ISSN 2347-1921

Continuous Generalized Hankel-type integral wavelet transformation

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ABSTRACT

Using the theory of Hankel-type convolution, continuous generalized Hankel-type wavelet integral transformation is defined. The generalized Hankel-type integral wavelet transformation is developed. Using the developed theory of generalized Hankel-type convolution, the generalized Hankel-type translation is introduced. Properties of the kernel $D_{\mu,\alpha,\beta,\nu}(x,y,z)$ are developed in the study. Using the properties of kernel, the generalized Hankel-type wavelet transformation is defined. The existence of the generalized Hankel-type integral wavelet transformation is given by a theorem. The boundedness and inversion formula for the generalized Hankel-type integral wavelet transformation is obtained. A basic wavelet which defines continuous generalized Hankel-type integral wavelet transformation, its admissibility conditions and the wavelet to the function is proved. Examples have been shown to explain the studied continuous generalized Hankel-type integral wavelet transformation.

INDEXING TERMS/KEYWORDS

Continuous generalized Hankel-type integral wavelet transformation; generalized Hankel-type transformation; Hankel-type Convolution.

SUBJECT CLASSIFICATION

44A20, 42C40, 46, 33C05

Council for Innovative Research
Peer Review Research Publishing System
Journal: JOURNAL OF ADVANCES IN MATHEMATICS
Vol.11, No.3
www.cirjam.com , editorjam@gmail.com

1. INTRODUCTION

Malgonde [1] investigated the following generalized Hankel-type integral transformation

$$F_1(t)=\left(F_{1,\mu,\alpha,\beta,\nu} f\right)(t)=\nu\beta\, t^{-1-2\alpha+2\nu}\int_0^\infty (xt)^\alpha J_\mu\left[\beta(xt)^\nu\right] f(x)\,dx \quad , \qquad (1.1)$$

$J_\mu(x)$ being the Bessel function of the first kind of order $\mu\geq -1/2$.

We define $L_p(0,\infty), 1\leq p\leq\infty,$ as the space of real measurable function ϕ on $(0,\infty)$ for which

$$\|\phi\|_{\mu,\nu,p}=\left(\int_0^\infty \left|x^{\mu\nu-\alpha}\phi(x)\right|^p \frac{dx}{x}\right)^{1/p}, 1\leq p<\infty$$

$$\|\phi\|_\infty = \operatorname*{ess\,sup}_{0<x<\infty}\left|x^{\mu\nu-\alpha}\phi(x)\right|<\infty .$$

For each $\phi\in L_1(0,\infty)$, generalized Hankel-type integral transformation of ϕ is defined by

$$\hat{\phi}(x)=\nu\beta\, t^{-1-2\alpha+2\nu}\int_0^\infty (xt)^\alpha J_\mu\left[\beta(xt)^\nu\right]\phi(t)dt \quad , 0<t<\infty.$$

From [1] we know that $\hat{\phi}(x)$ is bounded and continuous on $(0,\infty)$ and $\left\|\hat{\phi}(x)\right\|_\infty\leq\|\phi\|_1$.

If $f(x)$ is of bounded variation into a neighborhood of the point $x=x_0>0, \mu\geq -1/2$ and the integral $\int_0^\infty |f(x)| x^{\alpha-\nu/2}$ exists, then the inversion formula in [2] is given by

$$\lim_{R\to\infty}\nu\beta\, y_0^{-1-2\alpha+2\nu}\int_0^R (x_0 y)^\alpha J_\mu\left[\beta(x_0 y)^\nu\right] F_1(y)\,dy=\frac{1}{2}\left[f(x_0+0)+f(x_0-0)\right].$$

If $f(x)x^{-\alpha-\mu}$ and $F_2(y)y^{\mu-\alpha-1+2\nu}$ are in $L_1(0,\infty)$, for

$$F_1(t)=\left(F_{1,\mu,\alpha,\beta,\nu} f(x)\right)(t)=\nu\beta\, t^{-1-2\alpha+2\nu}\int_0^\infty (xt)^\alpha J_\mu\left[\beta(xt)^\nu\right] f(x)\,dx \quad ,$$

and

$$F_2(t)=\left(F_{2,\mu,\alpha,\beta,\nu} g(x)\right)(t)=\nu\beta\, t^{-1-2\alpha+2\nu}\int_0^\infty (xt)^\alpha J_\mu\left[\beta(xt)^\nu\right] g(x)\,dx \quad , \text{ for } \mu\geq -1/2,$$

the following mixed Parseval formula holds for F_1-transformation by [2];

$$\int_0^\infty f(x)g(x)\,dx=\int_0^\infty F_1(y)F_2(y)\,dy.$$

To define the generalized Hankel-type Convolution, we need to introduce generalized Hankel-type translation. Define

$$D_{\mu,\alpha,\beta,\nu}(x,y,z)$$
$$=\int_0^\infty t^{-\mu\nu-\alpha}\nu\beta\, t^{-1-2\alpha+2\nu}(xt)^\alpha J_\mu\left[\beta(xt)^\nu\right]\nu\beta\, t^{-1-2\alpha+2\nu}(yt)^\alpha J_\mu\left[\beta(yt)^\nu\right]\nu\beta\, t^{-1-2\alpha+2\nu}(zt)^\alpha J_\mu\left[\beta(zt)^\nu\right]dt. \qquad (1.2)$$

Properties of the kernel $D_{\mu,\alpha,\beta,\nu}(x,y,z)$:

Following [3] properties are established:

i) For $0<x,y<\infty$ and $0\le t<\infty$, we have

$$\int_0^\infty \nu\beta t^{\mu\nu+\alpha} z^{-1-2\alpha+2\nu}\ (zt)^\alpha J_\mu\left[\beta(zt)^\nu\right] D_{\mu,\alpha,\beta,\nu}(x,y,z)\,dz=(\nu\beta)^2(xy)^{-1-2\alpha+2\nu}(xt)^\alpha J_\mu\left[\beta(xt)^\nu\right](yt)^\alpha J_\mu\left[\beta(yt)^\nu\right]$$

Proof:

$$D_{\mu,\alpha,\beta,\nu}(x,y,z)=(\nu\beta)^3\int_0^\infty t^{-\mu\nu-\alpha}\ z^{-1-2\alpha+2\nu}(zt)^\alpha J_\mu\left[\beta(zt)^\nu\right]\left[x^{-1-2\alpha+2\nu}(xt)^\alpha J_\mu\left[\beta(xt)^\nu\right]y^{-1-2\alpha+2\nu}(yt)^\alpha J_\mu\left[\beta(yt)^\nu\right]\right]\ dz$$

$$=\int_0^\infty \nu\beta t^{-\mu\nu-\alpha} z^{-1-2\alpha+2\nu}\left[(\nu\beta)^2(xy)^{-1-2\alpha+2\nu}(xt)^\alpha J_\mu\left[\beta(xt)^\nu\right](yt)^\alpha J_\mu\left[\beta(yt)^\nu\right]\right]\ (zt)^\alpha J_\mu\left[\beta(zt)^\nu\right]dz$$

$$=t^{-\mu\nu-\alpha}F_{1,\mu,\alpha,\beta,\nu}\left\{(\nu\beta)^2(xy)^{-1-2\alpha+2\nu}(xt)^\alpha J_\mu\left[\beta(xt)^\nu\right](yt)^\alpha J_\mu\left[\beta(yt)^\nu\right]\right\}$$

$$F^{-1}_{1,\mu,\alpha,\beta,\nu}\left\{t^{\mu\nu+\alpha}D_{\mu,\alpha,\beta,\nu}(x,y,z)\right\}=(\nu\beta)^2(xy)^{-1-2\alpha+2\nu}(xt)^\alpha J_\mu\left[\beta(xt)^\nu\right](yt)^\alpha J_\mu\left[\beta(yt)^\nu\right]$$

Applying the inversion formula of generalized Hankel-type integral transformation to (1.2)

$$\int_0^\infty \nu\beta t^{\mu\nu+\alpha} z^{-1-2\alpha+2\nu}\ (zt)^\alpha J_\mu\left[\beta(zt)^\nu\right] D_{\mu,\alpha,\beta,\nu}(x,y,z)\,dz$$
$$=(\nu\beta)^2(xy)^{-1-2\alpha+2\nu}(xt)^\alpha J_\mu\left[\beta(xt)^\nu\right](yt)^\alpha J_\mu\left[\beta(yt)^\nu\right].$$

and hence the result. In particular, taking $t=0$, gives

ii) $\int_0^\infty \nu\beta t^{\mu\nu+\alpha} z^{-1-2\alpha+2\nu}\ (zt)^\alpha J_\mu\left[\beta(zt)^\nu\right] D_{\mu,\alpha,\beta,\nu}(x,y,z)\,dz=1,$

i.e. for which $x,y>0, D_{\mu,\alpha,\beta,\nu}(x,y,z)$ belongs to $L^1_{0,\alpha,\beta,\nu,\mu}(0,\infty)$.

iii) $0<x,y,z<\infty, D_{\mu,\alpha,\beta,\nu}(x,y,z)\ge 0$.

iv) $D_{\mu,\alpha,\beta,\nu}(x,y,z)=D_{\mu,\alpha,\beta,\nu}(y,x,z)=D_{\mu,\alpha,\beta,\nu}(z,x,y)=\ldots$

The generalized Hankel-type integral translation T_y of $\phi\in L_p(0,\infty), 1\le p\le\infty$, is defined by

$$T_y\phi(x)=\phi(x,y)=\int_0^\infty \phi(z)D_{\mu,\alpha,\beta,\nu}(x,y,z)\,dz,\ 0<x,y<\infty.$$

The map $y\to T_y\phi$ is continuous from $(0,\infty)$ into $(0,\infty)$.

Let $p,q,r\in[1,\infty)$ and $\frac{1}{r}=\frac{1}{p}+\frac{1}{q}-1.$ The generalized Hankel-type integral convolution of

$\phi\in L_p(0,\infty)$ and $\psi\in L_p(0,\infty)$ is defined by $(\phi\#\psi)(x)=\int_0^\infty \phi(x,y)\psi(y)\,dy.$

In [4] the integral is convergent for almost all $x, 0<x<\infty$ and $\|\phi\#\psi\|_r\le\|\phi\|_p\|\psi\|_q$.

Moreover, $p=\infty,$ then $(\phi\#\psi)(x)$ is defined for all $x, 0<x<\infty$ and is continuous.

If $\phi,\psi\in L_1(0,\infty)$, then $(\phi\#\psi)\wedge(t)=\hat{\phi}(t)\hat{\psi}(t),\ 0\le t<\infty.$

In this paper, in terms of the aforesaid generalized Hankel-type translation T_y and dilation D_a defined by

$$D_{\mu,\alpha,\beta,\nu,a}\phi(x,y)=a^{-2(\mu\nu-\alpha)-3\nu}\phi(x/a,y/a) \tag{1.3}$$

is a continuous generalized Hankel-type integral wavelet transformation is defined. Its continuity and boundedness properties are established. An inversion formula is obtained.

2. CONTINUOUS GENERALIZED HANKEL-TYPE INTEGRAL WAVELET TRANSFORMATION

Let $\psi\in L_p(0,\infty), 1\le p<\infty$ be given. For $b\ge 0$ and $a>0$ define the generalized Hankel-type integral wavelet transformation

$$\psi_{b,a}(x)=D_{\mu,\alpha,\beta,\nu,a}T_y\psi(x)=D_{\mu,\alpha,\beta,\nu,a}\psi(b,x)=a^{-2(\mu\nu-\alpha)-3\nu}\psi(b/a,x/a)$$
$$=a^{-2(\mu\nu-\alpha)-3\nu}\int_0^\infty D_{\mu,\alpha,\beta,\nu}(b/a,x/a,z)\psi(z)dz,$$

the integral being convergent by virtue of [8].

Using the wavelet $\psi_{b,a}$, define the generalized Hankel-type integral wavelet transformation,

$$\begin{aligned}H_{1,\alpha,\beta,\nu,\mu}(b,a)&=(H_{1,\alpha,\beta,\nu,\mu,\psi}f)(b,a)\\&=\langle f(t),\psi_{b,a}(t)\rangle\\&=\int_0^\infty f(t)\overline{\psi_{b,a}(t)}dt\\&=a^{-2(\mu\nu-\alpha)-3\nu}\int_0^\infty\int_0^\infty f(t)\overline{\psi(z)}D_{\mu,\alpha,\beta,\nu}(b/a,t/a,z)dzdt\end{aligned}$$

provided the integral is convergent.

The continuity of the generalized Hankel-type integral wavelet follows from the boundedness property of the generalized Hankel-type translation [5].

Lemma 1: Let $\psi\in L_p(0,\infty), 1\le p<\infty.$ Then for $y\ge 0,$ the map $y\to T_y f$ is continuous from $L_p(0,\infty)$ into $L_p(0,\infty)$. The function $\psi_{b,a}$ is defined almost everywhere on $[0,\infty)$, and $\|\psi_{b,a}(x)\|_p\le a^{(2(\mu\nu-\alpha)+3\nu)(1/p-1)}\|\psi\|_p$.

The existence of the generalized Hankel-type transformation is given by the following theorem.

Theorem 2. Let $f\in L_p(0,\infty)$ and $\psi\in L_p(0,\infty)$ with $1\le p,q<\infty$ and $\frac{1}{p}+\frac{1}{q}=1;\ H_{1,\alpha,\beta,\nu,\mu}(b,a)=(H_{1,\alpha,\beta,\nu,\mu,\psi}f)(b,a)$ be the continuous wavelet transform. Then

1) $(H_{1,\alpha,\beta,\nu,\mu}f)(b,a)$ is continuous on $(0,\infty)\times(0,\infty),$

2) $\|((H_{1,\alpha,\beta,\nu,\mu,\psi}f)(b,a)f)(b,a)\|_r\le a^{2(\mu\nu-\alpha)+3\nu}\|f\|_p\|\psi\|_q,\ \frac{1}{r}=\frac{1}{p}+\frac{1}{q}-1,\ 1\le p,q,r<\infty,$

3) $\|((H_{1,\alpha,\beta,\nu,\mu,\psi}f)(b,a)f)(b,a)\|_\infty\le a^{(2(\mu\nu-\alpha)+3\nu)\left(\frac{1}{q}-1\right)}\|f\|_p\|\psi\|_q,\ \frac{1}{p}+\frac{1}{q}=1.$

Proof.

1) Let (b_0, a_0) be an arbitrary but fixed point in $(0,\infty)\times(0,\infty)$. Then by Hölder's inequality,

$$\left|\left(H_{1,\alpha,\beta,\nu,\mu} f\right)(b,a)-\left(H_{1,\alpha,\beta,\nu,\mu} f\right)(b_0,a_0)\right|$$

$$\leq a^{-2(\mu\nu-\alpha)-3\nu}\int_0^\infty\int_0^\infty\left|f(t)\psi(z)\left[D_{\mu,\alpha,\beta,\nu}(b/a,t/a,z)-D_{\mu,\alpha,\beta,\nu}(b_0/a_0,t/a_0,z)\right]\right|dtdz$$

$$\leq a^{-2(\mu\nu-\alpha)-3\nu}\left[\int_0^\infty\int_0^\infty\left|f(t)\right|^p\left|D_{\mu,\alpha,\beta,\nu}(b/a,t/a,z)-D_{\mu,\alpha,\beta,\nu}(b_0/a_0,t/a_0,z)\right|dtdz\right]^{1/p}$$

$$\times\left[\int_0^\infty\int_0^\infty\left|\psi(z)\right|^q\left|D_{\mu,\alpha,\beta,\nu}(b/a,t/a,z)-D_{\mu,\alpha,\beta,\nu}(b_0/a_0,t/a_0,z)\right|dtdz\right]^{1/q}$$

Since by (9) $\int_0^\infty\left|D_{\mu,\alpha,\beta,\nu}(b/a,t/a,z)-D_{\mu,\alpha,\beta,\nu}(b_0/a_0,t/a_0,z)\right|dt\leq 2$, by dominated convergence theorem and continuity of $D_{\mu,\alpha,\beta,\nu}(b/a,t/a,z)$ in the variables b and a, we have $\lim_{\substack{b\to b_0\\ a\to a_0}}\left|\left(H_{1,\alpha,\beta,\nu,\mu} f\right)(b,a)-\left(H_{1,\alpha,\beta,\nu,\mu} f\right)(b_0,a_0)\right|=0$. This proves that $H(b,a)$ is continuous on $(0,\infty)\times(0,\infty)$.

2) $$\left\|\left(\left(H_{1,\alpha,\beta,\nu,\mu,\psi} f\right)(b,a)\, f\right)(b,a)\right\|_r\leq a^{2(\mu\nu-\alpha)+3\nu}\left\|f\right\|_p\left\|\psi\right\|_q,\ \frac{1}{r}=\frac{1}{p}+\frac{1}{q}-1,\ 1\leq p,q,r<\infty.$$

3) It can be proved using Hölder's inequality.

3. AN INVERSION FORMULA

In this section, we show that the function f can be recovered from its wavelet transform when the wavelet ψ satisfies admissibility condition.

Theorem 3. Let $\psi\in L^2(\mathrm{R}_+)$ be a basic wavelet which defines generalized Hankel-type wavelet integral transformation. Then, for $A_\psi=\int_0^\infty w^{-2(\mu\nu-\alpha)-3\nu}\left|\hat{\psi}(w)\right|^2 dw>0,$ we have

$$\int_0^\infty\int_0^\infty\left(\left(H_{1,\alpha,\beta,\nu,\mu,\psi} f\right)(b,a)\, f\right)(b,a)\overline{\left(\left(H_{1,\alpha,\beta,\nu,\mu,\psi} f\right)(b,a)\, g\right)(b,a)}\, a^{-2(\mu\nu-\alpha)-3\nu}dadb$$

$$=A_\psi\left\langle f,g\right\rangle \text{ for all } f,g\in L^2(\mathrm{R}_+).$$

Proof. The representation for $\left(H_{1,\alpha,\beta,\nu,\mu,\psi} f\right)(b,a)$, can be expressed as

$$\left(H_{1,\alpha,\beta,\nu,\mu,\psi} f\right)(b,a)$$

$$=\int_0^\infty\int_0^\infty f(t)\overline{\psi(z)}D_{\mu,\alpha,\beta,\nu}(b/a,t/a,z)\,dzdt$$

$$= a^{-2(\mu\nu-\alpha)-3\nu}\int_0^\infty\int_0^\infty \hat{f}(x/a)\overline{\psi(z)}\left[\begin{array}{l}\left\{\nu\beta\left(\frac{b}{a}\right)^{-1-2\alpha+2\nu}\left(\frac{xb}{a}\right)^{\alpha}J_\mu\left[\beta\left(\frac{xb}{a}\right)^{\nu}\right]\right\}\\ \left\{\nu\beta\, z^{-1-2\alpha+2\nu}(zax)^{\alpha}J_\mu\left[\beta(zax)^{\nu}\right]\right\}\end{array}\right]dxdz$$

$$= a^{-2(\mu\nu-\alpha)-3\nu}\int_0^\infty \hat{f}(x/a)\overline{\hat{\psi}(x)}\left\{\nu\beta\left(\frac{b}{a}\right)^{-1-2\alpha+2\nu}\left(\frac{xb}{a}\right)^{\alpha}J_\mu\left[\beta\left(\frac{xb}{a}\right)^{\nu}\right]\right\}dx$$

$$= \int_0^\infty \hat{f}(u)\overline{\hat{\psi}(au)}\left\{\nu\beta\, b^{-1-2\alpha+2\nu}(bu)^{\alpha}J_\mu\left[\beta(bu)^{\nu}\right]\right\}du$$

$$= \left(\hat{f}(u)\overline{\hat{\psi}(au)}\right)\wedge(b).$$

Parseval identity yields

$$\int_0^\infty\left(H_{1,\alpha,\beta,\nu,\mu,\psi}f\right)(b,a)\overline{\left(H_{1,\alpha,\beta,\nu,\mu,\psi}f\right)(b,a)}db$$

$$= \int_0^\infty\left(\hat{f}(u)\overline{\hat{\psi}(au)}\right)\wedge(b)\overline{\left(\hat{g}(u)\overline{\hat{\psi}(au)}\right)}\wedge(b)\,db$$

$$= \int_0^\infty\left(\hat{f}(u)\overline{\hat{\psi}(au)}\right)\overline{\left(\hat{g}(u)\overline{\hat{\psi}(au)}\right)}\,du.$$

Now multiplying by $a^{-2(\mu\nu-\alpha)-3\nu}da$ and integrating, we get

$$\int_0^\infty\int_0^\infty\left(H_{1,\alpha,\beta,\nu,\mu,\psi}f\right)(b,a)\overline{\left(H_{1,\alpha,\beta,\nu,\mu,\psi}f\right)(b,a)}db\; a^{-2(\mu\nu-\alpha)-3\nu}da$$

$$= \int_0^\infty\int_0^\infty\left(\hat{f}(u)\overline{\hat{\psi}(au)}\right)\overline{\left(\hat{g}(u)\overline{\hat{\psi}(au)}\right)}\,du.\; a^{-2(\mu\nu-\alpha)-3\nu}da$$

$$= \int_0^\infty \hat{f}(u)\overline{\hat{g}(u)}du\int_0^\infty \hat{\psi}(au)\overline{\hat{\psi}(au)}\;.\; a^{-2(\mu\nu-\alpha)-3\nu}da$$

$$= \int_0^\infty \hat{f}(u)\overline{\hat{g}(u)}du\int_0^\infty \left|\hat{\psi}(au)\right|^2\;.\; a^{-2(\mu\nu-\alpha)-3\nu}da$$

$$= \int_0^\infty \hat{f}(u)\overline{\hat{g}(u)}du\int_0^\infty \left|\hat{\psi}(w)\right|^2\;.\; w^{-2(\mu\nu-\alpha)-3\nu}dw$$

$$= A_\psi\langle f,g\rangle.$$

Notice that admissibility condition requires that $\hat{\psi}(0)=0$. If $\hat{\psi}$ is continuous, it follows that $\int_0^\infty \psi(x)dx=0.$ This justifies the wavelet to the function.

ISSN 2347-1921

Now consider

$\psi\left(\frac{b}{a},\frac{t}{a}\right)=\psi(x,y)$ by putting $\frac{b}{a}=x$ and $\frac{t}{a}=y$. Then $\psi(x,y)=\int_0^\infty \psi(z)D_{\mu,\alpha,\beta,\nu}(b/a,t/a,z)dz.$

Since

$$D_{\mu,\alpha,\beta,\nu}(x,y,z)=\int_0^\infty \xi^{-\mu\nu-\alpha}\begin{bmatrix}\nu\beta\, x^{-1-2\alpha+2\nu}(x\xi)^{\alpha} J_\mu\left[\beta(x\xi)^{\nu}\right] \\ \nu\beta\, y^{-1-2\alpha+2\nu}(y\xi)^{\alpha} J_\mu\left[\beta(y\xi)^{\nu}\right]\nu\beta\, z^{-1-2\alpha+2\nu}(z\xi)^{\alpha} J_\mu\left[\beta(z\xi)^{\nu}\right]d\xi\end{bmatrix}.$$

Substituting the expression becomes

$$\psi(x,y)=\int_0^\infty \psi(z)\left[\int_0^\infty \xi^{-\mu\nu-\alpha}\begin{Bmatrix}\nu\beta\, x^{-1-2\alpha+2\nu}(x\xi)^{\alpha} J_\mu\left[\beta(x\xi)^{\nu}\right] \\ \nu\beta\, y^{-1-2\alpha+2\nu}(y\xi)^{\alpha} J_\mu\left[\beta(y\xi)^{\nu}\right] \\ \nu\beta\, z^{-1-2\alpha+2\nu}(z\xi)^{\alpha} J_\mu\left[\beta(z\xi)^{\nu}\right]d\xi\end{Bmatrix}\right]dz$$

$$=\int_0^\infty\left(\int_0^\infty \psi(z)\nu\beta\, z^{-1-2\alpha+2\nu}(z\xi)^{\alpha} J_\mu\left[\beta(z\xi)^{\nu}\right]dz\right)\xi^{-\mu\nu-\alpha}\begin{Bmatrix}\nu\beta\, x^{-1-2\alpha+2\nu}(x\xi)^{\alpha} J_\mu\left[\beta(x\xi)^{\nu}\right] \\ \nu\beta\, y^{-1-2\alpha+2\nu}(y\xi)^{\alpha} J_\mu\left[\beta(y\xi)^{\nu}\right]\end{Bmatrix}d\xi$$

$$=\int_0^\infty \xi^{-\mu\nu-\alpha}\left\{\nu\beta\, x^{-1-2\alpha+2\nu}(x\xi)^{\alpha} J_\mu\left[\beta(x\xi)^{\nu}\right]\nu\beta\, y^{-1-2\alpha+2\nu}(y\xi)^{\alpha} J_\mu\left[\beta(y\xi)^{\nu}\right]\right\}\left(F_{1,\mu,\alpha,\beta,\nu}\psi\right)(\xi)d\xi.$$

Substitute $\frac{b}{a}=x$ and $\frac{t}{a}=y.$

$$\psi\left(\frac{b}{a},\frac{t}{a}\right)=\int_0^\infty \xi^{-\mu\nu-\alpha}\left\{\nu\beta\left(\frac{b}{a}\right)^{-1-2\alpha+2\nu}\left(\frac{b\xi}{a}\right)^{\alpha} J_\mu\left[\beta\left(\frac{b\xi}{a}\right)^{\nu}\right]\nu\beta\left(\frac{t}{a}\right)^{-1-2\alpha+2\nu}\left(\frac{t\xi}{a}\right)^{\alpha} J_\mu\left[\beta\left(\frac{t\xi}{a}\right)^{\nu}\right]\right\}\left(F_{1,\mu,\alpha,\beta,\nu}\psi\right)(\xi)d\xi.$$

$$\left(H_{1,\alpha,\beta,\nu,\mu,\psi}f\right)(b,a)$$

$$=a^{-2(\mu\nu-\alpha)-3\nu}\int_0^\infty \psi\left(\frac{b}{a},\frac{t}{a}\right)f(t)dt$$

$$=a^{-2(\mu\nu-\alpha)-3\nu}\int_0^\infty\left(\int_0^\infty \xi^{-\mu\nu-\alpha}\begin{Bmatrix}\nu\beta\left(\frac{b}{a}\right)^{-1-2\alpha+2\nu}\left(\frac{b\xi}{a}\right)^{\alpha} J_\mu\left[\beta\left(\frac{b\xi}{a}\right)^{\nu}\right] \\ \nu\beta\left(\frac{t}{a}\right)^{-1-2\alpha+2\nu}\left(\frac{t\xi}{a}\right)^{\alpha} J_\mu\left[\beta\left(\frac{t\xi}{a}\right)^{\nu}\right]\end{Bmatrix}\left(F_{1,\mu,\alpha,\beta,\nu}\psi\right)(\xi)d\xi\right)\times f(t)dt.$$

$$=a^{-2(\mu\nu-\alpha)-3\nu}\int_0^\infty \nu\beta\left(\frac{b}{a}\right)^{-1-2\alpha+2\nu}\left(\frac{b\xi}{a}\right)^{\alpha} J_\mu\left[\beta\left(\frac{b\xi}{a}\right)^{\nu}\right]\left(\int_0^\infty \nu\beta\left(\frac{t}{a}\right)^{-1-2\alpha+2\nu}\left(\frac{t\xi}{a}\right)^{\alpha} J_\mu\left[\beta\left(\frac{t\xi}{a}\right)^{\nu}\right]f(t)dt\right)$$
$$\times\xi^{-\mu\nu-\alpha}\left(F_{1,\mu,\alpha,\beta,\nu}\psi\right)(\xi)d\xi$$

$$=a^{-2(\mu\nu-\alpha)-3\nu}\int_0^\infty \nu\beta\left(\frac{b}{a}\right)^{-1-2\alpha+2\nu}\left(\frac{b\xi}{a}\right)^{\alpha} J_\mu\left[\beta\left(\frac{b\xi}{a}\right)^{\nu}\right]\left(F_{1,\mu,\alpha,\beta,\nu}f\right)\left(\frac{\xi}{a}\right)\times\xi^{-\mu\nu-\alpha}\left(F_{1,\mu,\alpha,\beta,\nu}\psi\right)(\xi)d\xi.$$

By substituting $\frac{\xi}{a}=x; d\xi=adx,$ the continuous generalized Hankel-type integral wavelet transform can be written as

ISSN 2347-1921

$$\left(H_{1,\alpha,\beta,\nu,\mu,\psi} f\right)(b,a) = a^{-2(\mu\nu-\alpha)-3\nu} \int_0^\infty \nu\beta \left(\frac{b}{a}\right)^{-1-2\alpha+2\nu} \left(\frac{b\xi}{a}\right)^{\alpha} J_\mu \left[\beta\left(\frac{b\xi}{a}\right)^{\nu}\right] \left(F_{1,\mu,\alpha,\beta,\nu} f\right)\left(\frac{\xi}{a}\right) \times \xi^{-\mu\nu-\alpha} \left(F_{1,\mu,\alpha,\beta,\nu}\psi\right)(\xi)\, d\xi.$$

$$= a^{-2(\mu\nu-\alpha)-3\nu} \int_0^\infty \nu\beta \left(\frac{b}{a}\right)^{-1-2\alpha+2\nu} (bx)^{\alpha} J_\mu\left[\beta(bx)^{\nu}\right]\left(F_{1,\mu,\alpha,\beta,\nu} f\right)(x) \times (ax)^{-\mu\nu-\alpha} \left(F_{1,\mu,\alpha,\beta,\nu}\psi\right)(ax)\, adx$$

$$= a^{-2(\mu\nu-\alpha)-3\nu} a^{-\mu\nu-\alpha+(1-2\nu+2\alpha)-1/2+1} \int_0^\infty \nu\beta\, b^{-1-2\alpha+2\nu} (bx)^{\alpha} J_\mu\left[\beta(bx)^{\nu}\right]\left(F_{1,\mu,\alpha,\beta,\nu} f\right)(x)\, x^{-\mu\nu-\alpha} \left(F_{1,\mu,\alpha,\beta,\nu}\psi\right)(ax)\, dx$$

$$= a^{-2(\mu\nu-\alpha)-3\nu} a^{-\mu\nu+\alpha-2\nu+\frac{3}{2}} \int_0^\infty \nu\beta\, b^{-1-2\alpha+2\nu} (bx)^{\alpha} J_\mu\left[\beta(bx)^{\nu}\right]\left(F_{1,\mu,\alpha,\beta,\nu} f\right)(x)\, x^{-\mu\nu-\alpha} \left(F_{1,\mu,\alpha,\beta,\nu}\psi\right)(ax)\, dx.$$

$$= a^{-3\mu\nu+3\alpha-5\nu+\frac{3}{2}} \int_0^\infty \nu\beta\, b^{-1-2\alpha+2\nu} (bx)^{\alpha} J_\mu\left[\beta(bx)^{\nu}\right]\left(F_{1,\mu,\alpha,\beta,\nu} f\right)(x)\, x^{-\mu\nu-\alpha} \left(F_{1,\mu,\alpha,\beta,\nu}\psi\right)(ax)\, dx.$$

$$D_{\mu,\alpha,\beta,\nu}(x,y,z) = \int_0^\infty \xi^{-\mu\nu-\alpha} \begin{bmatrix} \nu\beta\, x^{-1-2\alpha+2\nu} (x\xi)^{\alpha} J_\mu\left[\beta(x\xi)^{\nu}\right] \\ \nu\beta\, y^{-1-2\alpha+2\nu} (y\xi)^{\alpha} J_\mu\left[\beta(y\xi)^{\nu}\right] \\ \nu\beta\, z^{-1-2\alpha+2\nu} (z\xi)^{\alpha} J_\mu\left[\beta(z\xi)^{\nu}\right] d\xi \end{bmatrix}$$

$$x = \frac{b}{a};\ y = \frac{t}{a};$$

$$D_{\mu,\alpha,\beta,\nu}(b/a, t/a, z) = \int_0^\infty \xi^{-\mu\nu-\alpha} \begin{bmatrix} \nu\beta \left(\frac{b}{a}\right)^{-1-2\alpha+2\nu} \left(\frac{b}{a}\xi\right)^{\alpha} J_\mu\left[\beta\left(\frac{b}{a}\xi\right)^{\nu}\right] \\ \nu\beta \left(\frac{t}{a}\right)^{-1-2\alpha+2\nu} \left(\frac{t}{a}\xi\right)^{\alpha} J_\mu\left[\beta\left(\frac{t}{a}\xi\right)^{\nu}\right] \\ \nu\beta\, z^{-1-2\alpha+2\nu} (z\xi)^{\alpha} J_\mu\left[\beta(z\xi)^{\nu}\right] d\xi \end{bmatrix}$$

$$\xi = ax;\ d\xi = adx.$$

$$D_{\mu,\alpha,\beta,\nu}(b/a, t/a, z) = \int_0^\infty (ax)^{-\mu\nu-\alpha} \begin{bmatrix} \nu\beta \left(\frac{b}{a}\right)^{-1-2\alpha+2\nu} \left(\frac{b}{a}ax\right)^{\alpha} J_\mu\left[\beta\left(\frac{b}{a}ax\right)^{\nu}\right] \\ \nu\beta \left(\frac{t}{a}\right)^{-1-2\alpha+2\nu} (tx)^{\alpha} J_\mu\left[\beta(tx)^{\nu}\right] \\ \nu\beta\, z^{-1-2\alpha+2\nu} (zax)^{\alpha} J_\mu\left[\beta(zax)^{\nu}\right] adx \end{bmatrix}$$

$$D_{\mu,\alpha,\beta,\nu}(b/a, t/a, z) = \int_0^\infty (ax)^{-\mu\nu-\alpha} \begin{bmatrix} \nu\beta \left(\frac{b}{a}\right)^{-1-2\alpha+2\nu} (bx)^{\alpha} J_\mu\left[\beta(bx)^{\nu}\right] \\ \nu\beta \left(\frac{t}{a}\right)^{-1-2\alpha+2\nu} (tx)^{\alpha} J_\mu\left[\beta(tx)^{\nu}\right] \\ \nu\beta\, z^{-1-2\alpha+2\nu} (zax)^{\alpha} J_\mu\left[\beta(zax)^{\nu}\right] adx \end{bmatrix}$$

4. CONCLUSION

The applications of generalized Hankel-type integral wavelet transformation can be applied in signal processing, computer vision, seismology, turbulence, computer graphics, image processing, digital communication, approximation theory, numerical analysis and statistics.

ACKNOWLEDGMENTS

I am thankful for the support to this work by National Board of Higher Mathematics Ref No. 2/48(11) 2013/ NBHM/ (R.P.) / R& D II/979.

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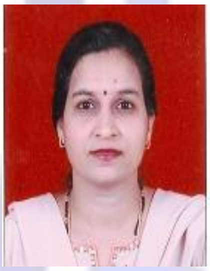
Author’ biography with Photo



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