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# Solving a Rough Interval Linear Fractional Programming problem

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## ABSTRACT

In this paper, A rough interval linear fractional programming( RILFP)problem is introduced. The RILFPproblems considered by incorporating rough interval in the objective function coefficients. This proved the RILFP problem can be converted to a rough interval optimization problem with rough interval objective which is upper and lower approximations are linear fractional whose bounds. Also there is a discussion for the solutions of this kind of optimization problem. An illustrative numerical example is given for the developed theory.

**Keywords:** Linear fractional; Rough interval; Rough interval function.

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## 1. INTRODUCTION

Fractional programming gains significant stature since many of the real world problems represented as fractional functions. These problems are often encountered in the situation such as return on investment, current ratio, actual capital to required capital. Linear fractional programming problem is one whose objective function are very useful in production planning, financial and corporate planning. Several methods to solve this problem have been proposed by Charnes and Cooper. The linear fractional programming is special class of fractional programming which can be transformed into a linear programming problem by the method of Charnesand Cooper (1962). Tantawy (2008), proposed a new method for solving linear fractional programming problems.Wu(2008), introduce four kinds of interval–valued optimization problems are formulated. Effati and Pakdaman (2012), introduced an interval–valued linear fractional programming (IVLFP) problem. They convert an IVLFP to an optimization problem with interval–valued objective function which its bounds are linear fraction function. Pawlak (1982),rough set theory is a new mathematical approach to imperfect knowledge. Kryskiewice (1998), rough set theory has found many interesting applications.Pal (2004), the rough set approach seems to be of fundamental importance to cognitive sciences, especially in the areas of machine learning , decision analysis, expert systems.Pawlak (1991), rough set theory, introduced by the author, expresses vagueness, not by means of membership, but employing a boundary region of a set.The theory of rough set deals with the approximation of an arbitrary subset of a universe by two definable or observable subsets called lower and upper approximations.Tsumoto (2004), using the concept of lower and upper approximation in rough sets theory, knowledge hidden in information systems may be unraveled and expressed in the form of decision rules. .Lu and Huang (2011), The concept of rough interval will be introduced to represent dual uncertain information of many parameters, and the associated solution method will be presented to solve rough interval fuzzy linear programming problems dual uncertain solutions.In this paper a proposed algorithm to solve rough interval linear fractional programming problem by separating them into four linear fractional programming problems and solve these problems . A numerical example is given for the sake of illustration.

## 2. Preliminaries

**Definition 2.1:**Suppose $I$ is the set of all compact intervals in the set of all real numbers $\mathcal{R}$. If $A \in I$ then we write $A = [a^L, a^U]$with $a^L \leq a^U$ and the following holds :

i. $A \geq 0$ iff$x_i \geq 0$ for all $x_i \in A$ .

ii. $A \leq 0$ iff$x_i \leq 0$ for all$x_i \in A$.

**Definition 2.2:**Let $X$ be denotea compact set of real numbers. A rough interval $X^R$ is defined as: $X^R = [X^{(LAI)} : X^{(UAI)}]$ where $X^{(LAI)} and X^{(UAI)}$are compact intervals denoted by lower and upper approximation intervals of $X^R$with$X^{(LAI)} \subseteq X^{(UAI)}$.

**Definition 2.3:**For the rough interval $X^R$the following holds:

i. $X^R \succcurlyeq 0$ iff$X^{(LAI)} \geq 0$ and$X^{(UAI)} \geq 0$

ii. $X^R \preccurlyeq 0$ iff$X^{(LAI)} \leq 0$ and $X^{(UAI)} \leq 0$ .

In this paperwe denote by$I^R$ the set of all rough intervals in $\mathcal{R}$.Suppose $A^R$, $B^R \in I^R$ we can write $A^R = [A^{LAI} : A^{UAI}]$and also$B^R = [B^{LAI} : B^{UAI}]$ where $A^{LAI} = [a^{-L}, a^{+L}]$, $B^{LAI} = [b^{-L}, b^{+L}]$and $a^{-L}, a^{+L}, b^{-L}, b^{+L} \in \mathcal{R}$ . Similarly we can defined $A^{UAI}$, $B^{UAI}$ .

**Definition 2.4:( see [9] ),**For rough interval $A^R$ , $B^R$ when $A^R \succcurlyeq 0$ $and B^R \succcurlyeq 0$ we can defined the operation on rough intervals as follows :

1) $A^R + B^R = [[A^{LAI} + B^{LAI}] : [A^{UAI} + B^{UAI}]]$

Such that : $[A^{LAI} + B^{LAI}] = [a^{-L} + b^{-L}, \ a^{+L} + b^{+L}]$ and

$$[A^{UAI} + B^{UAI}] = [a^{-U} + b^{-U}, \quad a^{+U} + b^{+U}]$$

2) $A^R - B^R = [[A^{LAI} - B^{LAI}] : [A^{UAI} - B^{UAI}]]$

Such that : $[A^{LAI} - B^{LAI}] = [a^{-L} - b^{+L}, \ a^{+L} - b^{-L}]$ and

$[A^{UAI} - B^{UAI}] = [a^{-U} - b^{+U}, \ a^{+U} - b^{-U}]$ .

3) $A^R \times B^R = [[A^{LAI} \times B^{LAI}] : [A^{UAI} \times B^{UAI}]]$

Such that:$[A^{LAI} \times B^{LAI}] = [a^{-L} \times b^{-L}, \ a^{+L} \times b^{+L}]$ and

$[A^{UAI} \times B^{UAI}] = [a^{-U} \times b^{-U}, \ a^{+U} \times b^{+U}]$ .

4) $A^R / B^R = [[A^{LAI} / B^{LAI}] : [A^{UAI} / B^{UAI}]]$

Such that :$[A^{LAI} / B^{LAI}] = [a^{-L} / b^{+L},\ a^{+L} / b^{-L}]$ and $[A^{UAI} / B^{UAI}] = [a^{-U} / b^{+U},\ a^{+U} / b^{-U}]$.

**Definition 2.5:**A function $f{:}\mathcal{R}^n \to I^R$ is called a rough interval function with $f^R(x) = [f^{(LAI)}(x) :\ f^{(UAI)}(x)]$where for every $x \in \mathcal{R}^n$, $f^{(LAI)}(x)$, $f^{(UAI)}(x)$ are lower and upper approximation interval valued functions.

**Proposition:**Let$f$be a rough interval function defined on $X \subset \mathcal{R}^n$ and $x_0 \in X$ Then $f$ is continuous at $x_0$ if and only if $f^{(LAI)}(x)$ and $f^{(UAI)}(x)$ are continuous at $x_0$ .

**Definition 2.6:**We define a linear fractional function $f(x)$ as follows :

$f(x) = \frac{cx+\alpha}{dx+\beta}$ (1)

Where $c, d, x \in \mathcal{R}^n$ , $\alpha, \beta \in \mathcal{R}$ .

## 3. Rough interval linear fractional programming

Consider the following linear fractional programming problem :

Maximize $f(x) = \frac{cx+\alpha}{dx+\beta}$(2)

Subject to : $Ax \le b$, $x \ge 0$

Where $c, d, x \in \mathcal{R}^n$ , $\alpha, \beta \in \mathcal{R}$ .

In the linear fractional programming problem (2), suppose that $c = (c_1, c_2, \dots, c_n)$ and $d = (d_1, d_2, \dots, d_n)$where$c_j$ , $d_j \in I^R$ , $j = 1,2,3 \dots n$.

We denoted $c_j^{LAI}$ and $d_j^{LAI}$ the lower bound of the rough interval $c_j$ *and* $d_j$ respectively

$$i.e\ \ c^{LAI} = (c_1^{LAI}, c_2^{LAI}, \dots, c_n^{LAI}),\qquad d^{LAI} = (d_1^{LAI}, d_2^{LAI}, \dots, d_n^{LAI})$$

Where $c_j^{LAI}$ and $d_j^{LAI}$ are interval with real scalars for $j = 1,2, \dots, n$ .

Similarly we can defined $c^{UAI}$ *and*, $d^{UAI}$ . Also we assume that$\alpha$, $\beta$are rough in the form $\alpha = [\alpha^{LAI}{:}\alpha^{UAI}]$, $\beta = [\beta^{LAI}{:}\beta^{UAI}]$ .

So we can rewrite (2) as follows :

(RILFPP)Maximize $f(x) = \frac{p^R(x)}{q^R(x)}$ (3)

Subject to : $Ax \le b$ , $x \ge 0$ .

Where $p^R(x)$ *and* $q^R(x)$ are a rough interval linear function defined as:

$p^R(x) = [p^{LAI}(x) :\ p^{UAI}(x)] = [c^{LAI}x + \alpha^{LAI} :\ c^{UAI}x + \alpha^{UAI}]$ $q^R(x) = [q^{LAI}(x) :\ q^{UAI}(x)] = [d^{LAI}x + \beta^{LAI} :\ d^{UAI}x + \beta^{UAI}]$ .

Now we can write equation (3) in the form :

$(\text{RILFPP})_1$Maximize $f(x) = \frac{[c^{LAI}x+\alpha^{LAI}\ :\ c^{UAI}x+\alpha^{UAI}]}{[d^{LAI}x+\beta^{LAI}\ :\ d^{UAI}x+\beta^{UAI}]}$(4)

Subject to : $Ax \le b$ , $x \ge 0$ .

From [9] problem(4) can be written as :

Maximize $f(x) = [\ \frac{c^{LAI}x+\alpha^{LAI}}{d^{LAI}x+\beta^{LAI}}\ :\ \frac{c^{UAI}x+\alpha^{UAI}}{d^{UAI}x+\beta^{UAI}}]$(5)

Subject to : $Ax \le b$ , $x \ge 0$ .

General problem (5) can be written as :

Maximize $f(x) = \left[\frac{[c^{-L}x+\alpha^{-L}\ ,\ c^{+L}x+\alpha^{+L}]}{[d^{-L}x+\beta^{-L}\ ,\ d^{+L}x+\beta^{+L}]} : \frac{[c^{-U}x+\alpha^{-U}\ ,\ c^{+U}x+\alpha^{+U}]}{[d^{-U}x+\beta^{-U}\ ,\ d^{+U}x+\beta^{+U}]}\right]$( 6)

Subject to : $Ax \le b$ , $x \ge 0$ .

Now using theorem 2-1 from [10] problem (6) can be written as :

$(\text{RILFPP})_2$Maximize $f(x) = \left[\left[\frac{c^{-L}x+\alpha^{-L}}{d^{+L}x+\beta^{+L}}\ ,\ \frac{c^{+L}x+\alpha^{+L}}{d^{-L}x+\beta^{-L}}\right]:\left[\frac{c^{-U}x+\alpha^{-U}}{d^{+U}x+\beta^{+U}}\ ,\ \frac{c^{+U}x+\alpha^{+U}}{d^{-U}x+\beta^{-U}}\right]\right]$ (7)

Subject to : $Ax \le b \ , x \ge 0$

**Theorem 3.1.** Anyrough interval linear fractional programming problemin the form $(\text{RILFPP})_{\mathbf{1}}$(see equation (4) ) under some assumptions can be converted to a rough interval linear fractional programming problem in the form $(\text{RILFPP})_2$ **(** see equation (7) ) **.**

**Proof .** The objective function in (4) is a quotient of two rough interval functions ($p^R(x)$ $and$ $q^R(x)$ ) .

To convert (4) to (5) we suppose that $0 \notin q^R(x)$ for each feasible point $x$, so we should $0 < q^{LAI}(x) \le q^{UAI}(x)$for each feasible point. Using the operation on a rough interval we have :

$f(x) = [\frac{c^{LAI}x+\alpha^{LAI}}{d^{LAI}x+\beta^{LAI}} : \frac{c^{UAI}x+\alpha^{UAI}}{d^{UAI}x+\beta^{UAI}}]$ .

Now we can using the operation on the interval and the theorem (2.1) [10] we have :$f(x) = \left[\left[\frac{c^{-L}x+\alpha^{-L}}{d^{+L}x+\beta^{+L}}, \frac{c^{+L}x+\alpha^{+L}}{d^{-L}x+\beta^{-L}}\right] : \left[\frac{c^{-U}x+\alpha^{-U}}{d^{+U}x+\beta^{+U}}, \frac{c^{+U}x+\alpha^{+U}}{d^{-U}x+\beta^{-U}}\right]\right]$ and this completes the proof.

**Definition 3.1:** A point $x^* \in X$ is said to bea optimal solution of optimization problem (RILFPP) equation (3) if there does not exist$x \in X$ such that$f(x^*) \le f(x)$ .

## 4.Algorithem solution for RILFPP :

We suppose algorithm to solve a RILFPP is as follows :

1. Convert any problem in the form of equation (7) .
2. We recompense the problem into four problems in the following form :

$$P_1 : \quad Maximize \quad \{ f^{+U}(x) = \frac{c^{+U}x+\alpha^{+U}}{d^{-U}x+\beta^{-U}} \}$$

Subject to : $Ax \le b \ , x \ge 0$ .

$$P_2 : \quad Maximize \quad \{ f^{-U}(x) = \frac{c^{-U}x+\alpha^{-U}}{d^{+U}x+\beta^{+U}} \}$$

Subject to :$\frac{c^{-U}x+\alpha^{-U}}{d^{+U}x+\beta^{+U}} \le maximize\ value\ of\ f^{+U}(x)$

$Ax \le b \ , \ x \ge 0.$

$$P_3 : \quad Maximize \quad \{ f^{+L}(x) = \frac{c^{+L}x+\alpha^{+L}}{d^{-L}x+\beta^{-L}} \}$$

Subject to : $maximize\ value\ of\ f^{-U}(x) \le \frac{c^{+L}x+\alpha^{+L}}{d^{-L}x+\beta^{-L}} \le maximize\ value\ of\ f^{+U}(x)$

$Ax \le b \ , \ x \ge 0.$

$$P_4 : \quad Maximize \quad \{ \ f^{-L}(x) = \frac{c^{-L}x+\alpha^{-L}}{d^{+L}x+\beta^{+L}} \}$$

Subject to : $maximize\ value\ of\ f^{-U}(x) \le \frac{c^{-L}x+\alpha^{-L}}{d^{+L}x+\beta^{+L}} \le maximize\ value\ of\ f^{+L}(x)$

$Ax \le b \ , x \ge 0$ .

3. Solving the problems $P_1$ , $P_2$ , $P_3$ and $P_4$ by transformation and using simplex method we obtain the optimal solution $x^*$ with the objective value $f(x^*) = [[f^{-L}(x^*), f^{+L}(x^*)] : [f^{-U}(x^*), f^{+U}(x^*)]]$ .

## 5. Numerical example:

**Example 5.1** Consider the following optimization problem:

$$\text{Minimize } f(x) = \frac{\left[\left[\frac{7}{2},\frac{9}{2}\right]:[3,5]\right]x_1 + [[2,3]:[1,4]]x_2 + [[8,10]:[7,11]]}{\left[\left[1,\frac{3}{2}\right]:\left[\frac{1}{2},2\right]\right]x_1 + \left[\left[\frac{3}{2},\frac{7}{4}\right]:[1,2]\right]x_2 + \left[\left[\frac{9}{2},\frac{11}{2}\right]:[4,6]\right]}$$

Subject to :$x_1 + 3x_2 \le 30$

$$-x_1 + 2x_2 \le 5$$

$$x_1, x_2 \ge 0 \quad .$$

**Solution :** First we can writhen the objective function on the form

$$f(x)=\frac{\left[\left[\frac{7}{2},\frac{9}{2}\right]x_1+[2,3]x_2+[8,10]\ :[3,5]x_1+[1,4]x_2+[7,11]\right]}{\left[\left[1,\frac{3}{2}\right]x_1+\left[\frac{3}{2},\frac{7}{4}\right]x_2+\left[\frac{9}{2},\frac{11}{2}\right]\ :\ \left[\frac{1}{2},2\right]x_1+[1,2]x_2+[4,6]\right]}$$

Now the objective function can be convert to as follows :

$$f(x)=\left[\frac{\left[\frac{7}{2},\frac{9}{2}\right]x_1+[2,3]x_2+[8,10]}{\left[1,\frac{3}{2}\right]x_1+\left[\frac{3}{2},\frac{7}{4}\right]x_2+\left[\frac{9}{2},\frac{11}{2}\right]}\ :\ \frac{[3,5]x_1+[1,4]x_2+[7,11]}{\left[\frac{1}{2},2\right]x_1+[1,2]x_2+[4,6]}\right]$$

$$f(x)=\left[\frac{\left[\frac{7}{2}x_1+2x_2+8\ ,\ \frac{9}{2}x_1+3x_2+10\right]}{\left[x_1+\frac{3}{2}x_2+\frac{9}{2}\ ,\ \frac{3}{2}x_1+\frac{7}{4}x_2+\frac{11}{2}\right]}:\right.$$

$$\left.:\frac{[3x_1+x_2+7\ ,\ 5x_1+4x_2+11\ ]}{\left[\frac{1}{2}x_1+x_2+4\ ,\ 2x_1+2x_2+6\ \right]}\right]$$

This objective function can be writhen :

$$f(x)=\left[\left[\frac{\frac{7}{2}x_1+2x_2+8}{\frac{3}{2}x_1+\frac{7}{4}x_2+\frac{11}{2}}\ ,\frac{\frac{9}{2}x_1+3x_2+10}{x_1+\frac{3}{2}x_2+\frac{9}{2}}\right]:\left[\frac{3x_1+x_2+7}{2x_1+2x_2+6}\ ,\frac{5x_1+4x_2+11}{\frac{1}{2}x_1+x_2+4}\right]\right].$$

Now we can solving four problems as follows :

$$P_1\ :\qquad Max\ \left\{f^{+U}(x)=\frac{5x_1+4x_2+11}{\frac{1}{2}x_1+x_2+4}\right\}$$

Subject to: $x_1+3x_2\ \le 30$

$$-x_1+2x_2\ \le 5$$

$$x_1,x_2\ge 0\quad .$$

Solving the problem $P_1$ by transformation and using simplex method we obtain the optimal solution $x_1=29.999,\ x_2=0$ with the objective value $f^{+U}(x)=8.47$

$$P_2\ :\qquad Max\{\ f^{-U}(x)=\frac{3x_1+x_2+7}{2x_1+2x_2+6}\}$$

Subject to : $x_1+3x_2\ \le 30$

$$-x_1+2x_2\ \le 5$$

$$\frac{3x_1+x_2+7}{2x_1+2x_2+6}\le 8.47$$

$$x_1,x_2\ge 0$$

Solving this problem $P_2$ by transformations and using simplex method we obtain the optimal solution $x_1=29.999\ ,x_2=0$ with the objective value $f^{-U}(x)=1.47$ .

$$P_3\ :\qquad Max\left\{f^{+L}(x)=\frac{\frac{9}{2}x_1+3x_2+10}{x_1+\frac{3}{2}x_2+\frac{9}{2}}\right\}$$

Subject to : $x_1+3x_2\ \le 30$

$-x_1+2x_2\ \le 5$

$$1.47 \leq \frac{\frac{9}{2}x_1 + 3x_2 + 10}{x_1 + \frac{3}{2}x_2 + \frac{9}{2}} \leq 8.47$$

$$x_1, x_2 \geq 0 \quad .$$

Solving this problem $P_3$ by transformation and using simplex method we obtain the optimal solution $x_1 = 29.99$ , $x_2 = 0$ with the objective value $f^{+L}(x) = 4.20$ .

$$P_4 : \quad Max\{ \ f^{-L}(x) = \frac{\frac{7}{2}x_1 + 2x_2 + 8}{\frac{3}{2}x_1 + \frac{7}{4}x_2 + \frac{11}{2}} \}$$

Subject to : $x_1 + 3x_2 \leq 30$

$-x_1 + 2x_2 \leq 5$

$$1.47 \leq \frac{\frac{7}{2}x_1 + 2x_2 + 8}{\frac{3}{2}x_1 + \frac{7}{4}x_2 + \frac{11}{2}} \leq 4.20$$

$x_1, x_2 \geq 0.$

Solving this problem $P_4$ by transformation and using simplex method we obtain the optimal solution $x_1 = 29.999$ , $x_2 = 0$ with the objective value $f^{-L}(x) = 2.24$

The optimal solutionof the original problem is $x^* \cong (30, 0)$ with the objectivevalue$f^* = [[2.24, 4.20] : [1.47, 8.47]]$ .

**Example 4.2.** consider the following optimization problem :

Maximize $f = \dfrac{\left[[1,3]:\left[\frac{1}{2},4\right]\right]x_1 + \left[[2,4]:\left[1,\frac{9}{2}\right]\right]x_2}{\left[\left[\frac{1}{2},\frac{3}{2}\right]:\left[\frac{1}{4},2\right]\right]x_1 + \left[\left[\frac{1}{2},\frac{3}{2}\right]:\left[\frac{1}{4},2\right]\right]x_2 + \left[[1,3]:\left[\frac{1}{3},\frac{7}{2}\right]\right]}$

Subject to : $x_1 - x_2 \geq 1$

$$2x_1 + 3x_2 \leq 15$$

$$x_1 \geq 3$$

$x_1, x_2 \geq 0$ .

**Solution :**We see the objective function can be written in the form

$$f(x) = \left[ \left[\frac{x_1 + 2x_2}{\frac{3}{2}x_1 + \frac{3}{2}x_2 + 3}, \frac{3x_1 + 4x_2}{\frac{1}{2}x_1 + \frac{1}{2}x_2 + 1}\right] : \left[\frac{\frac{1}{2}x_1 + x_2}{2x_1 + 2x_2 + \frac{7}{2}}, \frac{4x_1 + \frac{9}{2}x_2}{\frac{1}{4}x_1 + \frac{1}{4}x_2 + 3}\right] \right]$$

The optimal solution $x_1^* = 3.6$ , $x_2^* = 2.6$ with the objective value

$f^*(x) = [[0.72, 5.17] : [0.28, 13.87]]$ .

## 5. Conclusion .

In this paper, first we introduced two possible types equation (4),(7) of linear fractional programming problems with rough interval objective functions. Then we proved that we can convert the problem of the form (4) to the form (7). By solving (7) we obtained the optimal solution for original linear fractional programming problem with rough interval objective function.To find the beast solution we configure all four problems in the form linear fractional and then find a solution using approach for solving linear fractional programming problems.

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