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Design of Appropriate controller for synchronization of chaos in system of Rossler in presence of uncertainty parameters

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ABSTRACT

Many of the proposed approaches for nonlinear systems control are developed under the assumption that all involved parameters are known. However, the system uncertainties are always unavoidable and some system parameters cannot be exactly known in the most part of the cases. In this paper, a adaptive controlscheme is proposed to system is able to achieve the control objective regardless the parametric uncertainties of the model and the lack of a priori knowledge on the system. Also, is applied the adaptive sliding mode control for synchronization of chaos in system of Rossler in presence of unknown parameters. First, suppose that there are no parameter uncertainties in the system model. Then, this condition is removed while an adaptive sliding mode control system is designed. The advantages of obtaining such a motion are twofold: firstly the system behaves as a system of reduced order with respect to the original plant; and secondly the movement on the sliding surface of the system is insensitive to a particular kind of perturbation and model uncertainties.

Keywords

chaotic systems; synchronization; adaptive sliding mode control; uncertainties

Academic Discipline And Sub-Disciplines

Design of controller and synchronization of chaos

SUBJECT CLASSIFICATION

68U07, 68T45,68T40, 68N20, 68U35, 68N30, 68N99

TYPE (METHOD/APPROACH)

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1. INTRODUCTION

Chaos system has been widely researched in recent years, because of its many potential applications in daily life. Chaotic systems as a nonlinear system, can be considered as bounded unstable systems with high sensitivity to initial conditions. Synchronization of chaos is process in which two or more chaotic systems with distinct feature through a foreign force achieve to a set of common behaviors. Synchronization of interacting oscillators in biological systems has been widely studied over the last few years[6]. Classical phenomena such as mutual synchronization, entrainment and chaotic synchronization are now observed in many biological experiments and numerical simulations. Several methods have been applied to synchronize chaotic systems[7]. A number of methods based on master–slave pattern have been proposed. synchronization method with parametric adaptive control has been considered in [4]. A systematic design procedure to synchronize two identical generalized Lorenz chaotic systems based on a sliding mode control was studied by Lin and Yan. However, it did not consider the uncertain factors in practice. In fact, there are many uncertainty sources in the complex systems, such as unmodelled dynamics, ionic channel noise and external disturbances. Variable structure control (VSC) with sliding mode control was first proposed and elaborated by Emel'yanov and Taran, (1962); Emel'yanov, (1970); Utkin, (1974). Sliding mode control has developed into a general design control method applicable to a wide range of system types including nonlinear systems, MIMO systems, discrete time models, large-scale. The concept of sliding mode control based on the conceptions of variable structure control in which the second order system trajectories has been driven towards a line in the state space termed as the sliding line and enforcing the trajectories to the origin. In this paper, an adaptive sliding mode controller is used to achieve synchronization of two coupled chaotic systems in presence of uncertainty parameters. Then we use an adaptive algorithm to approximate the uncertainties and disturbances of the dynamical system. According to the Lyapunov stability theorem, the employed ASMC controller not only guarantees stability for the coupled system, but also assures the boundedness of synchronization errors[3], [5] [1], [9]. As a case study the method presented has been applied to Rossler system, Figure 1.

The Rossler system is given by

$$\dot{x}_1 = -x_2 - x_3$$

$$\dot{x}_2 = x_1 + a x_2$$

$$\dot{x}_3 = x_1 x_3 - c x_3 + b$$

where a , b , c are real constant.

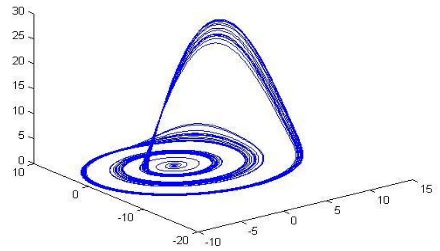

Figure 1: Rossler system

2. Synchronization and problem definition

Purpose of this section is that, by applying an appropriate control signal to the chaotic system, it behaves like a chaotic system synchronized with Uncertainties parameters. The first system is called master system and the second system that not fully known with uncertainty is called the slave system [10]. Note that, the control signal, enters into the second chaotic system, and the master system does not have any effect, it is called one-way synchoronization. This means that the dynamic of the slave system is synchronous with the dynamics of master system. Generally, in synchronization process, all state variables of slave system are synchronize with master system. To achieve this goal, the control algorithm based on adaptive and sliding mode control methods is designed.

For overview of synchronization the master and slave systems, respectively, is shown below

$$\dot{x}(t) = f(x,t)$$

$$\dot{y}(t) = g(y,t) + u \qquad (1)$$

In this relationship, t, Represents the time, $x \in R^n$, State vector of the master system, $y \in R^n$, State vector of the slave system, $f : R^n \times R^+ \to R^n$, and $g : R^n \times R^+ \to R^n$ are nonlinear functions.

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Now to define the synchronization error $e = x - y$. have

$$\dot{e} = f(x,t) - g(y,t) - u$$

Our goal is to find proper control functions u , such that

$$\lim_{t\to\infty}\|e\| = 0$$

Obviously, Rossler system is chaotic for some parameters by appling of 0-1 test (If K is close to zero, the system is not chaotic, and if is near one, the system is chaotic),TABLE 1, Figure 2 [6], [10].

Thus, this system is used for case study to examin controlling chaos in this system by applying of the proposed methods.

In order to investigate Synchronization behavior chaotic systems, the master and slave systems for Rossler system, respectively, is defined below

$$X_m :$$

$$\dot{x}_1 = -x_2 - x_3$$

$$\dot{x}_2 = x_1 + a_1 x_2 \qquad (2)$$

$$\dot{x}_3 = x_1 \; x_3 - c_1 x_3 \; + b_1$$

$$X_s :$$

$$\dot{x}_4 = -x_5 - x_6 + u_1$$

$$\dot{x}_5 = x_4 + a_2 x_5 + u_2 \qquad (3)$$

$$\dot{x}_6 = x_4 x_6 - c_2 x_6 + b_2 + u_3$$

where a_1, b_1 , c_1 , a_2, b_2 , c_2 are real constants.

In problem of synchronization of system explained in Equations (1), First system is considered as master system and second system as a slave system. However, by applying a suitable control signal on it, A state variables of system slave will converget to state variables of system master after a transient time.

Table 1 SYSTEM OF CHAOTIC ROSSLER

Coefficients	Data
$x_1(0)$	0.01
$x_2(0)$	0.01
$x_3(0)$	0.01
b	3
c	20
a	$0 \le a \le 0.085$
dt	0.001
K	$0.7655 \le k \le 0.8743$

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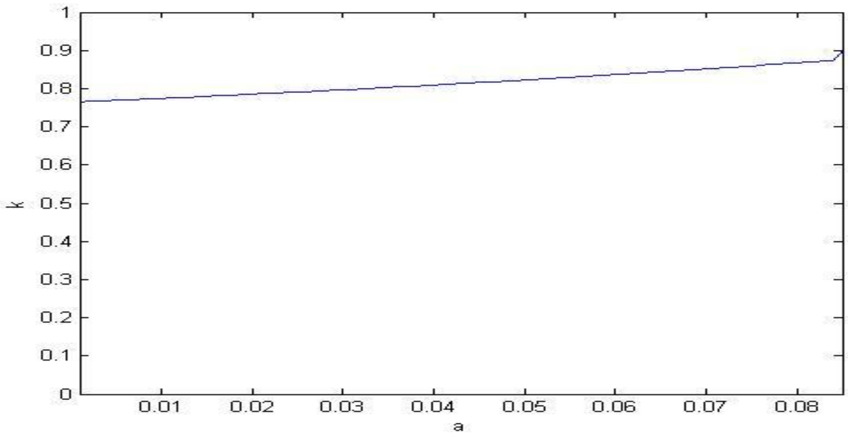

Figure 2: Appling of 0-1 test for system of chaotic Rossler

3. Synchronization with linearization method

Although in synchronization of chaos, Both systems are chaotic, so, for control of chaos through synchronization, is necessary to select the appropriate parameters, Periodic system of appropriate master obtain[2]. Now, with assumption

$a_1 = 0.125$, $b_1 = 0.21$, $c_1 = 5.82$

and

$a_2 = 0.21$, $b_2 = 0.21$, $c_2 = 5.82$

That for them the slave Rossler system is chaotic, to define the synchronization error for above master and slave systems, we have

$$E = X_s - X_m$$

$$\begin{aligned} \dot{e}_1(t) &= -e_2(t) - e_3(t) + u_1 \\ \dot{e}_2 &== e_1(t) + a_2 x_5(t) - a_1 x_2(t) + u_2 \\ \dot{e}_3 &= x_6(t)x_4(t) - x_3(t)x_1(t) - ce_3(t) + u_3 \end{aligned} \quad (4)$$

where $c = c_1 = c_2$.

Our goal is to find proper control functions u_i by linearization method such that

$$\lim_{t\to\infty} |e_i(t)| = 0 \,,\; i = 1,2,3 \quad (5)$$

Therefore define

$$\begin{aligned} u_1 &= 0 \\ u_2 &= -a_s x_5(t) + a_1 x_2(t) + \lambda e_2 \\ u_3 &= -x_6(t)x_4(t) + x_3(t)x_1(t) \end{aligned} \quad (6)$$

Theorem. the master systems X_m and slave systems X_s with The control law (6) for $c > 0$, $\lambda < 0$ Synchronized.

Proof. With replacement (6) in (4) have

$$\dot{e}_1 = -e_2 - e_3$$

$$\dot{e}_2 == e_1 + \lambda e_2$$

$$\dot{e}_3 = -ce_3$$

coefficient matrix of error equations and eigenvalues are as following

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$$\lambda_{1,2} = \frac{\lambda \pm \sqrt{\lambda^2 - 4}}{2}\ ,\ \lambda_3 = -c \qquad A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \lambda & 0 \\ 0 & 0 & -c \end{bmatrix}\ ,$$

If $\lambda < 0$ and $c > 0$ then all of eigenvalues of matrix A ,have negative real parts and system (4) Will be stable, and have

$$\lim_{t\to\infty} |e_i(t)| = 0\ ,\ i = 1,2,3$$

Simulation diagrams in Figures 3, 4, 5 and 6 are presented.

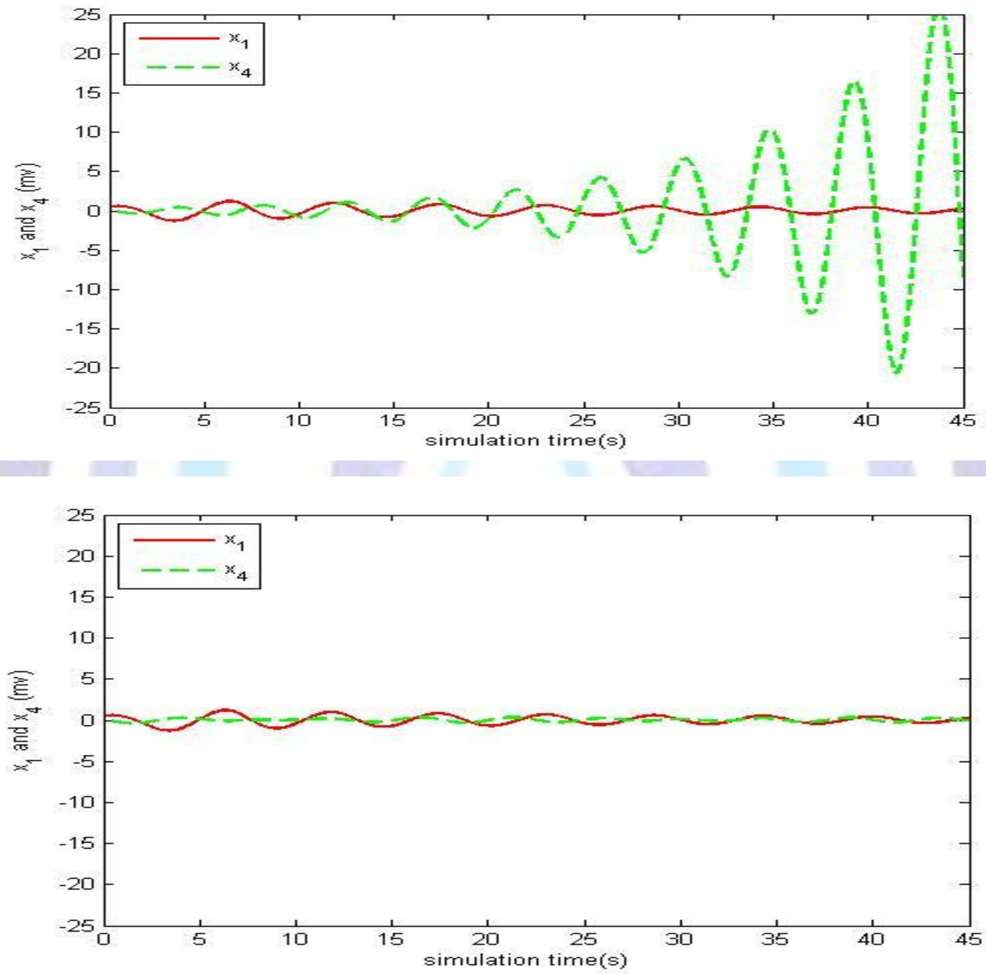
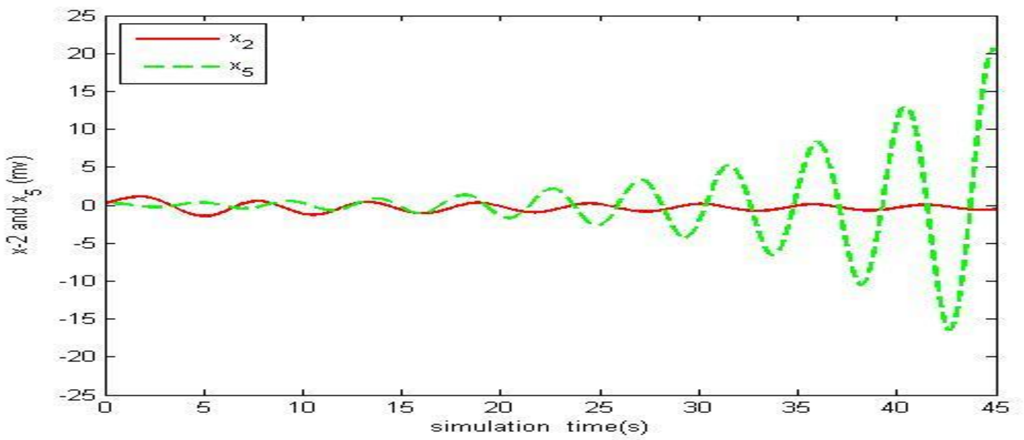


Figure 3: curve of x_1 and x_4 before and after synchronization



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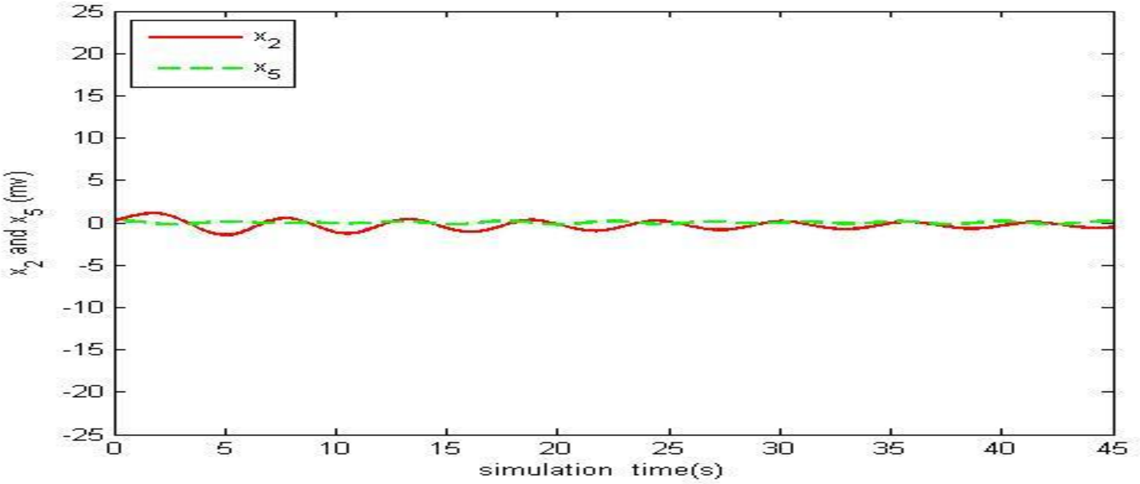


Figure 4: curve of x_2 and x_5 before and after synchronization

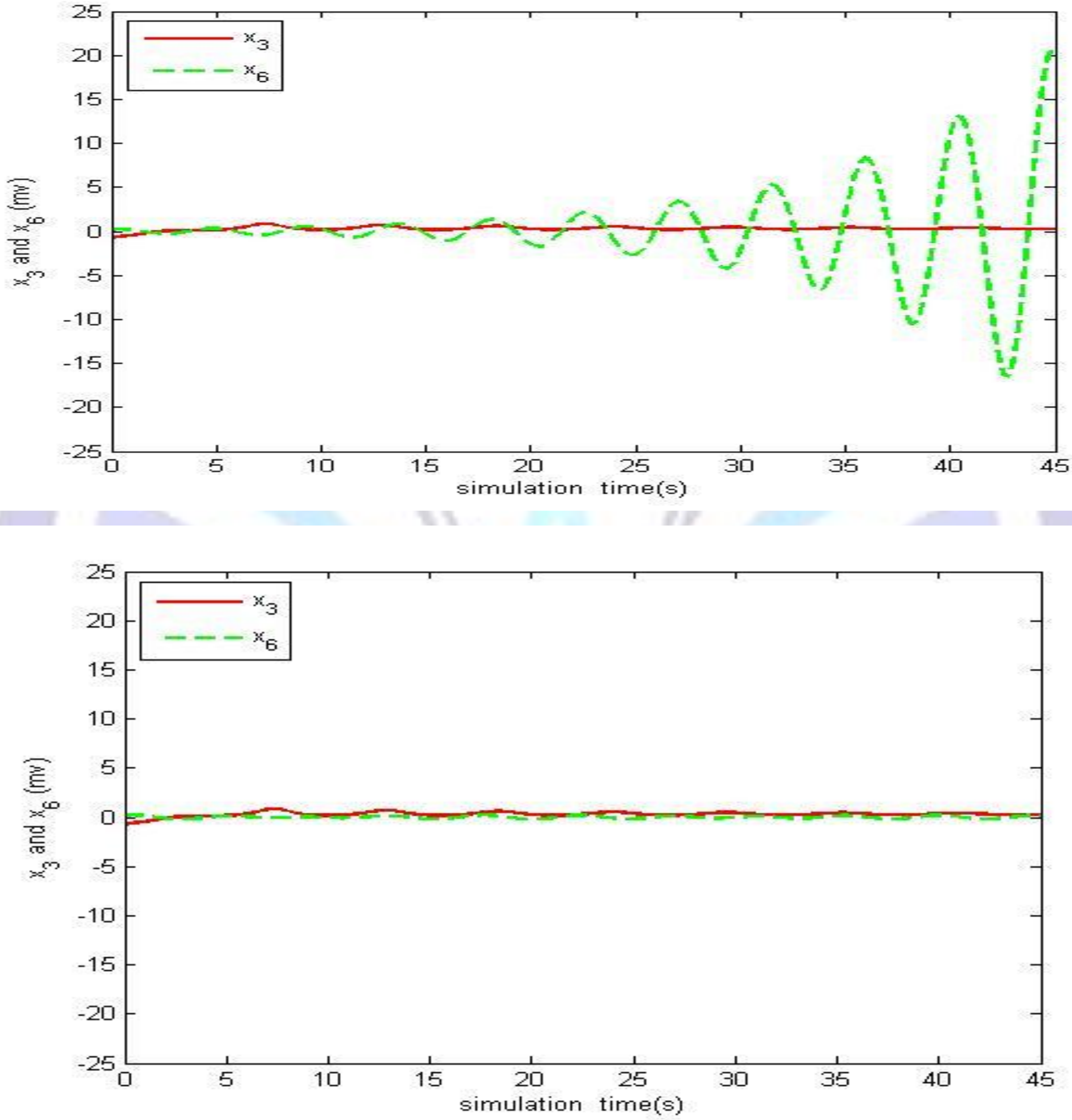
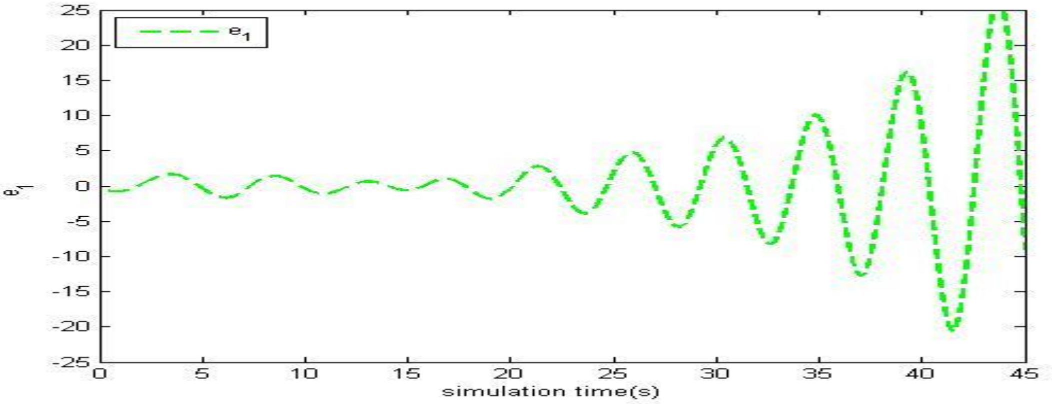


Figure 5: curve of x_3 and x_6 before and after synchronization



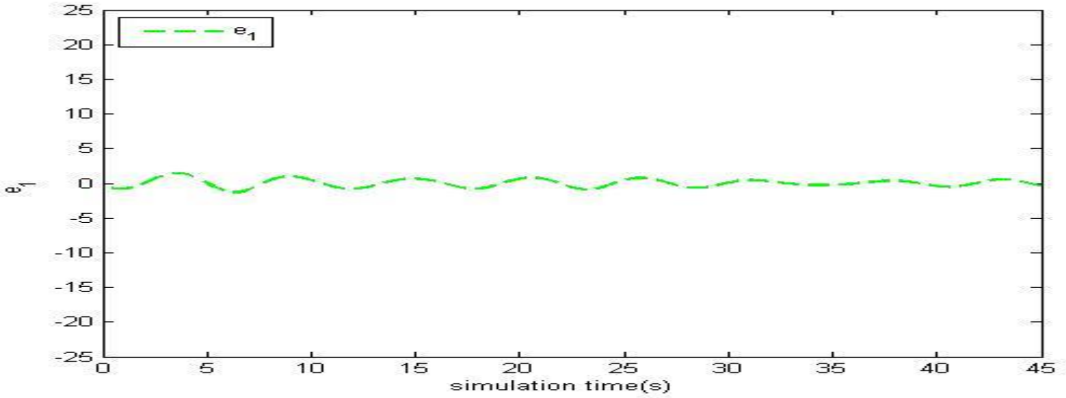


Figure 6: curve of error (e_1) before and after synchronization

4 Synchronization with adaptive control method

Because sometimes the system parameters are not known precisely and can change over time, thus making adaptive controller for chaos control is important. In this section, a method for chaos control of a class of chaotic systems through the adaptive synchronization is presented, that the parameters of the slave system are unknown. Therefore, master system parameters, choice from initial estimates of the parameters of slave system. The control rules are expressed based on the states of both the system and the estimated parameters, and estimation update laws of parameters are chosen such that the synchronization errors and estimation errors, converge to zero.

Suppose

$$\dot{X}_s(t) = \theta_1 X_s(t) + \theta_2 + f(X_s(t)) + U(t)$$

That $f(X_s(t))$ continuous function and θ_1 and θ_2 are unknown parameters. Error equations and the control rules are expressed as following

$$E = X_s - X_m$$

$$\dot{E} = \dot{X}_s - \dot{X}_m = -F(X_m, \theta') + f(X_s) + \theta_1 X_s + \theta_2 + u$$

$$U(t) = -KE(t) + F(X_m(t), \theta') - f(X_s(t))$$
$$-\hat{\theta}_1(t) X_s(t) - \hat{\theta}_2(t) \qquad K > 0$$

That $\hat{\theta}_1$ and $\hat{\theta}_2$ Respectively, the estimated parameters θ_1 , θ_2 and laws of update them are

$$\begin{cases} \dot{\hat{\theta}}_{ij}(ij) = \alpha_{ij}(t) x_j(t) \\ \dot{\hat{b}}_i(t) = \beta_i e_i(t) \end{cases} \qquad \alpha_{ij} \ , \ \beta_i > 0 \quad i, j = 1,2,\ldots,n$$

Based on nonlinear adaptive control method is described, the control and the parameters update laws to Rossler system, are selected as following that in this case, c_2 , b_2 , a_2 are unknown.

$$u_1 = e_2 + e_3 - k_1 e_1$$

$$u_2 == -e_1 - \hat{a}_2 x_5 + a_1 x_2 - k_2 e_2$$

$$u_3 = -x_6 x_4 + x_3 x_1 + \hat{c}_2 e_3 - c_1 x_3 - \hat{b}_2 + b_1 - k_3 e_3$$ and

$$\dot{\hat{a}}_2 = \lambda_1 x_5 e_2$$

$$\dot{\hat{b}}_2 == \lambda_2 e_3$$

$$\dot{\hat{c}}_3 = -\lambda_3 x_6 e_3$$

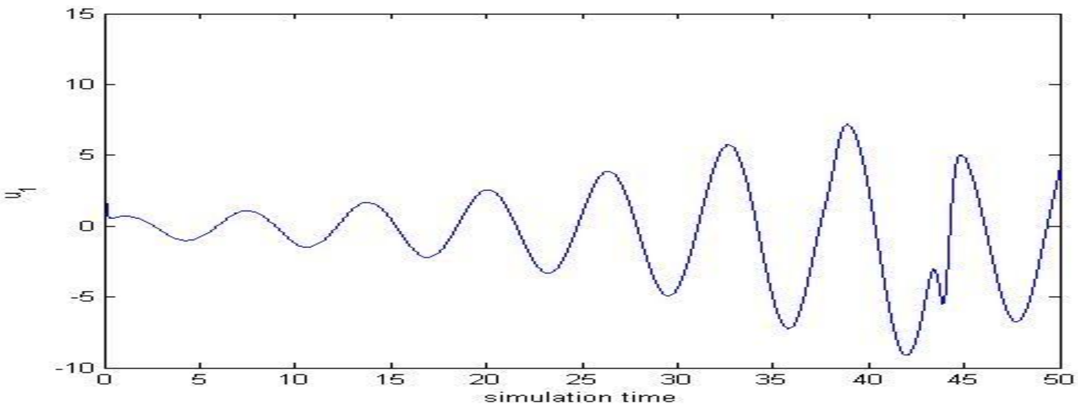
In section (6) with assumptions $k_1 = 10$, $k_2 = 17$, $k_3 = 21$

$$\lambda_1 = \lambda_2 = \lambda_3 = 10$$

$c_2 = 9$, $b_2 = 0.2$, $a_2 = 0.6$, $\hat{c}_2(0) = 5.82$, $\hat{b}_2(0) = 0.21$, $\hat{a}_2(0) = 0.21$

simulation results indicate that the synchronization error after about 4 seconds reaches zero, and the estimate of the unknown parameters converge to the true values, and the synchronization is performed[4], [8].

Simulation diagrams in Figures 7, 8,9 and 10 are presented.



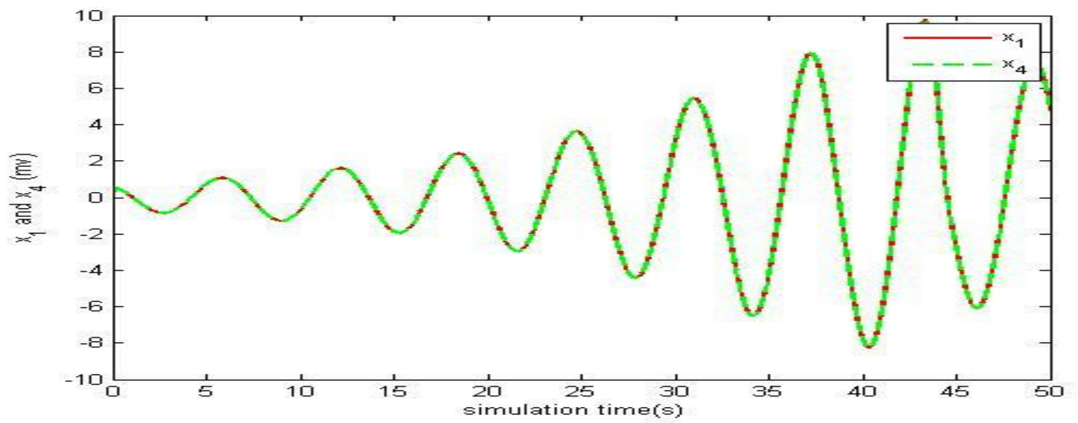


Figure 7: curve of x_1 and x_4 after synchronization by applying control signal of u_1

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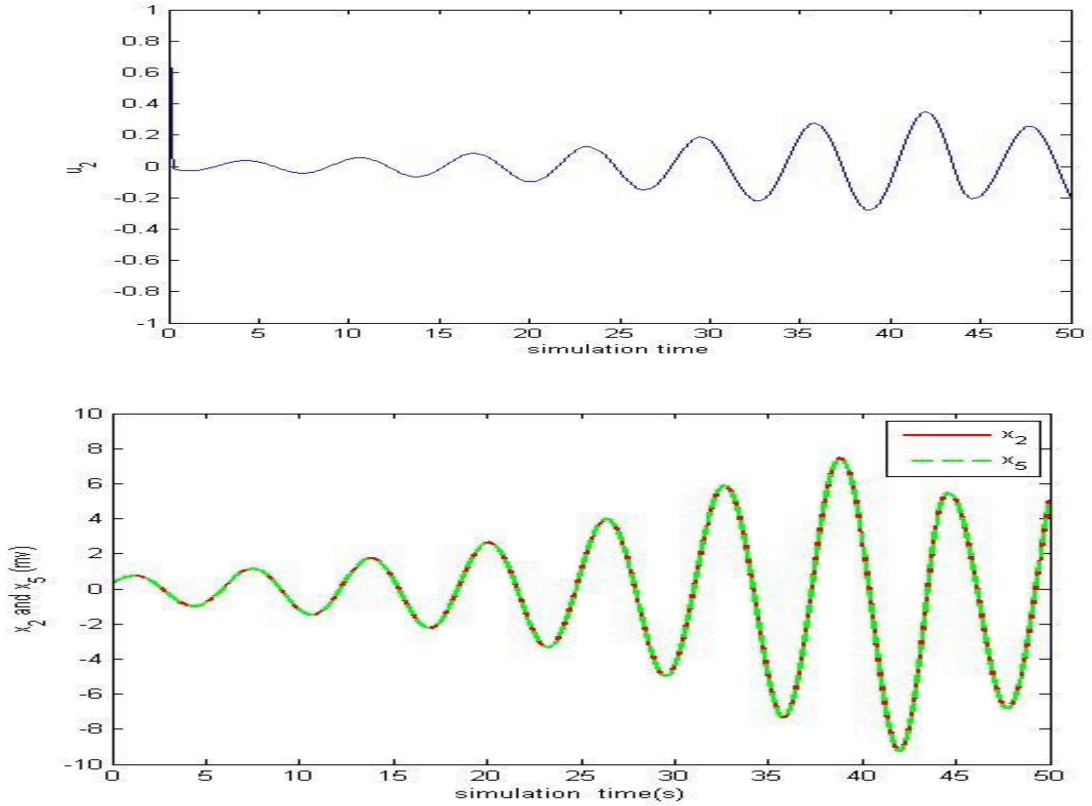

Figure 8: curve of x_2 and x_5 after synchronization by applying control signal of u_2

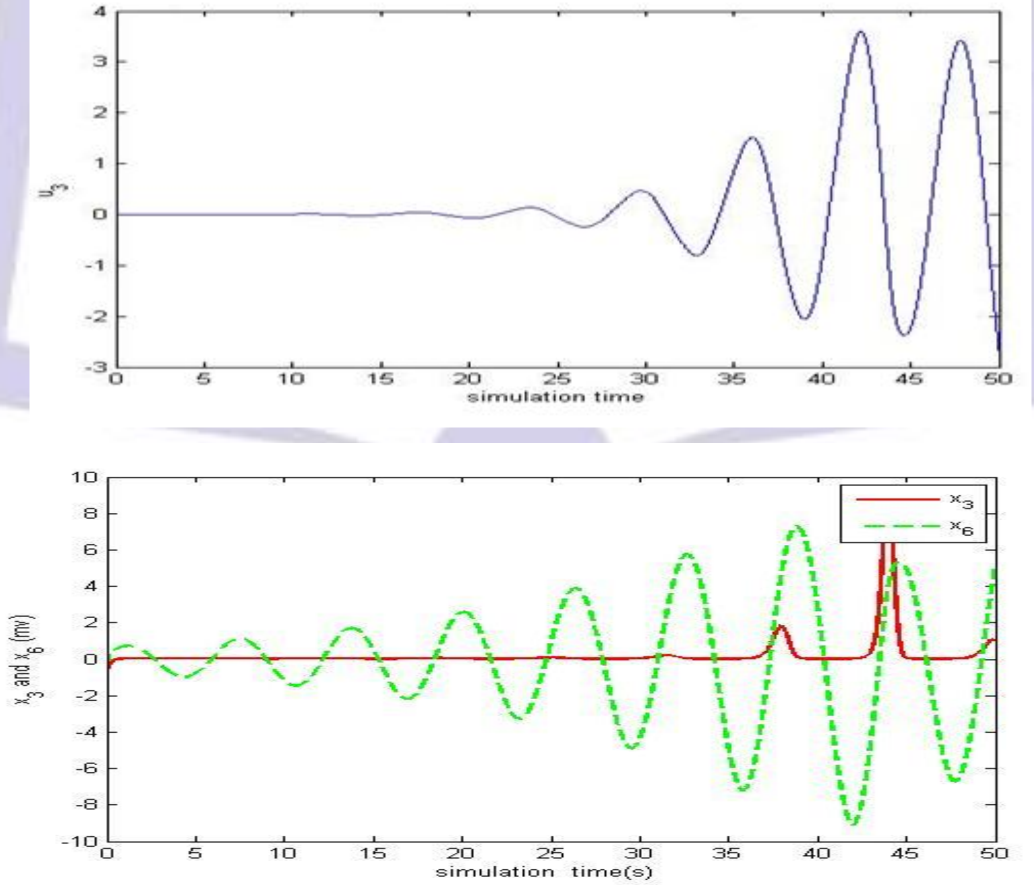

Figure 9: curve of x_3 and x_6 after synchronization by applying control signal of u_3

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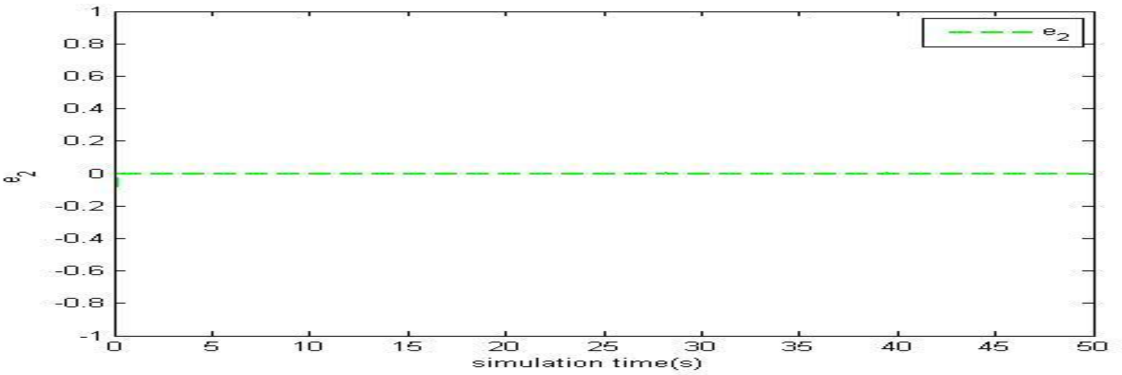

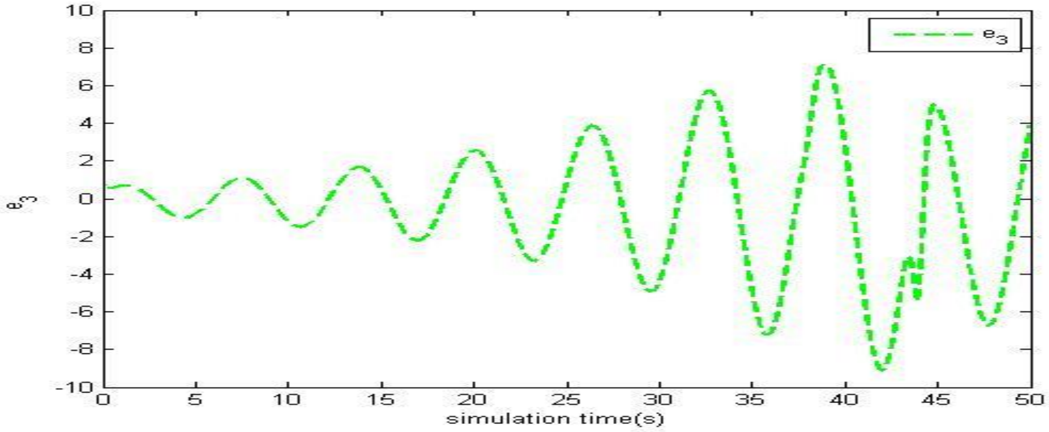

Figure 10: curves of error (e_2 and e_3) after synchronization

5. Synchronization with adaptive sliding mode control method

Essentially, sliding mode control utilizes discontinuous feedback control laws to force the system state to reach, and subsequently to remain on, a specified surface within the state space (the so-called sliding or switching surface). The system dynamic when confined to the sliding surface represent the controlled system behaviour. Therefore, first suppose that there are no parameter uncertainties in the system models. In order to solve synchronization problem, we can apply the idea of sliding-mode controller[1], [9].

Rossler system

Simulation results in section 3 show that, control input $u_3(t)$ is non-zero only for a short time. Thus, it can be ignored and the system can be controlled by a control input $u_2(t)$. Therfore define the synchronization error as $e_2(t)$. and time-varying surface $S(t)$. in order to design controller for the synchronization have to $\dot{s}(t)=0$. As mentioned in the previous sections, equations of error Rossler are as follow

$$\dot{e}_1(t)=-e_2(t)-e_3(t)$$

$$\dot{e}_2 == e_1(t)+a_2x_5(t)-a_1x_2(t)+u_2$$

$$\dot{e}_3 = x_6(t)x_4(t)-x_3(t)x_1(t)-ce_3(t)$$

However, we define sliding surface for Rossler systemas as following:

$$s(t)=e_2(t)+\lambda\int_0^t e_2(\tau)\,d\tau$$

$$\dot{s}(t)=\dot{e}_2(t)+\lambda\; e_2(t)=e_1(t)+a_2\,x_5(t)$$
$$-a_1\,x_2(t)+u_2\;+\lambda\,e_2(t)=0$$

Then

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$$u_2 = -e_1(t) - a_2 x_5(t) + a_1 x_2(t) - \lambda e_2(t) - k e_2(t)$$

To view stability of the error dynamics of the controller mentioned positive definite, Lyapunov function are defined as follows

$$v = \frac{s^2}{2}$$

With derivative

$$\dot{v} = s\dot{s} = -k e_2^2(t) - k\lambda\, e_2(t)\int_0^t e_2(\tau)\, d\tau$$

With take $\lambda = \varepsilon$ and $k > 0$ have $\dot{v} \le 0$. Thus, according to the Lyapunov stability theorem, since $\dot{v}$ is negative definite and v positive definite, once $t \to \infty$, then s tend asymptotically to zero. And the two systems through sliding mode controllers are synchronized with each other.

Simulation results indicate the issue is.

Simulation diagrams in Figures 11, 12,13 and 14 are presented.

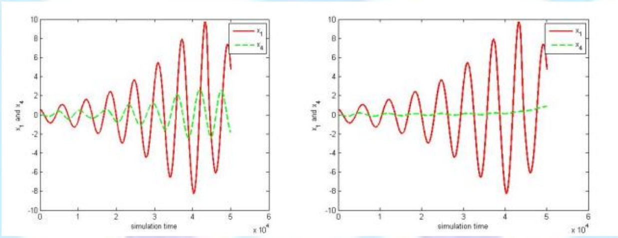

Figure 11: curve of x_1 and x_4 after synchronization by applying ASMC and AC method

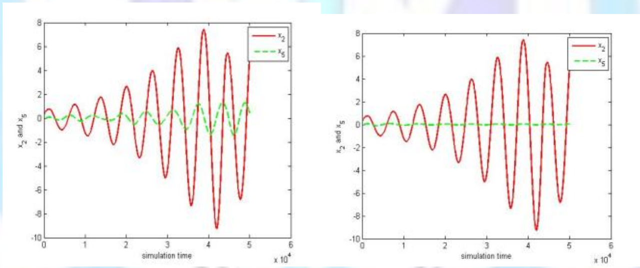

Figure 12: curve of x_2 and x_5 after synchronization by applying ASMC and AC method

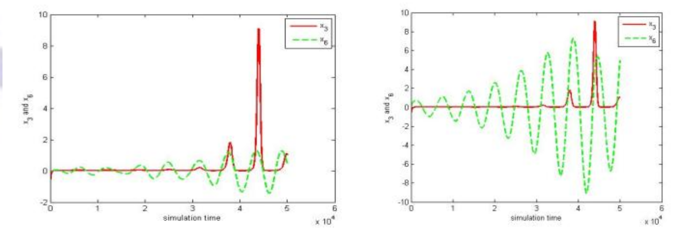

Figure 13: curve of x_3 and x_6 after synchronization by applying ASMC and AC method

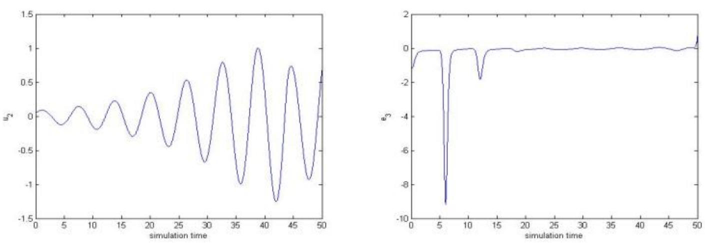

Figure 14: curves of error e_3 and curves of u_2

6. Conclusion

In this paper, an adaptive control scheme is proposed to system is able to achieve the control objective regardless the parametric uncertainties of the model and the lack of a priori knowledge on the system. Also, an adaptive sliding mode control algorithm was applied to design controller for synchronization of chaos in system of Rossler in presence of unknown parameters. Under the given parameter updating law, the error of the disturbance is bounded. This method can make the synchronization error convergent to zero. Finally, by choosing appropriate Lyapunov function, we examined the stability of the system The simulation results demonstrate the availability of the proposed synchronization control method. The proposed synchronization method can also be used to control other chaotic systems.

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