ONE CONSTRUCTION OF AN AFFINE PLANE OVER A CORPS

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ABSTRACT

In this paper, based on several meanings and statements discussed in the literature, we intend constuction a affine plane about a of whatsoever corps $(\mathbf{K},\oplus,\odot)$. His points conceive as ordered pairs (α, β), where α and β are elements of corps $(\mathbf{K},\oplus,\odot)$. Whereas straight-line in corps, the conceptualize by equations of the type $x \odot a \oplus y \odot b = c, a \neq 0_K$ or $b \neq 0_K$ the variables and coefficients are elements of that body. To achieve this construction we prove some theorems which show that the incidence structure **A=(Π, Λ, I)** connected to the corps **K** satisfies axioms A1, A2, A3 definition of affine plane. In all proofs rely on the sense of the corps as his ring and properties derived from that definition.

Keywords: The unitary ring, integral domain, zero division, corps, incdence structurse, point connected to a corp, straight line connected to a corp, affine plane.

Academic Discipline And Sub-Disciplines

Mathematics; Geometric Algebra; Affine Plane.

1. INTRODUCTION. GENERAL CONSIDERATIONS ON THE AFFINE PLANE AND THE CORPS

In this paper initially presented some definitions and statements on which the next material.

Let us have sets **Π, Λ, I,** where the two first are non-empty.

Definition 1.1: The incidence structure called a ordering trio **A=(Π, Λ, I)** where $\Pi \cap \Lambda = \emptyset$ and $I \subseteq \Pi \times \Lambda$.

Elements of sets Π we call points and will mark the capitalized alphabet, while those of the sets **Λ,** we call blocks (or straight line) and will mark minuscule alphabet. As in any binary relation, the fact $(P, \ell) \in I$ for $P \in \Pi$ and for $\ell \in \Lambda$, it will also mark P I ℓ and we will read, point P is incident with straight line ℓ or straight line ℓ there are incidents point P.

(See [3], [4], [5], [10], [11], [12], [13], [14], [15]).

Definition 1.2. ([3], [8], [16]) Affine plane called the incidence structure **A=(Π, Λ, I),** that satisfies the following axioms:

A1: For every two different points **P** and **Q** ∈ Π, there is one and only one straight line $\ell \in \Lambda$, passing of those points.

The straight line ℓ defined by points P and Q will mark the PQ.

A2: For a point **P** ∈Π, and straight line $\ell \in \Lambda$ such that $(P, \ell) \notin I$, there is one and only one straight line $\mathbf{m} \in \mathcal{L}$, passing the point P, and such that $\ell \cap r = \emptyset$.

A3: In $\mathcal{A}$ here are three non-incident points to a straight line.

A1 derived from the two lines different of $\mathcal{L}$ many have a common point, in other words two different straight lines of $\mathcal{L}$ or do not have in common or have only one common point.

In affine plane **A=(Π, Λ, I),** these statements are true.

Proposition 1.1. ([3], [5]) In affine plane **A=(Π, Λ, I),** there are four points, all three of which are not incident with a straight line (three points are called non-collinear).

Proposition 1.2. ([3], [6],]) In affine plane **A=(Π, Λ, I),** exists four different straight line.

Proposition 1.2. ([3], [8]) In affine plane **A=(Π, Λ, I),** every straight line is incident with at least two different points.

Proposition 1.3. ([3], [9]) In affine plane **A=(Π, Λ, I),** every point is incidents at least three of straight line.

Proposition 1.4. ([3]) On a finite affine plane **A=(Π, Λ, I),** every straight line contains the same number of points and in every point the same number of straight line passes. Furthermore, there is the natural number $n \in \mathbb{N}$, $n \geq 2$, such that:

1) In each of straight line $\ell \in \mathcal{L}$, the number of incidents is points with him is n.

2) For every point $P \in \mathcal{P}$, of affine plane **A=(Π, Λ, I),** it has exactly n + 1 straight line incident with him.

3) In a finite affine plane **A=(Π, Λ, I),** there are exactly n^2 points.

4) In a finite affine plane **A=(Π, Λ, I),** there are exactly $n^2 + n$ straight line.

The number n in Proposition 1.4, it called order of affine plane **A=(Π, Λ, I),** it is distinctly that the less order a finite affine plane, is n = 2. In a such affine plane it is with four points and six straight lines, shown in Fig.1.

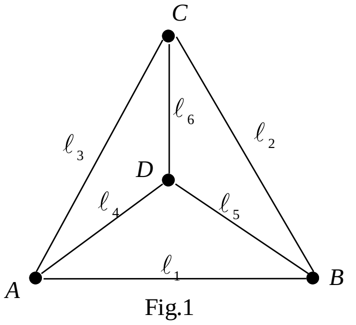

Fig.1

Definition 1.3. ([1]). The ring called structures $(B,\oplus,\odot)$, that has the properties:

1) structure $(B,\oplus)$, is an abelian group;

2) The second action $\odot$ It is associative ;

3) The second action $\odot$ is distributive of the first operation of the first $\oplus$.

In a ring $(B,\oplus,\odot)$ also included the action deduction – accompanying each (a, b) from B, sums

$$a \oplus (-b)$$

well

$$a \oplus (-b) = a - b$$

Proposition 1.5 ([1], [7]). In a unitary ring $(B,\oplus,\odot)$, having more than one element, the unitary element 1_B is different from 0_B.

Definition 1.4 ([1], [2]). Corp called rings $(K,\oplus,\odot)$ that has the properties:

1. K is at least one element different from zero.
2. $K^* = K - \{0_K\}$ it is a subset of the stable of K about multiplication;
3. $(K^*, \odot)$ is a group.

THEOREM 1.1. ([2]) If $(\mathbf{K},\oplus,\odot)$ **is the corp, then:**

1. **it is the unitary element (is the unitary ring);**
2. **there is no zero divisor (is integral domain);**
3. **They have single solutions in K equations $a \odot x = b$ and $x \odot a = b$, where $a \neq 0_K$ and b are two elements what do you want of K.**

2. TRANSFORMS OF A INCIDENCE STRUCTURES RELATING TO A CORPS IN A AFFINE PLANE

Definition 2.1. Let it be $(K,\oplus,\odot)$ a corps. A ordered pairs (α, β) by coordinates α, β ∈K, called point connected to the corp K.

Sets K^2 of points associated with corps K mark Π.

Definition 2.2. Let be a, b, c∈K.. Sets

$$\ell = \{(x, y) \in K^2 \mid x\odot a\oplus y\odot b=c,\ a \neq 0_K \text{ or } b \neq 0_K \} \tag{1}$$

called the straight line associated with corps K.

Equations $x\odot a\oplus y\odot b=c$, called equations of the straight line ℓ. Sets of straight lines connected to the body K mark Λ. It is evidently that

$$\Pi\cap\Lambda=\varnothing.$$

Definition 2.3. Will say that the point $P=(\alpha,\beta) \in\Pi$ is incident to straight line (1), if its coordinates verify equation of ℓ,

This means that if it is true equation $\alpha\odot a\oplus\beta\odot b=c$. This fact write down

$$P \in \ell.$$

Defined in this way is an incidence relations

$$\mathbf{I} \subseteq \Pi \times \Lambda,$$

such that $\forall(P, \ell)$, $\Pi I \Lambda \Leftrightarrow P \in \ell$. So even here, when pionts P is incidents with straight line ℓ, we will say otherwise point P is located at straight line ℓ, or straight line ℓ passes by points P.

It is thus obtained, connected to the corps **K** a incidence structure $A=(\Pi, \Lambda, I)$. Our intention is to study it.

According to (1), a straight line ℓ its having the equation

$$x \odot a \oplus y \odot b = c, \text{ where } a \neq 0_K \text{ or } b \neq 0_K. \tag{2}$$

Condition (2) met on three cases: 1) $a \neq 0_K$ and $b = 0_K$; 2) $a = 0_K$ and $b \neq 0_K$; 3) $a \neq 0_K$ and $b \neq 0_K$, that allow the separation of the sets Λ the straight lines of its three subsets Λ_0, Λ_1, Λ_2 as follows:

$$\boldsymbol{\mathcal{L}_0} = \{\ell \in \mathcal{L} \,|\, x \odot a \oplus y \odot b = c,\ \ a \neq 0_K \text{ and } b = 0_K\}; \tag{3}$$

$$\boldsymbol{\mathcal{L}_1} = \{\ell \in \mathcal{L} \,|\, x \odot a \oplus y \odot b = c,\ \ a = 0_K \text{ and } b \neq 0_K\}; \tag{4}$$

$$\boldsymbol{\mathcal{L}_2} = \{\ell \in \mathcal{L} \,|\, x \odot a \oplus y \odot b = c,\ \ a \neq 0_K \text{ and } b \neq 0_K\}. \tag{5}$$

Otherwise, subset $\boldsymbol{\mathcal{L}_0}$ is a sets of straight lines $\ell \in \mathcal{L}$ with equation

$$x \odot a = c, \text{ where } a \neq 0_K \Leftrightarrow x = d, \text{ where } d = c \odot a^{-1}; \tag{3'}$$

subset $\boldsymbol{\mathcal{L}_1}$ is a sets of straight lines $\ell \in \mathcal{L}$ with equation

$$y \odot b = c, \text{ where } b \neq 0_K \Leftrightarrow y = f, \text{ where } f = c \odot b^{-1}; \tag{4'}$$

Whereas subset $\boldsymbol{\mathcal{L}_2}$ is a sets of straight lines $\ell \in \mathcal{L}$ with equation

$$x \odot a \oplus y \odot b = c, \text{where } a \neq 0_K \text{ and } b \neq 0_K \Leftrightarrow y = x \odot k \oplus g; \tag{5'}$$

where $k = (-1_K) \odot a \odot b^{-1} \neq 0_K$, $g = c \odot b^{-1}$;

Hence the

- a straight line $\ell \in \boldsymbol{\mathcal{L}_0}$ is completely determined by the element $d \in K$ such that its equation is $x = d$,
- a straight line $\ell \in \boldsymbol{\mathcal{L}_1}$ is completely determined by the element $f \in K$ such that its equation is $y = f$ and
- a straight line $\ell \in \boldsymbol{\mathcal{L}_2}$ is completely determined by the elements $k \neq 0_K, g \in K$ such that its equation is $y = x \odot k \oplus g$.

From the above it is clear that $\Pi = \{\boldsymbol{\mathcal{L}_0}, \boldsymbol{\mathcal{L}_1}, \boldsymbol{\mathcal{L}_2}\}$ is a separation of the sets of straight lines $\boldsymbol{\mathcal{L}}$.

THEOREM 2.1. For every two distinct points P, Q $\in \Pi$, there exist only one straight line $\ell \in \Lambda$ that passes in those two points.

Proof. Let $P=(p_1, p_2)$ and $Q=(q_1, q_2)$. Fact that $P \neq Q$ means

$$(p_1, p_2) \neq (q_1, q_2). \tag{6}$$

Based on (6) we distinguish three cases:

1) $p_1 = q_1$ and $p_2 \neq q_2$;
2) $p_1 \neq q_1$ and $p_2 = q_2$;
3) $p_1 \neq q_1$ and $p_2 \neq q_2$;

Let's be straight line $\ell \in \Lambda$, yet unknown, according to (2), having the equation $x \odot a \oplus y \odot b = c$, where $a \neq 0_K$ or $b \neq 0_K$.

Consider the **case 1)** $p_1 = q_1$ and $p_2 \neq q_2$. From the fact P, Q $\in \ell$ we have:

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ q_1 \odot a \oplus q_2 \odot b = c \end{cases} \Leftrightarrow \begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_1 \odot a \oplus p_2 \odot b = q_1 \odot a \oplus q_2 \odot b \end{cases}$$

But $p_1 = q_1$ and $p_2 \neq q_2$, so, from the fact that $(K,\oplus)$ is abelian group, by Definition 1.4, we get

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_2 \odot b = q_2 \odot b \end{cases} \Leftrightarrow \begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ (p_2 - q_2) \odot b = 0_K \end{cases}$$

From above, according to Theorem 1.1, corps K is complete ring, so with no divisor 0_K, results

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ b = 0_K \end{cases} \Leftrightarrow \begin{cases} p_1 \odot a = c \\ b = 0_K \end{cases}$$

From this

a) if $p_1 = 0_K$, we get

$$\begin{cases} 0_K \odot a = c \\ b = 0_K \ (a \neq 0_K) \end{cases} \Leftrightarrow \begin{cases} c = 0_K \\ b = 0_K \ (a \neq 0_K) \end{cases}$$

.

According to this result, equation (2) takes the form $x \odot a = 0_K$, where $a \neq 0_K$, otherwise

$$x = 0_K \tag{7}$$

(since, being $a \neq 0_K$, it is element of group $(K^*, \odot)$, so $x \odot a = 0_K \Leftrightarrow x = 0_K \odot a^{-1} \Leftrightarrow x = 0_K$).

b) if $p_1 \neq 0_K$, and p_1 is element of group $(K^*, \odot)$, exists p_1^{-1}, that get the results:

$$\begin{cases} a = p_1^{-1} \odot c \\ b = 0_K \ (a \neq 0_K) \end{cases}$$

under which, equation (2) in this case take the form

$$x \odot p_1^{-1} \odot c = c, \text{ where } a \neq 0_K \Leftrightarrow x \odot p_1^{-1} = 1_K \Leftrightarrow x = p_1 \tag{7'}$$

Here it is used the right rules simplifying in the group $(K^*, \odot)$, with $c \neq 0_K$, because $a = p_1^{-1} \odot c$ and $a \neq 0_K$.

For two cases (7) and (7') notice that, when $p_1 = q_1$ and $p_2 \neq q_2$, there exists a unique straight line ℓ with equation $x = d$ of the form (3'), so a line $\ell \in \Lambda_0$.

Case 2) $p_1 \neq q_1$ and $p_2 = q_2$ is an analogous way and achieved in the conlusion and in this case there exists a unique straight line ℓ with equation $y = f$ of the form (4'), so a line $\ell \in \Lambda_1$.

Consider now the **case 3)** $p_1 \neq q_1$ and $p_2 \neq q_2$. From the fact P, Q $\in \ell$ we have:

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ q_1 \odot a \oplus q_2 \odot b = c \end{cases} \Leftrightarrow \begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_1 \odot a \oplus p_2 \odot b = q_1 \odot a \oplus q_2 \odot b \end{cases}. \tag{8}$$

The second equation can be written in the form

$$(p_1 - q_1) \odot a = (q_2 - p_2) \odot b, \text{ that bearing } a \neq 0_K \text{ and } b \neq 0_K \tag{9}$$

Regarding to the coordinates of point P we distinguish these four cases:

a) $p_1 = 0_K = p_2$. This bearing $q_1 \neq 0_K$ and $q_2 \neq 0_K$. In this conditions (8) take the form

$$\begin{cases} c = 0_K \\ a = -q_1^{-1} \odot q_2 \odot b \end{cases}.$$

According to this result, equation (2) take the form $x \odot (-q_1^{-1} \odot q_2 \odot b) \oplus y \odot b = 0_K$, where , according (9), $b \neq 0_K$. So, by the properties of group we have:

$$[x \odot (-q_1^{-1} \odot q_2) \oplus y] \odot b = 0_K \Leftrightarrow -x \odot (q_1^{-1} \odot q_2) \oplus y = 0_K \odot b^{-1} \Leftrightarrow$$
$$y = x \odot (q_1^{-1} \odot q_2), \text{ where } q_1^{-1} \odot q_2 \neq 0_K \tag{10}$$

b) $p_1 = 0_K \neq p_2$. This bearing $q_1 \neq 0_K$. In this conditions, system (8) take the form

$$\begin{cases} p_2 \odot b = c \\ q_1 \odot a = q_1^{-1} \odot (p_2 - q_2) \odot b \end{cases}.$$

−This result, give the equation (2) the form $x \odot [q_1^{-1} \odot (p_2 - q_2)] \odot b \oplus y \odot b = c$, where besides $c \neq 0_K$, by (9), the $b \neq 0_K$. So, by the properties of group we have:

$$x \odot [q_1^{-1} \odot (p_2 - q_2)] \odot b \oplus y \odot b = c \Leftrightarrow [x \odot q_1^{-1} \odot (p_2 - q_2) \oplus y] \odot b = c \Leftrightarrow$$

$$[x \odot q_1^{-1} \odot (p_2 - q_2) \oplus y] \odot p_2^{-1} \odot c = c \Leftrightarrow [x \odot q_1^{-1} \odot (p_2 - q_2) \oplus y] \odot p_2^{-1} = 1_K \Leftrightarrow$$

$$x \odot [q_1^{-1} \odot (p_2 - q_2)] \oplus y = p_2 \Leftrightarrow$$

$$y = x \odot [q_1^{-1} \odot (p_2 - q_2)] \oplus p_2, \quad \text{where } q_1^{-1} \odot (p_2 - q_2) \neq 0_K \tag{11}$$

c) $p_1 \neq 0_K = p_2$. This bearing $q_2 \neq 0_K$, and the system (8) take the form

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ (p_1 - q_1) \odot a = (q_2 - p_2) \odot b \end{cases}.$$

In a similar way b) it is shown that equation (2) take the form

$$y = x \odot [(q_1 - p_1)^{-1} \odot q_2] \oplus p_1 \odot (p_1 - q_1)^{-1} \odot q_2 \quad \text{where } (q_1 - p_1)^{-1} \odot q_2 \neq 0_K . \tag{12}$$

d) $p_1 \neq 0_K$ and $p_2 \neq 0_K$. We distinguish four subcases:

d₁) $q_1 = 0_K = q_2$. From the system (8) we have

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ q_1 \odot a \oplus q_2 \odot b = 0_K \end{cases} \Rightarrow c = 0_K \text{ and } a = -p_1^{-1} \odot p_2 \odot b$$

After e few transformations equation (2) take the form

$$y = x \odot (p_1^{-1} \odot p_2), \text{ where } p_1^{-1} \odot p_2 \neq 0_K \tag{13}$$

d₂) $q_1 = 0_K \neq q_2$. From the system (8) we have

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_1 \odot a \oplus p_2 \odot b = q_2 \odot b \end{cases} \Rightarrow q_2 \odot b = c \Rightarrow c = 0_K$$

and $a = p_1^{-1} \odot (q_2 - p_2) \odot b$, where $b = q_2^{-1} \odot c$.

After e few transformations equation (2) take the form

$$y = x \odot [p_1^{-1} \odot (p_2 - q_2)] \oplus q_2, \text{ ku } p_1^{-1} \odot (p_2 - q_2) \neq 0_K \tag{14}$$

d₃) $q_1 \neq 0_K = q_2$. In this conditions (8) bearing

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_1 \odot a \oplus p_2 \odot b = q_1 \odot a \end{cases} \Rightarrow q_1 \odot a = c \Rightarrow c \neq 0_K$$

And $b = p_2^{-1} \odot (q_1 - p_1) \odot a$, where $a = q_1^{-1} \odot c$.

After e few transformations equation (2) take the form

$$y = x \odot [q_1^{-1} \odot (p_1 - q_1) \odot p_2] \oplus q_1 \odot (q_1 - p_1)^{-1} \odot p_2, \text{ where } q_1^{-1} \odot (p_1 - q_1) \odot p_2 \neq 0_K \tag{15}$$

d₄) $q_1 \neq 0_K$ and $q_2 \neq 0_K$. If $c = 0_K$ system (8) have the form

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = 0_K \\ p_1 \odot a \oplus p_2 \odot b = q_1 \odot a \oplus q_2 \odot b \end{cases}.$$

After e few transformations results that the equation (2) have the form

$$y = x \odot [q_1^{-1} \odot (p_1 - q_1)^{-1} \odot (p_2 - q_2)], \quad \text{where } q_1^{-1} \odot (p_1 - q_1)^{-1} \odot (p_2 - q_2) \neq 0_K \tag{16}$$

If $c \neq 0_K$, system (8) , by multiplying both sides of his equations with c^{-1}, this is transform as follows:

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_1 \odot a \oplus p_2 \odot b = q_1 \odot a \oplus q_2 \odot b \end{cases} \Leftrightarrow \begin{cases} p_1 \odot a_1 \oplus p_2 \odot b_1 = 1_K \\ p_1 \odot a_1 \oplus p_2 \odot b_1 = q_1 \odot a_1 \oplus q_2 \odot b_1 \end{cases}$$

From this equation (2) take the form

$$y = x \odot [(p_1 - q_1)^{-1} \odot (p_2 - q_2)] \oplus b_1^{-1}, \text{ where } (p_1 - q_1)^{-1} \odot (p_2 - q_2) \neq 0_K \tag{17}$$

As conclusion, from the four cases (14), (15), (16) and (17), we notice that, when $p_1 \neq q_1$ and $p_2 \neq q_2$, there exists an unique straight line ℓ with equation $y = x \odot k \oplus g$ of the form (5'), so a line $\ell \in \Lambda_2$.

THEOREM 2.2. For a point P $\in \Pi$ and a straight line $\ell \in \Lambda$ such that $P \notin \ell$ exists only one straight line r $\in \Lambda$ passing the point P, and such that $\ell \cap r = \varnothing$.

Proof. Let it be $\boldsymbol{P} = (p_1, p_2)$. We distinguish cases:

a) $p_1 = 0_K$ and $p_2 = 0_K$;

b) $p_1 \neq 0_K$ and $p_2 = 0_K$;

c) $p_1 = 0_K$ and $p_2 \neq 0_K$;

d) $p_1 \neq 0_K$ and $p_2 \neq 0_K$;

The straight line, still unknown r, let us have equation

$$x \odot \alpha \oplus y \odot \beta = \gamma, ku\ \ \alpha \neq 0_K\ ose\ \ \beta \neq 0_K \tag{18}$$

For straight line ℓ, we distinguish these cases: **1)** $\ell \in \boldsymbol{\mathcal{L}_0}$; **2)** $\ell \in \boldsymbol{\mathcal{L}_1}$; **3)** $\ell \in \boldsymbol{\mathcal{L}_2}$

Case 1) $\ell \in \boldsymbol{\mathcal{L}_0}$**.** In this case it has equation $x = d$.

The fact that $P = (p_1, p_2) \notin \ell$, It brings to $p_1 \neq d$. But the fact that $\ell \cap r = \emptyset$, it means that there is no point $Q \in \mathcal{P}$, that $Q \in \ell$ and $Q \in r$, otherwise is this true

$$\forall Q \in \mathcal{P}, Q \notin \ell \cap r. \tag{19}$$

In other words there is no system solution

$$\begin{cases} x = d \neq p_1 \\ x \odot \alpha \oplus y \odot \beta = \gamma \end{cases}. \tag{19'}$$

since $P \in r$, that brings

$$p_1 \odot \alpha \oplus p_2 \odot \beta = \gamma, \text{ where } \alpha \neq 0_K \text{ and } \beta \neq 0_K \tag{20}$$

In case **a)** $p_1 = 0_K$ and $p_2 = 0_K$, from (20) it turns out that $\gamma = 0_K$,

Then equation (18) take the form

$$x \odot \alpha \oplus y \odot \beta = 0_K, \text{ where } \alpha \neq 0_K \text{ or } \beta \neq 0_K$$

- If $\alpha \neq 0_K$ *ore* $\beta = 0_K$, equation (18) take the form

$$x \odot \alpha = 0_K \Leftrightarrow x = 0_K$$

Determined so a straight line r with equation $x = 0_K$, that passing point $P = (0_K, 0_K)$, for which the system (19') no solution, after his appearance:

$$\begin{cases} x = d \neq 0_K \\ x = 0_K \end{cases}$$

- If $\alpha = 0_K$ *or* $\beta \neq 0_K$, equation (18) take the form

$$y \odot \beta = 0_K \Leftrightarrow y = 0_K$$

that defines a straight line r_1. In this case system (19') take the form

$$\begin{cases} x = d \neq 0_K \\ y = 0_K \end{cases},$$

which solution point $\mathrm{Q} = (\mathrm{d}, 0_\mathrm{K}) \in \ell \cap \mathrm{r}_1$. This proved that straight line r_1 It does not meet the demand $\ell \cap \mathrm{r}_1 = \emptyset$.

- If $\alpha \neq 0_\mathrm{K}$ ose $\beta \neq 0_\mathrm{K}$, equation (18) take the form

$$y \odot \beta = -x \odot \alpha \Leftrightarrow y = x \odot (-\alpha \odot \beta^{-1})$$

that defines a straight line r_2. In this case system (19') take the form

$$\begin{cases} x = d \neq 0_K \\ y = x \odot (-\alpha \odot \beta^{-1}) \end{cases},$$

which solution point $R = (d, -d \odot \alpha \odot \beta^{-1}) \in \ell \cap r_2$. Also straight line r_2 it does not meet the demand $\ell \cap r_1 = \emptyset$.

In this way we show that, when $\ell \in \mathcal{L}_0$ exist just a straight line r, whose equation is

$$x = 0_K$$

that satisfies the conditions of Theorem.

Conversely proved Theorem 2.2 is true for cases **2)** $\ell \in \mathcal{L}_1$ dhe **3)** $\ell \in \mathcal{L}_2$ **.**

THEOREM 2.3. **In the incidence structure A=(Π, Λ, I) connected to the corp K, there exists three points not in a straight line**.

Proof. From Proposition 1.5, since the corp K is unitary ring, this contains 0_K and $1_K \in K$, such that $0_K \neq 1_K$. It is obvious that the points $P = (0_K, 0_K), Q = (1_K, 0_K)$ and $R = (0_K, 1_K)$ are different points pairwise distinct $\mathcal{P}$. Since $P \neq Q$, and $0_K \neq 1_K$, by the case 2) of the proof of Theorem 2.1, results that the straight line $PQ \in \mathcal{L}_1$, so it have equation of the form $y = f$. Since $P \in PQ$ results that $f = 0_K$. So equation of PQ is $y = 0_K$. Easily notice that the point $R \notin PQ$.

Three Theorems 2.1, 2.2, 2.3 shows that an incidence structure **A=(Π, Λ, I)** connected to the corp **K,** satisfy three axioms **A1, A2, A3** of Definition 1.2 of an afine plane. As consequence we have

THEOREM 2.4. **An incidence structure A=(Π, Λ, I) connected to the corp K is an afine plane connected with that corp.**

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