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Fermat's last theorem: algebraic proof

James E. Joseph

Department of Mathematics Howard University

Washington, DC 20059 35 E Street NW #709 Washington, DC 20001

j122437@yahoo.com, jjoseph@Howard.edu

Abstract

In 1995, A, Wiles announced, using cyclic groups, a proof of Fermat's Last Theorem, which is stated as follows: If π is an odd prime and x, y, z are relatively prime positive integers, then $z^{\pi} \neq x^{\pi} + y^{\pi}$. In this note, a proof of this theorem is offered, using elementary Algebra. It is proved that if π is an odd prime and x, y, z are positive inyegera satisfying $z^{\pi} = x^{\pi} + y^{\pi}$, then $x, y,$ and z are each divisible by π.

Fermat [2010]Primary 11Yxx

The special case $z^4 = x^4 + y^4$ is impossible [1]. In view of this fact, it is only necessary to prove, if $x, y, z,$ are relatively prime positive integers, π is an odd prime, $z^{\pi} \neq x^{\pi} + y^{\pi}$ (In this article, the symbol π will represent an odd prime).

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Theorem 1 *If π is an odd prime and $x^{\pi}+y^{\pi}=z^{\pi}$, then*

1. $(x+y)^{\pi}-z^{\pi}\equiv 0\,(mod\,\pi^2),((z-x)^{\pi}-y^{\pi}\equiv 0\,(mod\,\pi^2);$
2. $x\equiv 0\,(mod\,\pi)(y\equiv 0\,(mod\,\pi))[z\equiv 0\,(mod\,\pi)].$

Theorem 1 is arrived at through the following two Lemmas.

Lemma 1 *If $x^{\pi}+y^{\pi}=z^{\pi}$, then* $x+y-z\equiv 0\,(mod\,\pi)(z-x-y\equiv 0\,(mod\,\pi)).$

Proof.It is obvious that $(x+y)^{\pi}-z^{\pi}\equiv 0\,(mod\,\pi),(z-y)^{\pi}-x^{\pi}\equiv 0\,(mod\,\pi),(z-x)^{\pi}-y^{\pi}\equiv 0\,(mod\,\pi)$

$$(x+y)^{\pi}-z^{\pi}=(x+y-z+z)^{\pi}-z^{\pi}=\sum_{0}^{\pi-1}C(\pi,k)(x+y-z)^{\pi-k}z^{k};$$

$$(x+y)^{\pi}-z^{\pi}-(x+y-z)^{\pi}=\sum_{1}^{\pi-1}C(\pi,k)(x+y-z)^{\pi-k}z^{k}.$$

Lemma 2 *If $x^{\pi}+y^{\pi}=z^{\pi}$, then*
$(x+y)^{\pi}-z^{\pi}\equiv 0\,(mod\,\pi^2)((z-x)^{\pi}-y^{\pi}\equiv 0\,(mod\,\pi^2))[(z-y)^{\pi}-x^{\pi}\equiv 0\,(mod\,\pi^2)].$

Proof. See proof of Lemma 1.

Theorem 2 *If $z^{\pi}=x^{\pi}+y^{\pi}$, then* $xy\equiv 0\,(mod\,\pi);xz\equiv 0\,(mod\,\pi);yz\equiv 0\,(mod\,\pi).$

Proof.

$$\sum_{1}^{\pi-1}C(\pi,k)x^{\pi-k}y^{k}=\sum_{1}^{\pi-1}C(\pi,k)(x+y-z)^{\pi-k}z^{k}\equiv 0\,(mod\,\pi^2);$$

for every $1\le k\le \pi-1, C(\pi,k)x^{\pi-k}y^{k}\equiv 0\,(mod\,\pi)$; so

$$xy^{\pi-1}\equiv 0\,(mod\,\pi);$$

$$(E1)\quad xy\equiv 0\,(mod\,\pi).$$

$$\sum_{1}^{\pi-1}C(\pi,k)x^{\pi-k}z^{k}=\sum_{1}^{\pi-1}C(\pi,k)(z-x-y)^{\pi-k}x^{k}\equiv 0\,(mod\,\pi^2);$$

$\sum_{1}^{\pi-1}C(\pi,k)x^{\pi-k}z^{k}\equiv 0\,(mod\,\pi^2);$ then for each integer $1\le k\le \pi-1, C(\pi,k)x^{\pi-k}z^{k}\equiv 0\,(mod\,\pi^2),$

$$xz^{\pi-1}\equiv 0\,(mod\,\pi);$$

$$(E2)\quad xz\equiv 0\,(mod\,\pi).$$

$$\sum_{1}^{\pi-1}C(\pi,k)y^{\pi-k}z^{k}=\sum_{1}^{\pi-1}C(\pi,k)(z-x-y)^{\pi-k}y^{k}\equiv 0\,(mod\,\pi^2);$$

$\sum_{1}^{\pi-1}C(\pi,k)y^{\pi-k}z^{k}\equiv 0\,(mod\,\pi^2);$ then for each integer $1\le k\le \pi-1, C(\pi,k)y^{\pi-k}z^{k}\equiv 0\,(mod\,\pi^2)$

$$yz^{\pi-1}\equiv 0\,(mod\,\pi);$$

$$(E3)\quad yz\equiv 0\,(mod\,\pi).$$

With the equivalences (E1), (E2), (E3), and the equivalence $x+y-z\equiv 0\,(mod\,\pi)$, comes $z\equiv 0\,(mod\,\pi), y\equiv 0\,(mod\,\pi), x\equiv 0\,(mod\,\pi).$

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Fermat' s Last Theorem. If π is an odd prime and $x, y, z,$ are relatively prime positive integers, then $z^{\pi} \neq x^{\pi} + y^{\pi}.$

Proof. The equivalences $x \equiv 0\,(mod\ \pi), y \equiv 0\,(mod\ \pi), z \equiv 0\,(mod\ \pi).$ hold

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