gz = \lim_{n\to\infty} gx_n = x$.

Step 5. We will prove that $x = y = z$.

Let $\theta(t) = M_{xy}(t) * M_{yz}(t) * M_{zx}(t)$ for all $t > 0$. Then, by condition (1),

$$M_{xy}(kt) = M_{F_{xyz}F_{yzx}}(kt) \geq M_{gxgy}(t)^a * M_{gygz}(t)^b * M_{gzgx}(t)^c =$$
$$= M_{yz}(t)^a * M_{zx}(t)^b * M_{xy}(t)^c \quad (20)$$

$$M_{yz}(kt) = M_{F_{yzx}F_{zxy}}(kt) \geq M_{gygz}(t)^a * M_{gzgx}(t)^b * M_{gxgy}(t)^c =$$
$$= M_{zx}(t)^a * M_{xy}(t)^b * M_{yz}(t)^c \quad (21)$$

$$M_{zx}(kt) = M_{F_{zxy}F_{xyz}}(kt) \geq M_{gzgx}(t)^a * M_{gxgy}(t)^b * M_{gygz}(t)^c =$$
$$= M_{xy}(t)^a * M_{yz}(t)^b * M_{zx}(t)^c \quad (22)$$

If we use these three inequalities at the same time,

$$\theta(kt) = M_{xy}(kt) * M_{yz}(kt) * M_{zx}(kt) \geq$$
$$\geq M_{yz}(t)^a * M_{zx}(t)^b * M_{xy}(t)^c * M_{zx}(t)^a * M_{xy}(t)^b * M_{yz}(t)^c * M_{xy}(t)^a * M_{yz}(t)^b * M_{zx}(t)^c$$
$$= M_{xy}(t)^c * M_{xy}(t)^b * M_{xy}(t)^a * M_{yz}(t)^a * M_{yz}(t)^c * M_{yz}(t)^b * M_{zx}(t)^b * M_{zx}(t)^a * M_{zx}(t)^c$$
$$\geq M_{xy}(t)^c . M_{xy}(t)^b . M_{xy}(t)^a * M_{yz}(t)^a . M_{yz}(t)^c . M_{yz}(t)^b * M_{zx}(t)^b . M_{zx}(t)^a . M_{zx}(t)^c$$
$$= M_{xy}(t)^{a+b+c} * M_{yz}(t)^{a+b+c} * M_{zx}(t)^{a+b+c} \geq$$

$$\geq M_{xy}(t) * M_{yz}(t) * M_{zx}(t) = \theta(t).$$

We find that $\theta(kt) \geq \theta(t)$ implies that $\theta(t) \geq \theta(t/k) \geq \theta(t/k^2) \geq \ldots \geq \theta(t/k^n)$ for all $t > 0$ and $n \geq 1$. By (20), (21) and (22),

$$M_{xy}(kt) \geq M_{yz}(t)^a * M_{zx}(t)^b * M_{xy}(t)^c \geq M_{yz}(t) * M_{zx}(t) * M_{xy}(t) = \theta(t) \geq \theta(t/k^n).$$

$$M_{yz}(kt) \geq M_{zx}(t)^a * M_{xy}(t)^b * M_{yz}(t)^c \geq M_{zx}(t) * M_{xy}(t) * M_{yz}(t) = \theta(t) \geq \theta(t/k^n).$$

$$M_{zx}(kt) \geq M_{xy}(t)^a * M_{yz}(t)^b * M_{zx}(t)^c \geq M_{xy}(t) * M_{yz}(t) * M_{zx}(t) = \theta(t) \geq \theta(t/k^n).$$

Letting $n \to \infty$, we have $\lim_{n\to\infty} \theta(t/k^n) = 1$ for all $t > 0$, and this means that $M_{xy}(kt) = M_{yz}(kt) = M_{zx}(kt) = 1$ for all $t > 0$, *i.e.*, $x = y = z$. The unicity of x follows from (1).

Remark 3.3 In the above theorem if we take ф (t) = kt then we can get more generalized form of the result.

Remark 3.4 The unicity of the coincidence point of F and g is not always true. For instance, if $F \equiv x_0$ is constant and $g \equiv x_0$ is also constant, then every $(x, y, z) \in X^3$ is a coincidence point of F and g.

Example 3.5 Let $X = \{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots\}$, $*$ = min, $M(x, y, t) = \frac{t}{|x-y|+t}$

for all $x, y \in X$, $t > 0$.

Then $(X, M, *)$ is a fuzzy metric space.

Let $g : X \to X$ and $F : X^3 \to X$ be defined as

$$g(x) = \begin{cases} 0, & x = 0 \\ 1, & x = \frac{1}{2n+1} \\ \frac{1}{2n+1}, & x = \frac{1}{2n} \end{cases}$$

$$F(x,y,z) = \begin{cases} \frac{1}{(2n+1)^4} & \text{when } (x,y,z) = (\frac{1}{2n}, \frac{1}{2n}, \frac{1}{2n}) \\ 0, & \text{others} \end{cases}$$

Let $x_n = y_n = \frac{1}{2n}$ then $g(x_n) = \to 0$ and $F(x_n, y_n\, z_n) = \frac{1}{(2n+1)^4} \to 0$, but

$$\lim_{n\to\infty} M(g(F(x_n, y_n, z_n)), F(g(x_n), g(y_n), g(z_n), t) = M(0, 1, t) \nrightarrow 0.$$

so g and F are not compatible. From $F_{xyz} = gx$, $F_{yzx} = gy$ and $F_{zxy} = gz$

we can get $(x, y, z) = (0,0,0)$,

and we have $gF_{xyz} = F_{g(x)g(y)g(z)}$ which implies that F and g are weakly compatible.

By the definition of M and taking $k = \frac{1}{2}$ and the result above, we can get that inequality (1).

Then all the conditions in Theorem 3.2 are satisfied, and 0 is the unique common fixed point.

Theorem 3.6 Theorem 3.2 remains true if the 'weakly compatible property' is replaced by any one (retaining the rest of the hypothesis) of the following:

(i) weakly commuting property, (ii) R-weakly commuting property, (iii) R-weakly commuting property of type (Af), (iv) R-weakly commuting property of type (Ag), (v) R-weakly commuting property of type (P).

Proof. Let u,v,w be three points in X so that Fuvw = gu and Fvwu = gv and Fwuv = gw. Taking $x_n = u$, $y_n = v$ and $z_n = w$ it is easy to show that F(gu,gv,gw) = gF(u,v,w) and F(gv,gw,gu) = gF(v,w,u) and F(gw,gu,gv) = gF(w,u,v) Now applying Theorem 3.2, we can conclude that F,g have a unique common fixed point. In case if (F,g) satisfies R-weakly commuting property, then there exists some R > 0 such that

$M(F(gx,gy,gz),gF(x,y,z),t) \geq M(F(x,y,z),gx,t/R)$,

$M(F(gy,gz,gx),gF(y,z,x),t) \geq M(F(y,z,x),gy,t/R)$

$M(F(gz,gx,gy),gF(z,x,y),t) \geq M(F(z,x,y),gz,t/R)$

for all x,y,z in X and t > 0.

Let u,v,w be three points in X so that Fuvw = gu and Fvwu = gv and Fwuv = gw, then it is easy to see that F and g commutes at u,v and w. Now applying Theorem 3.2, we can conclude that F and g have a unique common fixed point. Similarly, if pair (F,g) is weakly commuting, R-weakly commuting of type (Af), type (Ag), type (P) then it commutes at their points of coincidence. Now, in view of Theorem 3.2, in all the cases F and g have a unique common fixed point in X. This completes our proof.

Theorem 3.7. Theorem 3.2 remains true if the 'weakly compatible property' is replaced by any one (retaining the rest of the hypothesis) of the following: (i) weakly f-compatible, (ii) weakly g-compatible.

Proof. In case if the pair (F,g) satisfies weakly f-compatible property, then either

$\lim_{n\to\infty} gF(x_n, y_n, z_n) = F(x,y,z)$, $\lim_{n\to\infty} gF(y_n, z_n, x_n) = F(y,z,x)$, $\lim_{n\to\infty} gF(z_n, x_n, y_n) = F(z,x,y)$,

or

$\lim_{n\to\infty} ggx_n = F(x,y,z)$, $\lim_{n\to\infty} ggy_n = F(y,z,x)$, $\lim_{n\to\infty} ggz_n = F(z,x,y)$.

whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $\lim_{n\to\infty} F(x_n,y_n,z_n) = \lim_{n\to\infty} g(x_n) = x$,

$\lim_{n\to\infty} F(y_n,z_n,x_n) = \lim_{n\to\infty} g(y_n) = y$, $\lim_{n\to\infty} F(z_n,x_n,y_n) = \lim_{n\to\infty} g(z_n) = z$.

and

$\lim_{n\to\infty} F(gx_n,gy_n,gz_n) = \lim_{n\to\infty} F(F(x_n, y_n, z_n), F(y_n, z_n, x_n), F(z_n, x_n, y_n)) = F(x,y,z)$,

$\lim_{n\to\infty} F(gy_n,gz_n,gx_n) = \lim_{n\to\infty} F(F(y_n, z_n, x_n), F(z_n, x_n, y_n), F(x_n, y_n, z_n)) = F(y,z,x)$,

$\lim_{n\to\infty} F(gz_n,gx_n,gy_n) = \lim_{n\to\infty} F(F(z_n, x_n, y_n), F(x_n, y_n, z_n), F(y_n, z_n, x_n)) = F(z,x,y)$.

for some $x,y,z \in X$.

Let u,v,w be three points in X so that Fuvw = gu and Fvwu = gv and Fwuv = gw, then it is easy to see that F and g commutes at u,v and w. Now applying Theorem 3.2, we can conclude that F and g have a unique common fixed point. Similarly, the theorem holds good if the pair (F,g) weakly g- compatible.

ACKNOWLEDGMENTS

Our thanks to the experts who have contributed towards development of the paper.

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