ISSN 2347-1921

FIXED POINTS IN MENGER SPACE FOR WEAK COMPATIBLE MAPPINGS

V. K. Gupta[1], Arihant Jain[2] and Ramesh Bhinde[3]

[1]Department of Mathematics, Govt. Madhav Science College, Ujjain (M.P.) 4560India
[2]Department of Applied Mathematics, Shri Guru Sandipani Institute of Technology and Science, Ujjain (M.P.) 456 550 India
[3]Department of Mathematics, Govt. P. G. College, Alirajpur (M.P.) India

ABSTRACT. The purpose of this paper is to establish a unique common fixed point theorem for six self mappings using the concept of weak compatibility in Menger space which is an alternate result of Jain and Singh [5]. We also cited an example in support of our result.

Keywords and Phrases. Menger space, Common fixed points, Compatible maps, Weak Compatible mapping.

AMS Subject Classification (2000). Primary 47H10, Secondary 54H25.

Council for Innovative Research

Peer Review Research Publishing System

Journal: JOURNAL OF ADVANCES IN MATHEMATICS

Vol .11, No.2

www.cirjam.com , editorjam@gmail.com

1. INTRODUCTION.

There have been a number of generalizations of metric space. One such generalization is Menger space initiated by Menger [8]. It is a probabilistic generalization in which we assign to any two points x and y, a distribution function $F_{x,y}$. Schweizer and Sklar [11] studied this concept and gave some fundamental results on this space. Sehgal and Bharucha-Reid [12] obtained a generalization of Banach Contraction Principle on a complete Menger space which is a milestone in developing fixed point theory in Menger space.

Jungck and Rhoades [7] termed a pair of self maps to be coincidentally commuting or equivalently weakly compatible if they commute at their coincidence points. Sessa [13] initiated the tradition of improving commutativity in fixed point theorems by introducing the notion of weak commuting maps in metric spaces. Jungck [5] soon enlarged this concept to compatible maps. The notion of compatible mapping in a Menger space has been introduced by Mishra [9]. Interesting results in the field of Menger space have been discussed in Jain et. al. [2, 3, 4], Singh et. al. [14, 15], Cho et. al. [1], Patel et. al. [10] and so on.

In this paper a fixed point theorem for six self maps has been proved using the concept of weak compatibility which turns out to be an alternate result of Jain et. al. [5]. We also cited an example in support of our result.

2. Preliminaries.

Definition 2.1. [8] A mapping $\mathcal{F} : R \to R^+$ is called a *distribution* if it is non-decreasing left continuous with

$\inf \{ \mathcal{F}(t) \mid t \in R \} = 0$ and $\sup \{ \mathcal{F}(t) \mid t \in R\} = 1$.

We shall denote by L the set of all distribution functions while H will always denote the specific distribution function defined by

$$H(t) = \begin{cases} 0 & , \quad t \le 0 \\ 1 & , \quad t > 0 \end{cases}.$$

Definition 2.2. [2] A mapping $t : [0, 1] \times [0, 1] \to [0, 1]$ is called a *t-norm* if it satisfies the following conditions :

(t-1) $t(a, 1) = a$, $t(0, 0) = 0$;

(t-2) $t(a, b) = t(b, a)$;

(t-3) $t(c, d) \ge t(a, b)$; for $c \ge a$, $d \ge b$,

(t-4) $t(t(a, b), c) = t(a, t(b, c))$ for all $a, b, c, d \in [0, 1]$.

Definition 2.3. [2] A *probabilistic metric space (PM-space)* is an ordered pair $(X, \mathcal{F})$ consisting of a non empty set X and a function $\mathcal{F} : X \times X \to L$, where L is the collection of all distribution functions and the value of $\mathcal{F}$ at $(u, v) \in X \times X$ is represented by $F_{u, v}$. The function $F_{u,v}$ assumed to satisfy the following conditions:

(PM-1) $F_{u,v}(x) = 1$, for all $x > 0$, if and only if $u = v$;

(PM-2) $F_{u,v}(0) = 0$;

(PM-3) $F_{u,v} = F_{v,u}$;

(PM-4) If $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1$ then $F_{u,w}(x + y) = 1$,

for all $u,v,w \in X$ and $x, y > 0$.

Definition 2.4. [2] A *Menger space* is a triplet $(X, \mathcal{F}, t)$ where $(X, \mathcal{F})$ is a PM-space and t is a t-norm such that the inequality

(PM-5) $F_{u,w}(x + y) \ge t \{F_{u, v}(x), F_{v, w}(y) \}$, for all $u, v, w \in X$, $x, y \ge 0$.

Definition 2.5. [11] A sequence $\{x_n\}$ in a Menger space $(X, \mathcal{F}, t)$ is said to be *convergent* and *converges to a point* x in X if and only if for each $\varepsilon > 0$ and $\lambda > 0$, there is an integer $M(\varepsilon, \lambda)$ such that

$$F_{x_n, x}(\varepsilon) > 1 - \lambda \quad \text{for all } n \ge M(\varepsilon, \lambda).$$

Further the sequence $\{x_n\}$ is said to be *Cauchy sequence* if for $\varepsilon > 0$ and $\lambda > 0$, there is an integer $M(\varepsilon, \lambda)$ such that

$$F_{x_n, x_m}(\varepsilon) > 1- \lambda \quad \text{for all } m, n \ge M(\varepsilon, \lambda).$$

A Menger PM-space $(X, \mathcal{F}, t)$ is said to be *complete* if every Cauchy sequence in X converges to a point in X.

A complete metric space can be treated as a complete Menger space in the following way :

ISSN 2347-1921

Proposition 2.1.[3] If (X, d) is a metric space then the metric d induces mappings $\mathcal{F} : X \times X \to L$, defined by $F_{p,q}(x) = H(x - d(p, q))$, p, q $\in$ X, where

$$H(k) = 0, \quad \text{for } k \leq 0 \quad \text{and} \quad H(k) = 1, \quad \text{for } k > 0.$$

Further if, $t : [0,1] \times [0,1] \to [0,1]$ is defined by $t(a,b) = \min\{a, b\}$. Then (X, $\mathcal{F}$, t) is a Menger space. It is complete if (X, d) is complete.

The space (X, $\mathcal{F}$, t) so obtained is called the *induced Menger space.*

Proposition 2.2. [8] In a Menger space (X, $\mathcal{F}$, t) if $t(x, x) \geq x$, for all $x \in [0, 1]$ then $t(a, b) = \min\{a, b\}$, for all a, b $\in$ [0, 1].

Definition 2.6. [7] Self mappings A and S of a Menger space (X, $\mathcal{F}$, t) are said to be weak compatible if they commute at their coincidence points i.e. Ax = Sx for x $\in$ X implies ASx = SAx.

Definition 2.7. [9] Self mappings A and S of a Menger space (X, $\mathcal{F}$, t) are said to be *compatible* if $F_{ASx_n, SAx_n}(x) \to 1$ for all x > 0, whenever $\{x_n\}$ is a sequence in X such that Ax_n, $Sx_n \to u$ for some u in X, as $n \to \infty$.

Now, we give an example of pair of self maps (A, S) which are weak compatible but not compatible.

Example 2.1. Let (X, d) be a metric space where X = [0, 4] and (X, $\mathcal{F}$, t) be the induced Menger space with $F_{p,q}(\varepsilon) = H(\varepsilon - d(p, q))$, $\forall$ p, q $\in$ X and $\varepsilon > 0$.

Define self maps A and S as follows :

$$A(x) = \begin{cases} 4 - x, & \text{if } 0 \leq x < 2 \\ 4, & \text{if } 2 \leq x \leq 4, \end{cases} \qquad S(x) = \begin{cases} x, & \text{if } 0 \leq x < 2 \\ 4, & \text{if } 2 \leq x \leq 4 \end{cases}.$$

Taking $x_n = 2 - \frac{1}{n}$, we get $F_{Ax_n,2}(\varepsilon) = H\left(\varepsilon - \frac{2}{n}\right)$.

Hence, $\lim_{n\to\infty} F_{Ax_n,2}(\varepsilon) = 1$.

Thus, $Ax_n \to 2$. Similarly, $Sx_n \to 2$ as $n \to \infty$.

Again,

$$F_{ASx_n,SAx_n}(\varepsilon) = H\left(\varepsilon - \left(2 - \frac{1}{n}\right)\right).$$

$$\lim_{n\to\infty} F_{ASx_n,SAx_n}(\varepsilon) = H(\varepsilon - 2) \neq 1, \ \forall\, \varepsilon > 0.$$

Hence, (A, S) is not compatible. Also, set of coincident points of A and S is [2, 4].

Now, for any x $\in$ [2, 4], Ax = Sx = 4 and AS(x) = A(4) = 4 = S(4) = SA(x).

Remark 2.2. In view of above example, it follows that the concept of weak compatible maps is more general than that of compatible maps.

Proposition 2.3. Let $\{x_n\}$ be a Cauchy sequence in a Menger space (X, $\mathcal{F}$, t) with continuous t-norm t. If the subsequence $\{x_{2n}\}$ converges to x in X, then $\{x_n\}$ also converges to x.

Proof. As $\{x_{2n}\}$ converges to x, we have

$$F_{x_n,x}(\varepsilon) \geq t\left(F_{x_n,x_{2n}}\left(\frac{\varepsilon}{2}\right), F_{x_{2n},x}\left(\frac{\varepsilon}{2}\right)\right).$$

Then

$$\lim_{n\to\infty} F_{x_n,x}(\varepsilon) \geq t(1,1), \text{ which gives } \lim_{n\to\infty} F_{x_n,x}(\varepsilon) = 1, \ \forall\, \varepsilon > 0 \text{ and the result follows.}$$

Lemma 2.1. [15] Let $\{p_n\}$ be a sequence in a Menger space (X, $\mathcal{F}$, t) with continuous t-norm and $t(x, x) \geq x$. Suppose, for all x $\in$ [0, 1], there exists k $\in$ (0, 1) such that for all x > 0 and n $\in$ N,

$$F_{p_n, p_{n+1}}(kx) \geq F_{p_{n-1}, p_n}(x)$$

or ,

$$F_{p_n, p_{n+1}}(x) \geq F_{p_{n-1}, p_n}(k^{-1}x).$$

Then $\{p_n\}$ is a Cauchy sequence in X.

3. Main Result.

Theorem 3.1. Let A, B, S, T, L and M be self mappings on a Menger space $(X, \mathcal{F}, t)$ with continuous t-norm t satisfying :

(3.1) $L(X) \subseteq ST(X)$, $M(X) \subseteq AB(X)$;

(3.2) $AB = BA$, $ST = TS$, $LB = BL$, $MT = TM$;

(3.3) One of ST(X), M(X), AB(X) or L(X) is complete;

(3.4) The pairs (L, AB) and (M, ST) are weak compatible;

(3.5) for all $p, q \in X$, $x > 0$ and $0 < a < 1$,

$$[F_{Lp, Mq}(x) + F_{ABp, Lp}(x)][F_{Lp, Mq}(x) + F_{STq, Mq}(x)] \geq 4[F_{ABp, Lp}(x/\alpha)][F_{Mq, STq}(x)].$$

Then A, B, S, T, L and M have a unique common fixed point in X.

Proof. Let $x_0 \in X$. From condition (3.1) there exist $x_1, x_2 \in X$ such that

$$Lx_0 = STx_1 = y_0 \quad \text{and} \quad Mx_1 = ABx_2 = y_1.$$

Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$Lx_{2n} = STx_{2n+1} = y_{2n} \quad \text{and} \quad Mx_{2n+1} = ABx_{2n+2} = y_{2n+1} \quad \text{for } n = 0, 1, 2, \dots .$$

First of all, we show that $\{y_n\}$ is a Cauchy sequence in X.

Step 1. Putting $p = x_{2n}$, $q = x_{2n+1}$ for $x > 0$ in (3.5), we get

$$[F_{Lx_{2n}, Mx_{2n+1}}(x) + F_{ABx_{2n}, Lx_{2n}}(x)][F_{Lx_{2n}, Mx_{2n+1}}(x) + F_{STx_{2n+1}, Mx_{2n+1}}(x)]$$
$$\geq 4[F_{ABx_{2n}, Lx_{2n}}(x/\alpha)][F_{Mx_{2n+1}, STx_{2n+1}}(x)]$$

or, $$[F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)][F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n}, y_{2n+1}}(x)] \geq 4[F_{y_{2n-1}, y_{2n}}(x/\alpha)][F_{y_{2n+1}, y_{2n}}(x)]$$

or, $$2F_{y_{2n}, y_{2n+1}}(x)[F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)] \geq 4[F_{y_{2n-1}, y_{2n}}(x/\alpha)][F_{y_{2n+1}, y_{2n}}(x)]$$

or, $$F_{y_{2n}, y_{2n+1}}(x)[F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)] \geq 2[F_{y_{2n-1}, y_{2n}}(x/\alpha)][F_{y_{2n}, y_{2n+1}}(x)]$$

or, $$[F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)] \geq 2[F_{y_{2n-1}, y_{2n}}(x/\alpha)]$$

or, $$F_{y_{2n}, y_{2n+1}}(x) \geq F_{y_{2n-1}, y_{2n}}(x/\alpha). \tag{3.6}$$

Similarly,

$$F_{y_{2n-1}, y_{2n}}(x/\alpha) \geq F_{y_{2n-2}, y_{2n-1}}(x/\alpha^2). \tag{3.7}$$

From (3.6) and (3.7), it follows that

$$F_{y_{2n}, y_{2n+1}}(x) \geq F_{y_{2n-1}, y_{2n}}(x/\alpha) \geq F_{y_{2n-2}, y_{2n-1}}(x/\alpha^2).$$

By repeated application of above inequality, we get

$$F_{y_{2n}, y_{2n+1}}(x) \geq F_{y_{2n-1}, y_{2n}}(x/\alpha) \geq F_{y_{2n-2}, y_{2n-1}}(x/\alpha^2)$$
$$\geq \dots \geq F_{y_0, y_1}(x/\alpha^n).$$

Therefore, by lemma 2.1, $\{y_n\}$ is a Cauchy sequence in X.

Case I. ST(X) is complete. In this case $\{y_{2n}\} = \{STx_{2n+1}\}$ is a Cauchy sequence in ST(X), which is complete. Thus $\{y_{2n+1}\}$ converges to some $z \in ST(X)$. By proposition 2.3, we have

$$\{Mx_{2n+1}\} \to z \quad \text{and} \quad \{STx_{2n+1}\} \to z, \tag{3.8}$$

$$\{Lx_{2n}\} \to z \quad \text{and} \quad \{ABx_{2n}\} \to z. \tag{3.9}$$

As $z \in ST(X)$ there exists $v \in X$ such that $z = STv$.

Step I. Putting $p = x_{2n}$ and $q = v$ for $x > 0$ in (3.5), we get

$$[F_{Lx_{2n}, Mv}(x) + F_{ABx_{2n}, Lx_{2n}}(x)][F_{Lx_{2n}, Mv}(x) + F_{STv, Mv}(x)] \geq 4[F_{ABx_{2n},Lx_{2n}}(x/\alpha)][F_{Mv, STv}(x)].$$

Letting $n \to \infty$, we get

$$[F_{z, Mv}(x) + F_{z, z}(x)][F_{z, Mv}(x) + F_{z, Mv}(x)] \geq 4[F_{z, z}(x/\alpha)][F_{Mv, z}(x)],$$

i.e. $F_{z, Mv}(x) \geq 1$, yields $Mv = z$.

Hence, $STv = z = Mv$.

As (M, ST) is weakly compatible, we have

$$STz = Mz.$$

Step II. Putting $p = x_{2n}$, $q = z$ for $x > 0$ in (3.5), we get

$$[F_{Lx_{2n}, Mz}(x) + F_{ABx_{2n}, Lx_{2n}}(x)][F_{Lx_{2n}, Mz}(x) + F_{STz, Mz}(x)] \geq 4[F_{ABx_{2n}, Lx_{2n}}(x/\alpha)][F_{Mz, STz}(x)].$$

Letting $n \to \infty$ and using $STz = Mz$, we get

$$[F_{z, Mz}(x) + F_{z, z}(x)][F_{z, Mz}(x) + F_{Mz, Mz}(x)] \geq 4[F_{z, z}(x/\alpha)][F_{Mz, Mz}(x)],$$

i.e. $F_{z, Mz}(x) \geq 1$, yields $z = Mz$.

Step III. Putting $p = x_{2n}$ and $q = Tz$ for $x > 0$ in (3.5), we get

$$[F_{Lx_{2n}, MTz}(x) + F_{ABx_{2n}, Lx_{2n}}(x)][F_{Lx_{2n}, MTz}(x) + F_{STTz, MTz}(x)] \geq 4[F_{ABx_{2n}, Lx_{2n}}(x/\alpha)][F_{MTz, STTz}(x)].$$

As $MT = TM$ and $ST = TS$ we have $MTz = TMz = Tz$ and $ST(Tz) = T(STz) = Tz$.

Letting $n \to \infty$, we get

$$[F_{z,Tz}(x) + F_{z,z}(x)][F_{z,Tz}(x) + F_{Tz,Tz}(x)] \geq 4[F_{z,z}(x/\alpha)][F_{Tz,Tz}(x)],$$

i.e. $F_{z, Tz}(x) \geq 1$, yields $Tz = z$.

Now $STz = Tz = z$ implies $Sz = z$.

Hence $Sz = Tz = Mz = z$.

Step IV. As $M(X) \subseteq AB(X)$, there exists $w \in X$ such that

$$z = Mz = ABw.$$

Putting $p = w$ and $q = x_{2n+1}$ for $x > 0$ in (3.5), we get

$$[F_{Lw, Mx_{2n+1}}(x) + F_{ABw,Lw}(x)][F_{Lw,Mx_{2n+1}}(x) + F_{STx_{2n+1}, Mx_{2n+1}}(x)] \geq 4[F_{ABw, Lw}(x/\alpha)][F_{Mx_{2n+1}, STx_{2n+1}}(x)].$$

Letting $n \to \infty$, we get

$$[F_{Lw, z}(x) + F_{z, Lw}(x)][F_{Lw, z}(x) + F_{z, z}(x)] \geq 4[F_{z, Lw}(x/a)][F_{z, z}(x)],$$

i.e. $F_{Lw, z}(x) \geq 1$, yields $Lw = z$.

Therefore, $ABz = Lz$.

Step V. Putting $p = z$ and $q = x_{2n+1}$ for $x > 0$ in (3.5), we get

$$[F_{Lz, Mx_{2n+1}}(x) + F_{ABz, Lz}(x)][F_{Lz, Mx_{2n+1}}(x) + F_{STx_{2n+1}, Mx_{2n+1}}(x)] \geq 4[F_{ABz,Lz}(x/\alpha)][F_{Mx_{2n+1}, STx_{2n+1}}(x)].$$

Letting $n \to \infty$, we get

$$[F_{Lz, z}(x) + F_{z, Lz}(x)][F_{Lz, z}(x) + F_{z, z}(x)] \geq 4[F_{z, Lz}(x/\alpha)][F_{z, z}(x)],$$

i.e. $F_{Lz, z}(x) \geq 1$, yields $Lz = z$.

Therefore, $ABz = Lz = z$.

ISSN 2347-1921

Step VI. Putting $p = Bz$ and $q = x_{2n+1}$ for $x > 0$ in (3.5), we get

$$[F_{LBz,\, Mx_{2n+1}}(x) + F_{ABBz,\, LBz}(x)][F_{LBz,\, Mx_{2n+1}}(x) + F_{STx_{2n+1},\, Mx_{2n+1}}(x)]$$

$$\geq 4[F_{ABBz,LBz}(x/\alpha)][F_{Mx_{2n+1},\, STx_{2n+1}}(x)].$$

As $BL = LB$, $AB = BA$, so we have

$$L(Bz) = B(Lz) = Bz \quad \text{and} \quad AB(Bz) = B(ABz) = Bz.$$

Letting $n \to \infty$, we get

$$[F_{Bz,\, z}(x) + F_{Bz,\, Bz}(x)][F_{Bz,\, z}(x) + F_{z,\, z}(x)] \geq 4[F_{Bz,\, Bz}(x/\alpha)][F_{z,\, z}(x)],$$

i.e. $F_{Bz,\, z}(x) \geq 1$, yields $Bz = z$ and $ABz = z$ implies $Az = z$.

Therefore, $Az = Bz = Lz = z$.

Combining the results from different steps, we get

$$Az = Bz = Lz = Mz = Tz = Sz = z.$$

Hence, the six self maps have a common fixed point in this case.

Case when $L(X)$ is complete follows from above case as $L(X) \subseteq ST(X)$.

Case II. AB(X) is complete. This case follows by symmetry. As $M(X) \subseteq AB(X)$, therefore the result also holds when $M(X)$ is complete.

Uniqueness. Let u be another common fixed point of A, B, S, T, L and M; then $Au = Bu = Su = Tu = Lu = Mu = u$.

Putting $p = z$ and $q = u$ for $x > 0$ in (3.5), we get

$$[F_{Lz,\, Mu}(x) + F_{ABz,\, Lz}(x)][F_{Lz,\, Mu}(x) + F_{STu,\, Mu}(x)] \geq 4[F_{ABz,\, Lz}(x/\alpha)][F_{Mu,\, STu}(x)].$$

Letting $n \to \infty$, we get

$$[F_{z,\, u}(x) + F_{z,\, z}(x)][F_{z,\, u}(x) + F_{u,\, u}(x)] \geq 4[F_{z,\, z}(x/\alpha)][F_{u,\, u}(x)],$$

i.e. $F_{z,u}(x) \geq 1$, yields $z = u$.

Therefore, z is a unique common fixed point of A, B, S, T, L and M.

This completes the proof.

Remark 3.1. In view of proposition 2.2, $t(a, b) = \min\{a, b\}$. Thus, theorem 3.1 is an alternate result of Jain et. al. [5] reducing the compatibility of the pair (L, AB) to its weak-compatibility and dropping the condition of continuity in a Menger space with continuous t-norm.

Remark 3.2. If we take $B = T = I$, the identity map on X in theorem 3.1, then the condition (3.2) is satisfied trivially and we get

Corollary 3.1. Let A, S, L and M be self mappings on a Menger space $(X, \mathcal{F}, t)$ with continuous t-norm t satisfying :

(3.10) $L(X) \subseteq S(X)$, $M(X) \subseteq A(X)$;

(3.11) One of $S(X)$, $M(X)$, $A(X)$ or $L(X)$ is complete;

(3.12) The pairs (L, A) and (M, S) are weak compatible;

(3.13) for all $p, q \in X$, $x > 0$ and $0 < \alpha < 1$,

$$[F_{Lp,\, Mq}(x) + F_{Ap,\, Lp}(x)][F_{Lp,\, Mq}(x) + F_{Sq,\, Mq}(x)] \geq 4[F_{Ap,\, Lp}(x/\alpha)][F_{Mq,\, Sq}(x)].$$

Then A, S, L and M have a unique common fixed point in X.

Now, we give an example in support of Corollary 3.1.

Example 3.1. Let (X, d) be a metric space where $X = [0, 2]$ and $(X, \mathcal{F}, t)$ be the induced Menger space with $F_{p,q}(\varepsilon) = H(\varepsilon - d(p, q))$, $\forall\, p, q \in X$ and $\varepsilon > 0$.

Define self maps L, M, A and S as follows :

ISSN 2347-1921

$$L(x) = M(x) = \begin{cases} 0, & x \in \left[0, \frac{4}{5}\right] \\ 2 - x, & \text{otherwise}, \end{cases} \quad A(x) = \begin{cases} 0, & x \in \left[0, \frac{3}{4}\right] \\ 2 - x, & \text{otherwise} \end{cases} \text{ and } S(x) = \begin{cases} 0, & x \in \left[0, \frac{2}{3}\right] \\ 2 - x, & \text{otherwise}. \end{cases}$$

Then $L(X) = M(X) = \left[0, \frac{6}{5}\right)$, $A(X) = \left[0, \frac{5}{4}\right]$ and $S(X) = \left[0, \frac{4}{3}\right)$. Hence containment condition (3.10) is satisfied. Also, the pairs (L, A) and (M, S) are weak compatible and A(X) is complete. Thus all the conditions of Corollary 3.1 are satisfied and 0 is the unique common fixed point of self maps L, M, A and S.

If we take A = I, the identity map in Corollary 3.1, we get

Corollary 3.2. Let S, L and M be self mappings on a complete Menger space $(X, \mathcal{F}, t)$ satisfying :

(3.14) $L(X) \subseteq S(X)$;

(3.15) The pair (M, S) is weak compatible;

(3.16) for all $p, q \in X$, $x > 0$ and $0 < \alpha < 1$,

$$[F_{Lp,\, Mq}(x) + F_{p,\, Lp}(x)][F_{Lp,\, Mq}(x) + F_{Sq,\, Mq}(x)] \geq 4[F_{p,\, Lp}(x/\alpha)][F_{Mq,\, Sq}(x)].$$

Then S, L and M have a unique common fixed point in X.

If we take S = A = I, the identity map on X and writing $L = T_i$ and $M = T_j$ in Corollary 3.1, we get

Corollary 3.3. Let T_i and T_j be self mappings on a Menger space $(X, \mathcal{F}, t)$ with continuous t-norm t satisfying :

(3.17) for all $p, q \in X$, $x > 0$ and $0 < \alpha < 1$,

$$[F_{T_ip,\, T_jq}(x) + F_{p,\, T_ip}(x)][F_{T_ip,\, T_jq}(x) + F_{q,\, T_jq}(x)] \geq 4[F_{p,\, T_ip}(x/\alpha)][F_{T_jq,\, q}(x)].$$

Then T_i and T_j have a unique common fixed point in X.

References.

[1] Cho, Y.J., Murthy, P.P. and Stojakovik, M., Compatible mappings of type (A) and common fixed point in Menger space, Comm. Korean Math. Soc. 7 (2), (1992), 325-339.

[2] Jain, A. and Chaudhary, B., On common fixed point theorems for semi-compatible and occasionally weakly compatible mappings in Menger space, International Journal of Research and Reviews in Applied Sciences, Vol. 14 (3), (2013), 662-670.

[3] Jain, A. and Singh, B., Common fixed point theorem in Menger space through compatible maps of type (A), Chh. J. Sci. Tech. 2 (2005), 1-12.

[4] Jain, A. and Singh, B., A fixed point theorem in Menger space through compatible maps of type (A), V.J.M.S. 5(2), (2005), 555-568.

[5] Jain, A. and Singh, B., Common fixed point theorems in Menger spaces, The Aligarh Bull. of Maths. Vol. 25 (1), (2006), 23-31.

[6] Jungck, G., Compatible mappings and common fixed points, Internat. J. Math. and Math. Sci. 9(4), (1986), 771-779.

[7] Jungck, G. and Rhoades, B.E., Fixed points for set valued functions without continuity, Indian J. Pure Appl. Math. 29(1998), 227-238.

[8] Menger, K., Statistical metrics, Proc. Nat. Acad. Sci. USA. 28(1942), 535 -537.

[9] Mishra, S.N., Common fixed points of compatible mappings in PM-spaces, Math. Japon. 36(2), (1991), 283-289.

[10] Patel, R.N. and Patel, D., Fixed points of compatible mappings of type (A) on Menger space, V.J.M.S. 4(1), (2004), 185-189.

[11] Schweizer, B. and Sklar, A., Statistical metric spaces, Pacific J. Math. 10 (1960), 313-334.

[12] Sehgal, V.M. and Bharucha-Reid, A.T., Fixed points of contraction maps on probabilistic metric spaces, Math. System Theory 6(1972), 97- 102.

[13] Sessa, S., On a weak commutativity condition of mappings in fixed point consideration, Publ. Inst. Math. Beograd 32(46), (1982), 146-153.

[14] Singh, M., Sharma, R.K. and Jain, A., Compatible mappings of type (A) and common fixed points in Menger space, Vikram Math. J. 20 (2000), 68-78.

[15] Singh, S.L. and Pant, B.D., Common fixed point theorems in probabilistic metric spaces and extension to uniform spaces, Honam. Math. J. 6 (1984), 1-12.

ISSN 2347-1921

Biography -

Dr. V. K. Gupta presently working on the post of Professor and Head, Department of Mathematics, Govt. Madhav Science College, Ujjain. He has 33 years of teaching experience. His area of interest includes fixed point theory, mathematical modeling, number theory and approximation theory. Eight students have got Ph.D. under his supervision. He has authored five books. He has sent proposal to Govt. of India, Ministry of Science and Technology, New Delhi for declaring 22nd December as “Mathematics Day” and Year 2012 as “Mathematics Year”. He has published more than 100 research papers in national and international journals and referred many national and international journals.

Dr. Arihant Jain did his Post Graduation in Mathematics in the year 2000 from School of Studies in Mathematics, Vikram University, Ujjain. He has been awarded doctorate degree in Mathematics in the year 2007 from School of Studies in Mathematics, Vikram University, Ujjain on Fixed Point Theory. He has published 97 research papers in national and international journals of repute. He has a post graduate teaching experience of 7 years and graduate teaching experience of 6 years. Two students have got Ph.D. under his guidance. Presently, he is working on the post of Academic Dean, Professor and Head, Department of Applied Mathematics, Shri Guru Sandipani Institute of Technology and Science, Ujjain and actively engaged with his researchers.

Mr. Ramesh Bhinde did his M.Sc. in Mathematics from Devi Ahilya University, Indore in the year 2003. Presently, he is working on the post of Assistant Professor (Mathematics) at Govt. P. G. College, Alirajpur (M.P.).