Anti (Q, L)-Fuzzy Subhemirings of a Hemiring

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ABSTRACT:

In this paper, an attempt has been made to study the algebraic nature of an anti (Q, L)-fuzzy subhemirings of a hemi ring.

Indexing terms/Keywords

L-fuzzy set, anti (Q, L)-fuzzy subhemiring, pseudo anti (Q, L)-fuzzy coset.

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INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R; +; .). Some of them, in particular, about near rings and several kinds of semirings have been proved very useful. Semirings (also called half rings) are algebras (R; + ;.) which share the same properties as a ring excepting that (R; +) is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications the theory of automata and formal languages. An algebra (R; +;.) is said to be a semi ring (R; +) and (R; .) are semi groups satisfying a.(b+c)=a.b+a.c and (b+c).a=b.a+c.a for all a,b and c in R. A semiring R is said to be additively commutative if a+b=b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1.a=a=a.1 and a zero 0, defined by 0+a=a=a+0 and a.0=0=0.a for all a in R. A semiring R is said to be a hemi ring if it is additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh [12], several researchers explored the generalization of the concept of fuzzy sets. The notion of anti left h-ideals in hemi ring was introduced by Akram.M and K.H.Dar [1].The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan [6].Osman Kanzanci, Sultan Yamark and Serife Yilmaz in [13] introduced the notion of intuitionistic Q-fuzzification   of N-subgroups (sub near-rings) in a near–ring and investigated some related properties. A.Solairaju and R.Nagarajan have given a new structure in the construction of Q-fuzzy groups and subgroups [14] and [15]. In this paper are to be introduced some theorems in (Q,L)-fuzzy subhemirings of a hemiring.

1. PRELIMINARIES

1.1 Definition: Let X be a non-empty set and L= $(L, \leq)$ be a lattice with least element 0 and greastest element 1.

1.2 Definition: Let X be a non-empty set and Q be a non-empty set. A (Q, L)--fuzzy subset A of X is function $A: X \times Q \to L$

1.3 Definition:Let (R,+, .) be a hemiring. A (Q,L)-fuzzy subset A of R is said to be an anti (Q,L)-fuzzy subhemiring of R if it satisfies the following conditions:

(i) $\mu_A(x + y, q) \leq \left((\mu_A(x, q) \vee \mu_A(y, q) \right)$

(ii) $\mu_A(xy, q) \leq \left((\mu_A(x, q) \vee \mu_A(y, q) \right)$, for all x and y in R, and q in Q.

1.4 Definition: Let A and B be (Q, L)-fuzzy subsets of sets G and H respectively. The anti-product of A and B denoted by AxB is defined as AxB=$\{((x, y), q), \ \mu_{AXB}(x, y), q)\rangle /$ for all x in G and y in H & q in Q$\}$, where

$\mu_{AXB}(x, y), q) = \{(\mu_A(x, q) \vee \mu_B(y, q)\}.$

1.5 Definition: Let A be a (Q,L)-fuzzy subset in a set S, the anti-strongest relation (Q,L)-fuzzy relation on S, that is a (Q,L)- fuzzy relation on A is V given by $\mu_V((x, y), q) = \{(\mu_A(x, q) \vee \mu_B(y, q)\}$ for all x and y in S and q in Q.

1.6 Definition: Let (R, +,) and $(R', +, \ .)$ be any two hemirings. Let $f: R \to R'$ be any function and A be an anti (Q, L)-fuzzy subhemiring in R, V be an anti (Q, L)-fuzzy subhemiring in $f(R) = R'$, defined by $\mu_V(y, q) = \inf_{x \in f^{-1}(y)} \mu_A(x, q)$ for all x in R and y in R' and q in Q. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.7 Definition: Let A be an anti (Q, L)-fuzzy subhemiring of a hemiring (R, +, .) and a in R then the pseudo anti (Q, L)-fuzzy coset $(aA)^p$ is defined by $\left((a\mu_A)^p\right)(x, q) = p^{(a)}\mu_A(x, q)$, for every x in R, q in Q and for some p in P.

2. PROPERTIES OF ANTI (Q, L)-FUZZY SUBHEMIRINGS OF A HEMIRING

2.1 Theorem: Union of any two anti (Q,L)-fuzzy subhemiring of a hemiring R is an anti (Q,L)-fuzzy subhemiring of R.

Proof: Let A and B be any two anti (Q, L)-fuzzy subhemirings of a hemiring R and x and y in R. Let A =$\{((x, q), \mu_A(x, q) / x \in R \ \& q \in Q\}$ and B =$\{((x, q), \mu_B(x, q) / x \in R \ \& q \in Q\}$ and also Let

$C = AUB = \{((x, q), \mu_C(x, q)) / x \in R \ \& q \in Q\}$,where $\{(\mu_A (x, q) \ \vee \ \mu_B(x, q)\} = \mu_C(x, q)$, Now,

$\mu_C(x + y, q) \leq \ \{(\mu_A(x, q) \vee \mu_A(y, q) \vee ((\mu_B(x, q) \vee \mu_B(y, q))\} \leq \left(\mu_C(x, q) \vee \mu_C(y, q) \right)$.Therefore,

$\mu_C(x + y, q) \leq \left(\mu_C(x, q) \vee \mu_C(y, q) \right)$, for all x and y in R and q in Q. And. $\mu_C(xy , q) \leq$

$\left\{ \left(\mu_A(x, q) \ \vee \ \mu_A(y, q) \right) \vee \left((\mu_B(x, q) \ \vee \ \mu_B(y, q) \right) \right\} \leq \left(\mu_C (x, q) \ \vee \ \mu_C (y, q) \right)$ Therefore, $\mu_C(xy, q) \leq \left(\mu_C(x, q) \ \vee \ \mu_C(y, q) \right)$, for

all x and y in R and q in Q. Therefore C is an anti (Q, L)-fuzzy subhemiring of a hemiring R.

2.2 Theorem: The Union of a family of anti (Q,L)-fuzzy subhemiring of hemiring R is an anti (Q,L)-fuzzy subhemiring of R.

Proof: It is trivial.

2.3 Theorem: If A and B are two anti (Q,L)-fuzzy subhemirings of the hemirings R_1 and R_2 respectively,then anti product AXB is an anti (Q,L)-fuzzy subhemiring of $R_1 X R_2$.

Proof: Let A and B be two anti (Q,L)- fuzzy subhemirings of the hemirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then $((x_1, y_1), q)$ and $((x_2, y_2), q)$ are in $R_1 X R_2$.Now,

$$\mu_{AXB}[(x_1, y_1), q) + (x_2, y_2), q)] \leq \left\{\left((\mu_A(x_1, q) \vee \mu_A(x_2, q)) \vee \left(\mu_B(y_1, q) \vee \mu_B(y_2, q)\right)\right)\right\} \leq \left(\mu_{AXB}(x_1, y_1), q) \vee \mu_{AXB}(x_2, y_2), q)\right).$$

Therefore, $\mu_{AXB}[(x_1, y_1), q) + (x_2, y_2), q)] \leq \left(\mu_{AXB}(x_1, y_1), q) \vee \mu_{AXB}(x_2, y_2), q)\right)$. Also,

$$\mu_{AXB}[(x_1, y_1), q)(x_2, y_2), q)] \leq \left\{(\mu_A(x_1, q) \vee \mu_A(x_2, q)\} \vee ((\mu_B(y_1, q) \vee \mu_B(y_2, q))\right\} \leq \left(\mu_{AXB}(x_1, y_1), q) \vee \mu_{AXB}(x_2, y_2), q)\right).$$

Therefore,$\mu_{AXB}[(x_1, y_1), q)(x_2, y_2), q)] \leq \left(\mu_{AXB}(x_1, y_1), q) \vee \mu_{AXB}(x_2, y_2), q)\right)$.Hence AXB is an anti (Q,L)-fuzzy subhemiring of a hemiring $R_1 X R_2$.

2.4 Theorem: Let A be a (Q, L)-fuzzy subset of a hemiring R and V be the anti-strongest fuzzy relation of R. Then A is an anti (Q, L)-fuzzy subhemiring of R if and only if V is an anti (Q, L)-fuzzy subhemiring of RxR.

Proof: Suppose that A is an anti (Q,L)-fuzzy subhemiring of a hemiring R. Then for any $X = (x_1, x_2)$ and $Y = (y_1, y_2)$ are in $R \times R$. We have $\mu_V(XY, q) \leq \left\{(\mu_A(x_1y_1, q) \vee \mu_A(x_2y_2, q))\right\} \leq$

$$\left\{\left(\mu_A(x_1, q) \vee \mu_A(y_1, q)\right) \vee ((\mu_A(x_2, q) \vee \mu_A(y_2, q))\right\} \leq (\mu_V((x_1, x_2), q) \vee \mu_V((y_1, y_2), q) \leq (\mu_V(X, q) \vee \mu_V(Y, q)$$

for all X and Y in R x R and q in Q. Therefore, $\mu_V(XY, q) \leq \left(\mu_V(X, q) \vee \mu_V(Y, q)\right)$, for all X and Y in R x R and q in Q. This proves that V is an anti (Q, L)-fuzzy subhemiring of a hemiring of R x R. Conversely assume that V is an anti (Q, L)-fuzzy subhemiring of a hemiring of R x R, then for any $X = (x_1, x_2)$ and $Y = (y_1, y_2)$ are in R x R we have $\left\{(\mu_A(x_1 + y_1, q) \vee \mu_A(x_2 + y_2, q)\right\} = \mu_V(X + Y, q) \leq \left(\mu_V(X, q) \vee \mu_V(Y, q)\right) = (\mu_V((x_1, x_2), q) \vee \mu_V((y_1, y_2), q) =$

$\left(\mu_A(x_1, q) \vee \mu_A(y_1, q)\right) \vee ((\mu_A(x_2, q) \vee \mu_A(y_2, q))$ If $x_2 = 0, y_2 = 0$ we get $\mu_A(x_1 + y_1, q) \leq \left(\mu_A(x_1, q) \vee \mu_A(y_1, q)\right)$

for all x_1 and y_1 in R .And, $\left\{(\mu_A(x_1y_1, q) \vee \mu_A(x_2y_2, q)\right\} = \mu_V(XY, q) \leq (\mu_V(X, q) \vee \mu_V(Y, q)) =$

$\left(\mu_V((x_1, x_2), q)\right) \vee \mu_V((y_1, y_2), q) = (\{\mu_A(x_1, q) \vee \mu_A(x_2, q)\} \vee \{\mu_A(y_1, q) \vee \mu_A(y_2, q)\})$. If $x_2 = 0, y_2 = 0$ we get

$\mu_A(x_1y_1, q) \leq \left(\mu_A(x_1, q) \vee \mu_A(y_1, q)\right)$ for all x_1 and y_1 in R.Therefore A is an anti(Q,L)-fuzzy subhemiring of R.

2.5 Theorem: If A is an anti (Q,L)-fuzzy subhemiring of a hemiring (R, +, .) if and only if $\mu_A(x + y, q) \leq (\mu_A(x, q) \vee \mu_A(y, q)$, $\mu_A(xy, q) \leq (\mu_A(x, q) \vee \mu_A(y, q)$ for all x and y in R.

Proof: It is trivial.

2.6 Theorem: If A is an anti (Q,L)-fuzzy subhemiring of a hemiring (R, +, .),then $H = \{x/x \in R : \mu_A(x, q) = 0\}$ is either empty or is a subhemiring of R.

Proof: It is trivial.

2.7 Theorem: Let A is an anti (Q,L)-fuzzy subhemiring of a hemiring (R, +, .).If $\mu_A(x + y, q) = 1$, then either $\mu_A(x, q) = 1$ or $\mu_A(y, q) = 1$, for all x and y in R.

Proof: It is trivial.

2.8 Theorem: Let A is an anti (Q,L)-fuzzy subhemiring of a hemiring (R, +, .),then the pseudo anti (Q,L)-fuzzy coset $(aA)^p$ is anti (Q,L)-fuzzy subhemiring of a hemiring R, for every a in R.

Proof: Let A is an anti (Q,L)-fuzzy subhemiring of a hemiring R. For every x and y in R, we have

$((a\mu_A)^p)(x+y,q) \le p(a)\left(\mu_A(x,q) \vee \mu_A(y,q)\right) \in \left(p(a)\mu_A(x,q) \vee p(a)\mu_A(y,q)\right) = (a\mu_A)^p(x,q) \vee (a\mu_A)^p(y,q).$ Therefore,

$((a\mu_A)^p)(x+y,q) \le \left((a\mu_A)^p(x,q) \vee (a\mu_A)^p(y,q)\right).$ Now, $((a\mu_A)^p)(xy,q) \le p(a)\left(\mu_A(x,q)\vee\mu_A(y,q)\right) = \left(p(a)\mu_A(x,q)\vee p(a)\mu_A(y,q)\right) = \left((a\mu_A)^p(x,q)\vee(a\mu_A)^p(y,q)\right).$

Therefore, $\left((a\mu_A)^p\right)(xy,q) \le \left((a\mu_A)^p(x,q) \vee (a\mu_A)^p(y,q)\right).$ Hence $(aA)^p$ is an anti (Q,L)-fuzzy subhemiring of a hemiring R .

2.9 Theorem: Let (R, +,.) and ($R^{'}$, +, .) be any two hemirings. The homomorphic image of an anti (Q, L)-fuzzy subhemiring of R is an anti (Q, L)-fuzzy subhemiring of $R^{'}$.

Proof: Let $f:R \to R^{'}$ be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all x and y in R. Let V= f(A), where A is an anti (Q,L) fuzzy subhemiring of R. Now, for $f(x)$, $f(y)$ in $R^{'}$, $\mu_V((f(x),q) + (f(y),q)) \le \mu_A(x+y,q) \le (\mu_A(x,q)\vee\mu_A(y,q))$, which implies that $\mu_V(f(x) + (f(y),q)) \le (\mu_V\left((f(x),q) \vee \mu_V(f(y),q)\right).$ Again, $\mu_V((f(x),\ q)(f(y),q)) \le \mu_A(xy,q) \le (\mu_A(x,q)\vee\mu_A(y,q))$, which implies that $\mu_V((f(x)(f(y)),q)) \le \left(\mu_V(f(x),q) \vee \mu_V(f(y),q)\right).$ Hence V is an anti (Q,L)-fuzzy subhemiring of hemiring $R^{'}$.

2.10 Theorem: Let (R, +, .) and ($R^{'}$, +,.) be any two hemirings. The homomorphic preimage of an anti (Q, L)-fuzzy subhemiring of $R^{'}$ is an anti (Q, L)-fuzzy subhemiring of R.

Proof: Let $f:R \to R^{'}$ be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let V= f(A) where V is an anti (Q,L) fuzzy subhemiring of $R^{'}$. Now,

for x,y in R, $\mu_A(x+y,q) = \mu_V((f(x) + f(y)),q)) \le (\mu_V(f(x),q) \vee \mu_V(f(y),q) = \left(\mu_A(x,q)\vee\mu_A(y,q)\right)$ which implies that $\mu_A((x+y,q)) \le \left(\mu_A(x,q)\vee\mu_A(y,q).\right)$ Again, $\mu_A((xy,q)) = \mu_V((f(x),q)(f(y),q)) \le (\mu_V(f(x),\ q)\vee\mu_V(f(y),q) = (\mu_A(x,q)\vee\mu_A(y,q)$

which implies that $\mu_A((xy,q)) \le \left(\mu_A(x,q)\vee\mu_A(y,q)\right).$ Hence A is an anti (Q, L)-fuzzy subhemiring of hemiring R.

2.11 Theorem: Let (R, +, .) and ($R^{'}$, +, .) be any two hemirings. The anti-homomorphic image of an anti (Q,L)-fuzzy subhemiring of R is an anti (Q,L)-fuzzy subhemiring of $R^{'}$.

Proof: Let $f:R \to R^{'}$ be a anti- homomorphism. Then, $f(x+y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$,for all x and y in R. Let V= f(A) where A is an anti (Q,L)- fuzzy subhemiring of R.

Now, for $f(x),f(y)$ in $R^{'}$, $\mu_V((f(x) + f(y)),q)) \le \mu_A(y+x,q) \le (\mu_A(y,q)\vee\mu_A(x,q) = (\mu_A(x,q)\vee\mu_A(y,q)$, which implies that $\mu_V((f(x) + f(y)),q)) \le (\mu_V\left((f(x),q)\vee\mu_V(f(y),q)\right).$ Again, $\mu_V((f(x),q)(f(y),q)) \le \mu_A(yx,q) \le \left(\mu_A(y,q)\vee\mu_A(x,q)\right) = (\mu_A(x,q)\vee\mu_A(y,q)$

which implies that $\mu_V((f(x)f(y)),q)) \le (\mu_V\left((f(x),q)\vee\mu_V(f(y),q)\right).$ Hence V is an anti (Q, L)-fuzzy subhemiring of hemiring $R^{'}$.

2.12 Theorem: Let (R, +,.) and ($R^{'}$, +, .) be any two hemirings. The anti-homomorphic preimage of an anti (Q, L)-fuzzy subhemiring of $R^{'}$ is an anti (Q,L)-fuzzy subhemiring of R.

Proof: Let $V = f(A)$ where V is an anti $(Q,L)-$ fuzzy subhemiring of $R^{'}$. Let x and y in R. Then

$\mu_A(\ (x+y,q)) = \mu_V((f(x) + f(y)),q) \le (\mu_V\ (f(y),q) \vee \mu_V\ (f(x),q) = (\mu_A(x,q)\vee\mu_A(y,q)$,which implies that $\mu_A((x+y,q)) \le (\mu_A(x,\ q)\vee\mu_A\ (y,\ q).$ Again, $\mu_A((xy,q)) = \mu_V((f(x)f(y)),q) \le (\mu_V(f(y),q)\vee\mu_V(f(x),q) = (\mu_A(x,q)\vee\mu_A(y,q),$

which implies that $\mu_A((xy,q)) \le (\mu_A(x,\ q)\vee\mu_A(y,q).$ Hence A is an anti (Q, L)-fuzzy subhemiring of hemiring R.

In the following Theorem∘ is the composition operation of functions:

2.13 Theorem: Let A be an anti (Q, L)-fuzzy subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H. Then A∘ f is an anti (Q, L)-fuzzy subhemiring of R.

Proof: Let x and y in R. Then we have, $(\mu_A \circ f)\ ((x+y),q)) = \mu_A((f(x) + f(y)),q) \le (\mu_A(\ f(x),q) \vee \mu_A(f(y),q) \le$

$(\mu_A \circ f)(x, q) \vee (\mu_A \circ f)(y, q)$, which implies that $(\mu_A \circ f)((x + y), q)) \leq (\mu_A \circ f)(x, q) \vee (\mu_A \circ f)(y, q)$. And,

$(\mu_A \circ f)((xy, q)) = \mu_A((f(x), q)(f(y), q) \leq (\mu_A(f(x), q) \vee \mu_A(f(y), q) \leq (\mu_A \circ f)(x, q) \vee (\mu_A \circ f)(y, q)$,

which implies that $(\mu_A \circ f)((xy, q)) \leq (\mu_A \circ f)(x, q) \vee (\mu_A \circ f)(y, q)$. Therefore A∘f is an anti(Q, L)-fuzzy subhemiring of hemiring R.

2.14 Theorem: Let A be an anti (Q, L)-fuzzy subhemiring of hemiring H and f is an anti- isomorphism from a hemiring R onto H. Then A∘ f is an anti (Q,L)-fuzzy subhemiring of R.

Proof: Let x and y in R. Then we have, $(\mu_A \circ f)((x + y, q)) = \mu_A(f(y) + f(x), q) \leq (\mu_A(f(x), q) \vee \mu_A(f(y), q) \leq$

$(\mu_A \circ f)(x, q) \vee (\mu_A \circ f)(y, q)$, which implies that $(\mu_A \circ f)((x + y, q)) \leq ((\mu_A \circ f)(x, q) \vee (\mu_A \circ f)(y, q))$. And,

$(\mu_A \circ f)((xy), q)) = \mu_A((f(y) f(x)), q) \leq (\mu_A(f(x), q) \vee \mu_A(f(y), q) \leq (\mu_A \circ f)(x, q) \vee (\mu_A \circ f)(y, q)$ which implies that

$(\mu_A \circ f)((xy, q)) \leq (\mu_A \circ f)(x, q) \vee (\mu_A \circ f)(y, q)$. Therefore A∘f is an anti (Q, L)-fuzzy subhemiring of hemiring R.

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