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On the sulutions of a fractional differemtial equation

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Abstruct

We have showed the results obtained in [1] are incorrect and the fractional complex transform is invalid to the fractional differential equation which contain modified Riemann-Liouville fractional derivative.

Keyword:

fractional complex transform; fractional differential equation; modified Riemann-Liouville fractional derivative.

1. INTRUDUCTION

Fractional differential equations are generalizations of classical differential equation. During the past decades, fractional differential equations appeared more and more frequently in various research areas of science and engineering [1]. Therefore, many authors want to find the exact analytical solutions or approximate analytical solutions of some fractional differential equations using different ways [2].

In [1], by using fractional complex transform, Zhang et al. studied the following frctional differential equation:

$$\partial_t^\alpha u(x,t) = k\partial_x^\beta u(x,t), \quad t \in R^+, x \in R \qquad (1)$$

Subject to the initial condition

$$u(x,0) = 2x, \qquad (2)$$

where k is a positive coefficient, $0<\alpha<1, 0<\beta<1,$ $u(x,t)$ is the real-valued variable function, ∂_t^α and ∂_x^β are modified Riemann-Liouville fractional derivatives.

They obtained the following exact solution to the initial problems (1) and (2):

$$u(x,t) = 2\left(\frac{\Gamma(1+\beta)}{p}\right)^{\frac{1}{\beta}}\left(\frac{px^p}{\Gamma(1+\beta)} + \frac{qt^\alpha}{\Gamma(1+\alpha)}\right)^{\frac{1}{\beta}}, \qquad (3)$$

where p and q are free parameters satisfying $kp-q=0.$

In this note, we will showed that above solution (3) is not solution of the problem (1)-(2) by counterexample, and therefore we can assert the fractional complem transform is false to the initial problem.

2. Main result

We first recall the dedinition of modified Riemann-Liouville fractional derivatives.

Dedinition: Let $f(x)$ is a continuous but not necessarily differentiable function, then its fractional derivative of order $\alpha\ (0<\alpha<1)$ is defined by the following expression [2]:

$$\partial^{\alpha} f(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-\xi)^{-\alpha}(f(\xi)-f(0))d\xi. \qquad (4)$$

From the above Definition, we can check the results obtaind in [1].

For simplicity, we take $\alpha=\frac{1}{3}, \beta=\frac{1}{2}, k=1,$ and $p=q=1.$

Thus the initial problem (1)-(2) becomes

$$\begin{cases}\partial^{1/3}u(x,t)=\partial^{1/2}u(x,t),\\ u(x,0)=2x,\end{cases} \qquad (5)$$

and the solution (3) becomes

$$u(x,t)=2x+\frac{4\Gamma(\frac{3}{2})}{\Gamma(\frac{4}{3})}x^{\frac{1}{2}}t^{\frac{1}{3}}+\frac{2\Gamma^2(\frac{3}{2})}{\Gamma^2(\frac{4}{3})}t^{\frac{2}{3}}. \qquad (6)$$

But, by the formula (4), we have

$$\partial_t^{\frac{1}{3}}u(x,t)=2\Gamma(\frac{1}{2})x^{\frac{1}{2}}+\frac{9\Gamma^2(\frac{1}{2})\Gamma(\frac{2}{3})}{\Gamma^3(\frac{1}{3})}t^{\frac{1}{3}},$$

and

$$\partial_x^{\frac{1}{2}}u(x,t)=\frac{4}{\Gamma(\frac{1}{2})}x^{\frac{1}{2}}+\frac{3\Gamma^2(\frac{1}{2})}{\Gamma(\frac{1}{3})}t^{\frac{1}{3}}.$$

From the above two expressions, we obtain :

$$\partial_t^{\frac{1}{3}}u(x,t)\neq\partial_x^{\frac{1}{2}}u(x,t).$$

This shows that the function (6) does not the solution of the problem (5). Moreover, we see that the function (3) does not the solution of the problem (1)-(2), so that the fractional complex transform is invalid to the fractional differential equation which contain modified Riemann-Liouville fractional derivative.

3. Conclusion

We have showed the results obtained in [1] are incorrect and the fractional complex transform Is invalid to the problem (1)-(2).

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