ISSN 2347-1921

Evaluation of reliability parameters of a system having three independent components with repair facility

V.K. Pathak[1] and H.L. Manker[2]
Assistant Professor, BCS Govt. P.G. College, Dhamtari(C.G.),
Assistant Professor, Govt.G.S.G P.G. College, Balod(C.G.),

ABSTRACT

Barlow & Prochan [1] were first to study a complex system taking the component failure and repair times as Independent of each other. In recent years, many papers on reliability such as Li [2] used multi-state weighted k- out- of- n systems to analyze repairable systems with arbitrary failure time distributions. Exponential distribution plays an important role in the study of system with repair. In order to predict and estimate or optimize the probability of survival and the mean life, it is essential to take exponential distribution. Earlier, Goel et al[8] have done similar reliability analysis taking units in three different modes. Rander et-al [6] has evaluated the cost analysis of two dissimilar cold standby systems with preventive maintenance and replacement of standby units. A pioneer work in this field was done by Gopalan [3] and Osaki [5] by performing analysis of warm standby system and parallel system with bivariate exponential life respectively. Earlier, Pathak et al [10 & 11] studied reliability parameters of a main unit with its supporting units and also compared the results with two different distributions. We define semi-up mode as the case when the one particular unit is not able to operate due to error in other units which makes these units non-operative. In this paper an attempt has been made by authors by incorporating the concept of semi-up mode and tried to obtain the reliability parameters of working system taking three independent components.

Keywords: Regenerative Point, MTSF, Availability, Busy Period, Semi-up Mode.

Academic Discipline: Mathematics

Subject Classification: 53A04, 53A05

Council for Innovative Research

Peer Review Research Publishing System

Journal: JOURNAL OF ADVANCES IN MATHEMATICS

Vol .11, No.2

www.cirjam.com , editorjam@gmail.com

ISSN 2347-1921

2. System Description about the model:

The system consists of three independent components namely A, B & C. Here component A are independent on other two components B and C and the system is operable when atleast two of the components are in operation and the system is semi operable when at least one of the components B or C are in operation. As soon as a job arrives, all the components work with load. It is assumed that only one job is taken for processing at a time. There is a single repairman who repairs the failed units on first come first served basis. Using regenerative point technique several system characteristics such as transition probabilities, mean sojourn times, availability and busy period of the repairman are evaluated. In the end the expected profit is also calculated.

3. Assumptions used in the model:

a. The system consists of three independent components A, B and C.

b. There is a single repairman which repairs the failed units whenever called for service.

c. The repairs are done on FCFS basis .

d. All units work as new after repair.

e. After random period of time the whole system goes to preventive maintenance.

f. The failure rates of all the units are taken to be exponential whereas the repair time distributions are arbitrary.

g. Switching devices are perfect and instantaneous.

4. Symbols and Notations:

p_{ij} = Transition probabilities from S_i to S_j

μ_i = Mean sojourn time at time t

E_0 =State of the system at epoch t=0

E=set of regenerative states $S_0 - S_7$

$q_{ij}(t)$ = Probability density function of transition time from S_i to S_j

$Q_{ij}(t)$ = Cumulative distribution function of transition time from S_i to S_j

$\pi_i(t)$ = Cdf of time to system failure when starting from state $E_0 = S_i \in E$

$\mu_i(t)$ = Mean Sojourn time in the state $E_0 = S_i \in E$

$B_i(t)$ = Repairman is busy in the repair at time t / $E_0 = S_i \in E$

$r_1 / r_2 / r_3 / r_4$ =Constant repair rate of Main unit A /Unit B/Unit C/Unit B or C

$\alpha / \beta / \gamma$ =Failure rate of Main unit A /Unit B/Unit C

$g_1 / g_2 / g_3 / g_4$ =Probability density function of repair time of Main unit A/Unit B/Unit C/Unit B or C

$\overline{G}_1 / \overline{G}_2 / \overline{G}_3 / \overline{G}_4$ =Cumulative distribution function of repair time of Main unit A/Unit B/Unit C/Unit B or C

a(t) = Probability density function of preventive maintenance .

b(t) = Probability density function of preventive maintenance completion time.

$\overline{A}(t)$ = Cumulative distribution functions of preventive maintenance.

$\overline{B}(t)$ = Cumulative distribution functions of preventive maintenance completion time.

$\boxed{s}$ **=** Symbol for Laplace -stieltjes transforms**.**

$\boxed{c}$ **=** Symbol for Laplace-convolution.

5. Symbols used for states of the system:

$A_0 / A_g / A_r / A_{wr}$ -- Component 'A' under operation/good and non-operative mode/ repair mode /waiting for repair

$B_0 / B_r / B_g$ -- Component 'B' under operation/repair/ good and non-operative mode

$C_0 / C_r / C_g$ -- Component 'C' under operation/repair/good and non-operative mode

P.M. -- System under preventive maintenance.

Up states: $S_0 = (A_0, B_0, C_0); S_2 = (A_0, B_r, C_0); S_3 = (A_0, B_0, C_r)$

Semi Up states: $S_1 = (A_r, B_0, C_0); S_4 = (A_{wr}, B_r, C_0); S_5 = (A_{wr}, B_0, C_r)$

Down States: $S_6 = (S.D.); S_7 = (P.M.)$

6. Transition Probabilities:

Simple probabilistic considerations yield the following non-zero transition probabilities:

1. $Q_{01}(t) = \int_0^t \alpha\, e^{-(\alpha+\beta+\gamma)t} \overline{A}(t)dt$

2. $Q_{02}(t) = \int_0^t \beta\, e^{-(\alpha+\beta+\gamma)t} \overline{A}(t)dt$

3. $Q_{03}(t) = \int_0^t \gamma\, e^{-(\alpha+\beta+\gamma)t} \overline{A}(t)dt$

4. $Q_{10}(t) = \int_0^t e^{-(\beta+\gamma)t} g_1(t)dt$

5. $Q_{14}(t) = \int_0^t \beta e^{-(\beta+\gamma)t} \overline{G}_1(t)dt$

6. $Q_{15}(t) = \int_0^t \gamma e^{-(\beta+\gamma)t} \overline{G}_1(t)dt$

7. $Q_{20}(t) = \int_0^t e^{-\alpha t} g_2(t)dt$

8. $Q_{24}(t) = \int_0^t \alpha e^{-\alpha t} \overline{G}_2(t)dt$

9. $Q_{30}(t) = \int_0^t e^{-\alpha t} g_3(t)dt$

10. $Q_{35}(t) = \int_0^t \alpha e^{-\alpha t} \overline{G}_3(t)dt$

11. $Q_{41}(t) = \int_0^t e^{-\gamma t} g_2(t)dt$

12. $Q_{46}(t) = \int_0^t \gamma e^{-\gamma t} \overline{G}_2(t)dt$

13. $Q_{51}(t) = \int_0^t e^{-\beta t} g_3(t)dt$

14. $Q_{56}(t) = \int_0^t \beta e^{-\beta t} \overline{G}_3(t)dt$

15. $Q_{60}(t) = \int_0^t g_4(t)dt$

16. $Q_{70}(t) = \int_0^t b(t)dt$

17. $Q_{07}(t) = \int_0^t a(t) e^{-(\alpha+\beta+\gamma)t} dt$

Now letting $t \to \infty$, we get $\lim_{t\to\infty} Q_{ij}(t) = p_{ij}$

18. $p_{01} = \int_0^\infty \alpha\, e^{-(\alpha+\beta+\gamma)t} \overline{A}(t)dt = \dfrac{\alpha}{(\alpha+\beta+\gamma)}[1 - a^*(\alpha+\beta+\gamma)]$,

19. $p_{02} = \int_0^\infty \beta e^{-(\alpha+\beta+\gamma)t} \overline{A}(t)dt = \frac{\beta}{(\alpha+\beta+\gamma)}[1 - a^*(\alpha+\beta+\gamma)]$

20. $p_{03} = \int_0^\infty \gamma e^{-(\alpha+\beta+\gamma)t} \overline{A}(t)dt = \frac{\gamma}{(\alpha+\beta+\gamma)}[1 - a^*(\alpha+\beta+\gamma)]$, 21.

$p_{10} = \int_0^\infty e^{-(\beta+\gamma)t} g_1(t)dt = g_1^*(\beta+\gamma)$

22. $p_{14} = \int_0^\infty \beta e^{-(\beta+\gamma)t} \overline{G}_1(t)dt = \frac{\beta}{(\beta+\gamma)}[1 - g_1^*(\beta+\gamma)]$, 23.

$p_{15} = \int_0^\infty \gamma e^{-(\beta+\gamma)t} \overline{G}_1(t)dt = \frac{\gamma}{(\beta+\gamma)}[1 - g_1^*(\beta+\gamma)]$

24. $p_{20} = \int_0^\infty e^{-\alpha t} g_2(t)dt = g_2^*(\alpha)$,

25. $p_{24} = \int_0^\infty \alpha e^{-\alpha t} \overline{G}_2(t)dt = 1 - g_2^*(\alpha)$

26. $p_{30} = \int_0^\infty e^{-\alpha t} g_3(t)dt = g_3^*(\alpha)$,

27. $p_{35} = \int_0^\infty \alpha e^{-\alpha t} \overline{G}_3(t)dt = 1 - g_3^*(\alpha)$

28. $p_{41} = \int_0^\infty e^{-\gamma t} g_2(t)dt = g_2^*(\gamma)$,

29. $p_{46} = \int_0^\infty \gamma e^{-\gamma t} \overline{G}_2(t)dt = 1 - g_2^*(\gamma)$

30. $p_{51} = \int_0^\infty e^{-\beta t} g_3(t)dt = g_3^*(\beta)$,

31. $p_{56} = \int_0^\infty \beta e^{-\beta t} \overline{G}_2(t)dt = 1 - g_3^*(\beta)$

32 . $p_{60}(t) = \int_0^\infty g_4(t)dt = 1$

33. $p_{70}(t) = \int_0^\infty b(t)dt = 1$

34. $p_{60} = p_{70} = 1$

[6.1-6.34]

It is easy to see that

$p_{01} + p_{02} + p_{03} + p_{07} = 1$, $p_{10} + p_{14} + p_{15} = 1$, $p_{20} + p_{24} = 1$, $p_{30} + p_{35} = 1$, $p_{41} + p_{46} = 1$,

$p_{51} + p_{56} = 1$

[6.35-6.40]

And mean sojourn time are given by

41. $\mu_0 = \frac{1}{(\alpha+\beta+\gamma)}[1 - a^*(\alpha+\beta+\gamma)]$,

42. $\mu_1 = \frac{1}{\beta+\gamma}[1 - g_1^*(\beta+\gamma)]$,

43. $\mu_2 = \frac{1}{\alpha}[1 - g_2^*(\alpha)]$,

44. $\mu_3 = \frac{1}{\alpha}[1 - g_3^*(\alpha)]$

45. $\mu_4 = \frac{1}{\gamma}[1 - g_2^*(\gamma)]$,

46. $\mu_5 = \frac{1}{\beta}[1 - g_3^*(\beta)]$

47. $\mu_6 = \int_0^\infty \overline{G}_4(t)dt$

48. $\mu_7 = \int_0^\infty \overline{B}(t)dt$

[6.41-6.48]

We note that the Laplace-stieltjes transform of $Q_{ij}(t)$ is equal to Laplace transform of $q_{ij}(t)$

i.e. $\tilde{Q}_{ij}(s) = \int_0^\infty e^{-st} Q_{ij}(t)dt = L\{Q_{ij}(t)\} = q_{ij}^*(s)$

[6.49]

50. $\tilde{Q}_{01}(s) = \int_0^\infty \alpha e^{-(s+\alpha+\beta+\gamma)t} \overline{A}(t)dt = \frac{\alpha}{s+\alpha+\beta+\gamma}[1 - a^*(s+\alpha+\beta+\gamma)]$

51. $\tilde{Q}_{02}(s) = \int_0^\infty \beta e^{-(s+\alpha+\beta+\gamma)t} \overline{A}(t)dt = \frac{\beta}{s+\alpha+\beta+\gamma}[1 - a^*(s+\alpha+\beta+\gamma)]$

52. $\tilde{Q}_{03}(s) = \int_0^\infty \gamma e^{-(s+\alpha+\beta+\gamma)t} \overline{A}(t)dt = \frac{\gamma}{s+\alpha+\beta+\gamma}[1 - a^*(s+\alpha+\beta+\gamma)]$

53. $\tilde{Q}_{07}(s) = \int_0^\infty e^{-(s+\alpha+\beta+\gamma)t} a(t)dt = a^*(s+\alpha+\beta+\gamma)$

54. $\tilde{Q}_{10}(s) = \int_0^\infty e^{-st} e^{-(\beta+\gamma)t} g_1(t)dt = g_1^*(s+\beta+\gamma)$

55. $\tilde{Q}_{14}(s) = \int_0^\infty \beta e^{-st} e^{-(\beta+\gamma)t} \overline{G}_1(t)dt = \frac{\beta}{s+\beta+\gamma}[1 - g_1^*(s+\beta+\gamma)]$

56. $\tilde{Q}_{15}(s) = \int_0^\infty \gamma e^{-st} e^{-(\beta+\gamma)t} \overline{G}_1(t)dt = \frac{\gamma}{s+\beta+\gamma}[1 - g_1^*(s+\beta+\gamma)]$

57. $\tilde{Q}_{20}(s) = \int_0^\infty e^{-(s+\alpha)t} g_2(t)dt = g_2^*(s+\alpha)$

58. $\tilde{Q}_{24}(s) = \int_0^\infty \alpha e^{-(s+\alpha)t} \overline{G}_2(t)dt = \frac{\alpha}{s+\alpha}[1 - g_2^*(s+\alpha)]$

59. $\tilde{Q}_{30}(s) = \int_0^\infty e^{-(s+\alpha)t} g_3(t)dt = g_3^*(s+\alpha)$

60. $\tilde{Q}_{35}(s) = \int_0^\infty (\alpha) e^{-(s+\alpha)t} \overline{G}_3(t)dt = \frac{\alpha}{s+\alpha}[1 - g_3^*(s+\alpha)]$

61. $\tilde{Q}_{41}(s) = \int_0^\infty e^{-(s+\gamma)t} g_2(t)dt = g_2^*(s+\gamma)$

62. $\tilde{Q}_{46}(s)=\int_0^\infty \gamma e^{-(s+\gamma)t}\overline{G}_2(t)dt=\frac{\gamma}{s+\gamma}[1-g_2^{*}(s+\gamma)]$

63. $\tilde{Q}_{51}(s)=\int_0^\infty e^{-(s+\beta)t}g_3(t)dt=g_3^{*}(s+\beta)$

64. $\tilde{Q}_{56}(s)=\int_0^\infty \beta e^{-(s+\beta)t}\overline{G}_3(t)dt=\frac{\beta}{s+\beta}[1-g_3^{*}(s+\beta)]$

65. $\tilde{Q}_{60}(s)=\int_0^\infty g_4(t)dt=g_4^{*}(s)$

66. $\tilde{Q}_{70}(s)=\int_0^\infty e^{-st}b(t)dt=b^{*}(s)$

[6.50-6.66]

We define m_{ij} as follows:-

$$m_{ij}=-[\frac{d}{ds}\tilde{Q}_{ij}(s)]_{s=0}=-Q_{ij}^{/}(0)$$

[6.67]

It can to show that

$$m_{01}+m_{02}+m_{03}+m_{07}=\mu_0; m_{10}+m_{14}+m_{15}=\mu_1; m_{20}+m_{24}=\mu_2; m_{30}+m_{35}=\mu_3\ ;$$

$$m_{41}+m_{46}=\mu_4\ ; m_{51}+m_{56}=\mu_5$$

[6.68-6.73]

7. Mean time to System failure-

Time to system failure can be regarded as the first passage time to the failed state. To obtain it we regard the down state as absorbing. Using the argument as for the regenerative process, we obtain the following recursive relations.

$$\pi_0(t)=Q_{01}(t)\boxed{s}\ \pi_1(t)+Q_{02}(t)\boxed{s}\ \pi_2(t)+Q_{03}(t)\boxed{s}\ \pi_3(t)+Q_{07}(t)$$

$$\pi_1(t)=Q_{10}(t)\boxed{s}\ \pi_0(t)+Q_{14}(t)\boxed{s}\ \pi_4(t)+Q_{15}(t)\boxed{s}\ \pi_5(t)$$

$$\pi_2(t)=Q_{20}(t)\boxed{s}\ \pi_0(t)+Q_{24}(t)\boxed{s}\ \pi_4(t)$$

$$\pi_3(t)=Q_{30}(t)\boxed{s}\ \pi_0(t)+Q_{35}(t)\boxed{s}\ \pi_5(t)$$

$$\pi_4(t)=Q_{41}(t)\boxed{s}\ \pi_1(t)+Q_{46}(t)$$

$$\pi_5(t)=Q_{51}(t)\boxed{s}\ \pi_1(t)+Q_{56}(t)$$

[7.1-7.6]

Taking Laplace -stieltjes transform of above equations and writing in matrix form.

We get
$$\begin{vmatrix} 1 & -\tilde{Q}_{01} & -\tilde{Q}_{02} & -\tilde{Q}_{03} & 0 & 0 \\ -\tilde{Q}_{10} & 1 & 0 & 0 & -\tilde{Q}_{14} & -\tilde{Q}_{15} \\ -\tilde{Q}_{20} & 0 & 1 & 0 & -\tilde{Q}_{24} & 0 \\ -\tilde{Q}_{30} & 0 & 0 & 1 & 0 & -\tilde{Q}_{35} \\ 0 & -\tilde{Q}_{41} & 0 & 0 & 1 & 0 \\ 0 & -\tilde{Q}_{51} & 0 & 0 & 0 & 1 \end{vmatrix} \begin{bmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \end{bmatrix} = \begin{bmatrix} \tilde{Q}_{07} \\ 0 \\ 0 \\ 0 \\ \tilde{Q}_{46} \\ \tilde{Q}_{56} \end{bmatrix}$$

$$D_1(s) = \begin{vmatrix} 1 & -\tilde{Q}_{01} & -\tilde{Q}_{02} & -\tilde{Q}_{03} & 0 & 0 \\ -\tilde{Q}_{10} & 1 & 0 & 0 & -\tilde{Q}_{14} & -\tilde{Q}_{15} \\ -\tilde{Q}_{20} & 0 & 1 & 0 & -\tilde{Q}_{24} & 0 \\ -\tilde{Q}_{30} & 0 & 0 & 1 & 0 & -\tilde{Q}_{35} \\ 0 & -\tilde{Q}_{41} & 0 & 0 & 1 & 0 \\ 0 & -\tilde{Q}_{51} & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\begin{aligned} D_1(s) = {} & (1-\tilde{Q}_{14}\tilde{Q}_{41}-\tilde{Q}_{15}\tilde{Q}_{51})-\tilde{Q}_{01}\tilde{Q}_{10}-\tilde{Q}_{02}\tilde{Q}_{10}\tilde{Q}_{24}\tilde{Q}_{41}-\tilde{Q}_{02}\tilde{Q}_{20}(1-\tilde{Q}_{14}\tilde{Q}_{41}-\tilde{Q}_{15}\tilde{Q}_{51}) \\ & -\tilde{Q}_{03}\tilde{Q}_{10}\tilde{Q}_{35}\tilde{Q}_{51}-\tilde{Q}_{03}\tilde{Q}_{30}(1-\tilde{Q}_{14}\tilde{Q}_{41}-\tilde{Q}_{15}\tilde{Q}_{51}) \end{aligned}$$

[7.7]

And $N_1(s) = \begin{vmatrix} \tilde{Q}_{07} & -\tilde{Q}_{01} & -\tilde{Q}_{02} & -\tilde{Q}_{03} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\tilde{Q}_{14} & -\tilde{Q}_{15} \\ 0 & 0 & 1 & 0 & -\tilde{Q}_{24} & 0 \\ 0 & 0 & 0 & 1 & 0 & -\tilde{Q}_{35} \\ \tilde{Q}_{46} & -\tilde{Q}_{41} & 0 & 0 & 1 & 0 \\ \tilde{Q}_{56} & -\tilde{Q}_{51} & 0 & 0 & 0 & 1 \end{vmatrix}$

$$\begin{aligned} N_1(s) = {} & \tilde{Q}_{07}(1-\tilde{Q}_{14}\tilde{Q}_{41}-\tilde{Q}_{15}\tilde{Q}_{51})+\tilde{Q}_{01}(\tilde{Q}_{14}\tilde{Q}_{46}+\tilde{Q}_{15}\tilde{Q}_{56}) \\ & -\tilde{Q}_{02}\tilde{Q}_{24}(-\tilde{Q}_{46}+\tilde{Q}_{15}\tilde{Q}_{46}\tilde{Q}_{51}-\tilde{Q}_{15}\tilde{Q}_{41}\tilde{Q}_{56}Q_{51}) \\ & +\tilde{Q}_{03}\tilde{Q}_{35}(\tilde{Q}_{56}+\tilde{Q}_{14}\tilde{Q}_{46}\tilde{Q}_{51}-\tilde{Q}_{14}\tilde{Q}_{41}\tilde{Q}_{56}Q_{41}) \end{aligned}$$

[7.8]

Now letting $s \to 0$ we get

$$D_1(0) = (1-p_{02}p_{20}-p_{03}p_{30})(1-p_{14}p_{14}-p_{15}p_{51})-p_{01}p_{10}-p_{02}p_{01}p_{24}p_{41}-p_{03}p_{01}p_{35}p_{51}$$

[7.9]

The mean time to system failure when the system starts from the state S_0 is given by

MTSF= $E(T) = -[\frac{d}{ds}\tilde{\pi}_0(s)]_{s=0} = \frac{D_1^{/}(0) - N_1^{/}(0)}{D_1(0)}$

[7.10]

To obtain the numerator of the above equation, we collect the coefficients of relevant of m_{ij} in $D_1^{/}(0) - N_1^{/}(0)$.

Coeff. of $(m_{01} = m_{02} = m_{03} = m_{07}) = 1 - p_{14}p_{14} - p_{15}p_{51}$

Coeff. of $(m_{10}) = p_{01} + p_{02}p_{24}p_{41} + p_{03}p_{35}p_{51}$

Coeff. of $(m_{20} = m_{24}) = p_{02}(1 - p_{14}p_{14} - p_{15}p_{51})$

Coeff. of $(m_{30} = m_{35}) = p_{03}(1 - p_{14}p_{14} - p_{15}p_{51})$

Coeff. of $(m_{41} = m_{46}) = p_{02}p_{24}(1 - p_{15}p_{51}) + p_{03}p_{35}p_{14}p_{51} + p_{01}p_{14}$

Coeff. of $(m_{51} = m_{56}) = p_{03}p_{35}(1 - p_{14}p_{41}) + p_{02}p_{24}p_{15}p_{41} + p_{01}p_{15}$

[7.11-7.16]

From equation [7.9]

MTSF= $E(T) = -[\frac{d}{ds}\tilde{\pi}_0(s)]_{s=0} = \frac{D_1^{/}(0) - N_1^{/}(0)}{D_1(0)}$

$$= \frac{\mu_0 L_0 + \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3 + \mu_4 L_4 + \mu_5 L_5}{(1 - p_{02}p_{20} - p_{03}p_{30})(1 - p_{14}p_{14} - p_{15}p_{51}) - p_{01}p_{10} - p_{02}p_{01}p_{24}p_{41} - p_{03}p_{01}p_{35}p_{51}}$$

[7.17]

Where

$$L_0 = 1 - p_{14}p_{14} - p_{15}p_{51}$$
$$L_1 = p_{01} + p_{02}p_{24}p_{41} + p_{03}p_{35}p_{51}$$
$$L_2 = p_{02}(1 - p_{14}p_{14} - p_{15}p_{51})$$
$$L_3 = p_{03}(1 - p_{14}p_{14} - p_{15}p_{51})$$
$$L_4 = p_{02}p_{24}(1 - p_{15}p_{51}) + p_{03}p_{35}p_{14}p_{51} + p_{01}p_{14}$$
$$L_5 = p_{03}p_{35}(1 - p_{14}p_{41}) + p_{02}p_{24}p_{15}p_{41} + p_{01}p_{15}$$

8. Availability Analysis:

Let $M_i(t)(i = 0,1,2,3,4,5)$ denote the probability that system is initially in regenerative state $S_i \in E$ is up at time t without passing through any other regenerative state or returning to itself through one or more non regenerative states .i.e. either it continues to remain in regenerative S_i or a non regenerative state including itself . By probabilistic arguments, we have the following recursive relations

$$M_0(t) = e^{-(\alpha+\beta+\gamma)t}\overline{A}(t), \quad M_1(t) = \overline{G}_1(t), \quad M_2(t) = e^{-\alpha t}\overline{G}_2(t), \quad M_3(t) = e^{-\alpha t}\overline{G}_3(t),$$
$$M_4(t) = e^{-\gamma t}\overline{G}_4(t), M_5(t) = e^{-\beta t}\overline{G}_5(t)$$

[8.1-8.6]

Recursive relations giving point wise availability $A_i(t)$ given as follows:

$$A_0(t) = M_0(t) + \sum_{i=1,2,3,7} q_{0i}(t) \boxed{c} A_i(t) \quad ; \qquad A_1(t) = M_1(t) + \sum_{i=0,4.5} q_{1i}(t) \boxed{c} A_i(t) ;$$

$$A_2(t) = M_2(t) + \sum_{i=0,4} q_{2i}(t) \boxed{c} A_i(t) \quad ; \qquad A_3(t) = M_3(t) + \sum_{i=0,5} q_{3i}(t) \boxed{c} A_i(t) \; ;$$

$$A_4(t) = M_4(t) + \sum_{i=1,6} q_{4i}(t) \boxed{c} A_i(t) \quad ; \qquad A_5(t) = M_5(t) + \sum_{i=1,6} q_{5i}(t) \boxed{c} A_i(t) \; ;$$

$$A_6(t) = q_{60}(t) \boxed{c} A_0(t) \quad ; \qquad A_7(t) = q_{70}(t) \boxed{c} A_0(t) ;$$

[8.7-8.14]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{8X8}[A_0^*, A_1^*, A_2^*, A_3^*, A_4^*, A_5^* A_6^*, A_7^*]' = [M_0^*, M_1^*, M_2^*, M_3^*, M_4^*, M_5^*, 0, 0]'$$

[8.15]

Where q_{8X8} =

$$\begin{vmatrix} 1 & -q_{01}^* & -q_{02}^* & -q_{03}^* & 0 & 0 & 0 & -q_{07}^* \\ -q_{10}^* & 1 & 0 & 0 & -q_{14}^* & -q_{15}^* & 0 & 0 \\ -q_{20}^* & 0 & 1 & 0 & -q_{24}^* & 0 & 0 & 0 \\ -q_{30}^* & 0 & 0 & 1 & 0 & -q_{35}^* & 0 & 0 \\ 0 & -q_{41}^* & 0 & 0 & 1 & 0 & -q_{46}^* & 0 \\ 0 & -q_{51}^* & 0 & 0 & 0 & 1 & -q_{56}^* & 0 \\ -q_{60}^* & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -q_{70}^* & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

Therefore

$$D_2(s) = \begin{vmatrix} 1 & -q_{01}^* & -q_{02}^* & -q_{03}^* & 0 & 0 & 0 & -q_{07}^* \\ -q_{10}^* & 1 & 0 & 0 & -q_{14}^* & -q_{15}^* & 0 & 0 \\ -q_{20}^* & 0 & 1 & 0 & -q_{24}^* & 0 & 0 & 0 \\ -q_{30}^* & 0 & 0 & 1 & 0 & -q_{35}^* & 0 & 0 \\ 0 & -q_{41}^* & 0 & 0 & 1 & 0 & -q_{46}^* & 0 \\ 0 & -q_{51}^* & 0 & 0 & 0 & 1 & -q_{56}^* & 0 \\ -q_{60}^* & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -q_{70}^* & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$D_2(s)=(1-q_{14}^{*}q_{41}^{*}-q_{15}^{*}q_{51}^{*})-q_{01}^{*}(q_{10}^{*}+q_{14}^{*}q_{46}^{*}q_{60}^{*}+q_{15}^{*}q_{56}^{*}q_{60}^{*})$$
$$-q_{02}^{*}(q_{10}^{*}q_{24}^{*}q_{41}^{*}+q_{20}^{*}+q_{24}^{*}q_{46}^{*}q_{60}^{*}-q_{14}^{*}q_{41}^{*}q_{20}^{*}-q_{15}^{*}q_{20}^{*}q_{51}^{*}$$
$$+q_{15}^{*}q_{24}^{*}q_{41}^{*}q_{56}^{*}q_{60}^{*}-q_{15}^{*}q_{24}^{*}q_{46}^{*}q_{51}^{*}q_{60}^{*})$$
$$-q_{03}^{*}(q_{10}^{*}q_{35}^{*}q_{51}^{*}+q_{30}^{*}+q_{35}^{*}q_{56}^{*}q_{60}^{*}-q_{14}^{*}q_{41}^{*}q_{30}^{*}-q_{15}^{*}q_{30}^{*}q_{51}^{*}$$
$$-q_{14}^{*}q_{35}^{*}q_{41}^{*}q_{56}^{*}q_{60}^{*}+q_{14}^{*}q_{35}^{*}q_{46}^{*}q_{51}^{*}q_{60}^{*})$$
$$-q_{07}^{*}q_{70}^{*}(1-q_{14}^{*}q_{41}^{*}-q_{15}^{*}q_{51}^{*})$$

If $s\to 0$ we get $D_2(0)=0$ which is true

[8.16]

$$\text{Now } N_2(s)=\begin{vmatrix} M_0^{*} & -q_{01}^{*} & -q_{02}^{*} & -q_{03}^{*} & 0 & 0 & 0 & -q_{07}^{*} \\ M_1^{*} & 1 & 0 & 0 & -q_{14}^{*} & -q_{15}^{*} & 0 & 0 \\ M_2^{*} & 0 & 1 & 0 & -q_{24}^{*} & 0 & 0 & 0 \\ M_3^{*} & 0 & 0 & 1 & 0 & -q_{35}^{*} & 0 & 0 \\ M_4^{*} & -q_{41}^{*} & 0 & 0 & 1 & 0 & -q_{46}^{*} & 0 \\ M_5^{*} & -q_{51}^{*} & 0 & 0 & 0 & 1 & -q_{56}^{*} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

Solving this Determinant, we get

$$N_2(s)=M_0^{*}(1-q_{14}^{*}q_{41}^{*}-q_{15}^{*}q_{51}^{*})+M_1^{*}(q_{01}^{*}+q_{02}^{*}q_{24}^{*}q_{41}^{*}+q_{03}^{*}q_{35}^{*}q_{51}^{*})$$
$$+M_2^{*}q_{02}^{*}(1-q_{14}^{*}q_{41}^{*}-q_{15}^{*}q_{51}^{*})+M_3^{*}q_{03}^{*}(1-q_{14}^{*}q_{41}^{*}-q_{15}^{*}q_{51}^{*})$$
$$+M_4^{*}(q_{01}^{*}q_{14}^{*}+q_{02}^{*}q_{24}^{*}-q_{02}^{*}q_{24}^{*}q_{15}^{*}q_{51}^{*}+q_{03}^{*}q_{35}^{*}q_{51}^{*}q_{14}^{*})$$
$$+M_5^{*}(q_{01}^{*}q_{15}^{*}+q_{03}^{*}q_{35}^{*}-q_{03}^{*}q_{35}^{*}q_{14}^{*}q_{41}^{*}+q_{02}^{*}q_{24}^{*}q_{51}^{*}q_{14}^{*})$$

[8.17]

If $s\to 0$ we get

$$N_2(0)=\mu_0(1-p_{14}p_{14}-p_{15}p_{51})+\mu_1(p_{01}+p_{02}p_{24}p_{41}+p_{03}p_{35}p_{51})$$
$$+\mu_2p_{02}(1-p_{14}p_{14}-p_{15}p_{51})+\mu_3p_{03}(1-p_{14}p_{14}-p_{15}p_{51})$$
$$+\mu_4[p_{02}p_{24}(1-p_{15}p_{51})+p_{03}p_{35}p_{14}p_{51}+p_{01}p_{14}]$$
$$+\mu_5[p_{03}p_{35}(1-p_{14}p_{41})+p_{02}p_{24}p_{15}p_{41}+p_{01}p_{15}]$$

[8.18]

To find the value of $D_2^{/}(0)$ we collect the coefficient m_{ij} in $D_2(s)$ we get

Coeff. of $(m_{01}=m_{02}=m_{03}=m_{07})=(1-p_{14}p_{14}-p_{15}p_{51})=L_0$

Coeff. of $(m_{10}=m_{14}=m_{15})=\ (p_{01}+p_{02}p_{24}p_{41}+p_{03}p_{35}p_{51})=L_1$

Coeff. of $(m_{20} = m_{24}) = p_{02}(1 - p_{14}p_{14} - p_{15}p_{51}) = L_2$

Coeff.of $(m_{30} = m_{35}) = p_{03}(1 - p_{14}p_{14} - p_{15}p_{51}) = L_3$

Coeff.f $(m_{41} = m_{46}) = [p_{02}p_{24}(1 - p_{15}p_{51}) + p_{03}p_{35}p_{14}p_{51} + p_{01}p_{14}] = L_4$

Coeff. of $(m_{51} = m_{56}) = [p_{03}p_{35}(1 - p_{14}p_{41}) + p_{02}p_{24}p_{15}p_{41} + p_{01}p_{15}] = L_5$

Coeff. of $m_{60} = p_{01}(1 - p_{10} - p_{14}p_{41} - p_{15}p_{51}) + p_{02}p_{24}(1 - p_{41} + p_{51}p_{41} - p_{15}p_{51})$
$+ p_{03}p_{35}(1 - p_{51} + p_{14}p_{51} - p_{14}p_{41}) = L_6$

Coeff. of $m_{70} = p_{07}(1 - p_{14}p_{14} - p_{15}p_{51}) = L_7$

[8.19-8.26]

Thus the solution for the steady-state availability is given by

$$A_0^*(\infty) = \underset{t\to\infty}{Lim}\, A_0^*(t) = \underset{S\to 0}{Lim}\, sA_0^*(s) = \frac{N_2(0)}{D_2^{/}(0)} = \frac{\mu_0 L_0 + \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3 + \mu_4 L_4 + \mu_5 L_5}{\sum_{i=0,1,2,3,4,5,6,7} \mu_i L_i}$$

[8.27]

9. BUSY PERIOD ANALYSIS:

(a) Let $W_i(t)$ $(i = 1,2,3,4,5)$ denote the probability that the repairman is busy initially with repair in regenerative state S_i and remain busy at epoch t without transiting to any other state or returning to itself through one or more regenerative states.

By probabilistic arguments we have

$W_1(t) = \overline{G}_1(t), W_2(t) = \overline{G}_2(t), W_3(t) = \overline{G}_3(t)$, $W_4(t) = \overline{G}_4(t)$ $W_5(t) = \overline{G}_5(t)$
[9.1-9.5]

Developing similar recursive relations as in availability, we have

$B_0(t) = \sum_{i=1,2,3,7} q_{0i}(t) \boxed{c}\ B_i(t)$; $B_1(t) = W_1(t) + \sum_{i=0,4,5} q_{1i}(t) \boxed{c}\ B_i(t)$;

$B_2(t) = W_2(t) + \sum_{i=0,4} q_{2i}(t) \boxed{c}\ B_i(t)$; $B_3(t) = W_3(t) + \sum_{i=0,5} q_{3i}(t) \boxed{c}\ B_i(t)$;

$B_4(t) = W_4(t) + \sum_{i=1,6} q_{4i}(t) \boxed{c}\ B_i(t)$; $B_5(t) = W_5(t) + \sum_{i=1,6} q_{5i}(t) \boxed{c}\ B_i(t)$;

$B_6(t) = q_{60}(t) \boxed{c}\ B_0(t)$; $B_7(t) = q_{70}(t) \boxed{c}\ B_0(t)$

[9.6-9.13]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$q_{8X8}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*, B_6^*, B_7^*]^{/} = [0, W_1^*, W_2^*, W_3^*, W_4^*, W_5^*, 0, 0]^{/}$
[9.14]

Where q_{8x8} is denoted by [8.15] and therefore $D_2^{/}(s)$ is obtained as in the expression of availability.

Now $N_3(s) = \begin{vmatrix} 0 & -q_{01}^* & -q_{02}^* & -q_{03}^* & 0 & 0 & 0 & -q_{07}^* \\ W_1^* & 1 & 0 & 0 & -q_{14}^* & -q_{15}^* & 0 & 0 \\ W_2^* & 0 & 1 & 0 & -q_{24}^* & 0 & 0 & 0 \\ W_3^* & 0 & 0 & 1 & 0 & -q_{35}^* & 0 & 0 \\ W_4^* & -q_{41}^* & 0 & 0 & 1 & 0 & -q_{46}^* & 0 \\ W_5^* & -q_{51}^* & 0 & 0 & 0 & 1 & -q_{56}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$

Solving this Determinant, In the long run, we get the value of this determinant after putting $s \to 0$ is

$$\begin{aligned} N_3(0) &= \mu_1(p_{01} + p_{02}p_{24}p_{41} + p_{03}p_{35}p_{51}) \\ &\quad + \mu_2 p_{02}(1 - p_{14}p_{14} - p_{15}p_{51}) + \mu_3 p_{03}(1 - p_{14}p_{14} - p_{15}p_{51}) \\ &\quad + \mu_4[p_{02}p_{24}(1 - p_{15}p_{51}) + p_{03}p_{35}p_{14}p_{51} + p_{01}p_{14}] \\ &\quad + \mu_5[p_{03}p_{35}(1 - p_{14}p_{41}) + p_{02}p_{24}p_{15}p_{41} + p_{01}p_{15}] \\ &= \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3 + +\mu_4 L_4 + \mu_5 L_5 = \sum_{i=1,2,3,4,5} \mu_i L_i \end{aligned}$$

[9.15]

Thus the fraction of time for which the repairman is busy with repair of the failed unit is given by:

$$B_0^{1^*}(\infty) = \underset{t\to\infty}{Lim}\, B_0^{1^*}(t) = \underset{s\to 0}{Lim}\, B_0^{1^*}(s) = \frac{N_3(0)}{D_2^{/}(0)} = \frac{\sum_{i=1,2,3,4,5} \mu_i L_i}{\sum_{i=0,1,2,3,4,5,6,7} \mu_i L_i}$$

[9.16]

(b)Busy period of the Repairman in preventive maintenance in time (0, t], By probabilistic arguments we have

$$W_7(t) = \overline{B}(t)$$

[9.17]

Similarly developing similar recursive relations as in 9(a), we have

$B_0(t) = \sum_{i=1,2,3,7} q_{0i}(t) \boxed{c} B_i(t)$; $B_1(t) = \sum_{i=0,4,5} q_{1i}(t) \boxed{c} B_i(t)$;

$B_2(t) = \sum_{i=0,4} q_{2i}(t) \boxed{c} B_i(t)$; $B_3(t) = \sum_{i=0,5} q_{3i}(t) \boxed{c} B_i(t)$;

$B_4(t) = \sum_{i=1,6} q_{4i}(t) \boxed{c} B_i(t)$; $B_5(t) = \sum_{i=1,6} q_{5i}(t) \boxed{c} B_i(t)$;

$B_6(t) = q_{60}(t) \boxed{c} B_0(t)$; $B_7(t) = W_7(t) + q_{70}(t) \boxed{c} B_0(t)$;

[9.18-9.25]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

ISSN 2347-1921

$$q_{8X8}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*, B_6^*, B_7^*]' = [0,0,0,0,0,0,0,W_7^*]'$$

[9.26]

Where q_{8x8} is denoted by [8.15] and therefore $D_2'(s)$ is obtained as in the expression of availability.

Now $N_4(s) = \begin{vmatrix} 0 & -q_{01}^* & -q_{02}^* & -q_{03}^* & 0 & 0 & 0 & -q_{07}^* \\ 0 & 1 & 0 & 0 & -q_{14}^* & -q_{15}^* & 0 & 0 \\ 0 & 0 & 1 & 0 & -q_{24}^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -q_{35}^* & 0 & 0 \\ 0 & -q_{41}^* & 0 & 0 & 1 & 0 & -q_{46}^* & 0 \\ 0 & -q_{51}^* & 0 & 0 & 0 & 1 & -q_{56}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ W_7^* & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$

Solving this Determinant, In the long run, we get the value of this determinant after putting $s \to 0$ is

$$N_4(0) = \mu_7 p_{07}(1 - p_{14}p_{14} - p_{15}p_{51}) = \mu_7 L_7$$

[9.27]

Thus the fraction of time for which the system is under preventive maintenance is given by:

$$B_0^{2^*}(\infty) = \lim_{t\to\infty} B_0^{2^*}(t) = \lim_{s\to 0} sB_0^{2^*}(s) = \frac{N_4(0)}{D_2'(0)} = \frac{\mu_7 L_7}{\sum_{i=0,1,2,3,4,5,6,7} \mu_i L_i}$$

[9.28]

(c)Busy period of the Repairman in Shut Down repair in time (0, t], By probabilistic arguments we have

$$W_6(t) = \overline{G}_6(t)$$

[9.29]

Similarly developing similar recursive relations as in 9(b), we have

$B_0(t) = \sum_{i=1,2,3,7} q_{0i}(t) \boxed{c} B_i(t)$; $B_1(t) = \sum_{i=0,4,5} q_{1i}(t) \boxed{c} B_i(t)$;

$B_2(t) = \sum_{i=0,4} q_{2i}(t) \boxed{c} B_i(t)$; $B_3(t) = \sum_{i=0,5} q_{3i}(t) \boxed{c} B_i(t)$;

$B_4(t) = \sum_{i=1,6} q_{4i}(t) \boxed{c} B_i(t)$; $B_5(t) = \sum_{i=1,6} q_{5i}(t) \boxed{c} B_i(t)$;

$B_6(t) = W_6(t) + q_{60}(t) \boxed{c} B_0(t)$; $B_7(t) = q_{70}(t) \boxed{c} B_0(t)$;

[9.30-9.37]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{8X8}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*, B_6^*, B_7^*]' = [0,0,0,0,0,0,W_6^*,0]'$$

Where q_{9x9} is denoted by [8.15] and therefore $D_2'(s)$ is obtained as in the expression of availability.

Now $N_5(s) = \begin{vmatrix} 0 & -q_{01}^{\,*} & -q_{02}^{\,*} & -q_{03}^{\,*} & 0 & 0 & 0 & -q_{07}^{\,*} \\ 0 & 1 & 0 & 0 & -q_{14}^{\,*} & -q_{15}^{\,*} & 0 & 0 \\ 0 & 0 & 1 & 0 & -q_{24}^{\,*} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -q_{35}^{\,*} & 0 & 0 \\ 0 & -q_{41}^{\,*} & 0 & 0 & 1 & 0 & -q_{46}^{\,*} & 0 \\ 0 & -q_{51}^{\,*} & 0 & 0 & 0 & 1 & -q_{56}^{\,*} & 0 \\ W_6^{\,*} & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$

In the long run, we get the value of this determinant after putting $s \to 0$ is

$$N_5(0) = \mu_6[p_{01}(1 - p_{10} - p_{14}p_{41} - p_{15}p_{51}) + p_{02}p_{24}(1 - p_{41} + p_{51}p_{41} - p_{15}p_{51}) + p_{03}p_{35}(1 - p_{51} + p_{14}p_{51} - p_{14}p_{41})] = \mu_6 L_6$$

[9.29]

Thus the fraction of time for which the system is under shut down is given by:

$$B_0^{3^*}(\infty) = \underset{t\to\infty}{Lim}\, B_0^{3^*}(t) = \underset{s\to 0}{Lim}\, sB_0^{3^*}(s) = \frac{N_5(0)}{D_2^{/}(0)} = \frac{\mu_6 L_6}{\sum\limits_{i=0,1,2,3,4,5,6,7} \mu_i L_i}$$

[9.30]

10. Particular cases:

When all repair time distributions are n-phase Erlangian distributions i.e.

Density function $g_i(t) = \dfrac{nr_i(nr_i t)^{n-1} e^{-nr_i t}}{n-1!}$ And Survival function $\overline{G}_j(t) = \sum\limits_{j=0}^{n-1} \dfrac{(nr_i t)^j e^{-nr_i t}}{j!}$

[10.1-10.2]

And other distributions are negative exponential

$$a(t) = \theta e^{-\theta t}, b(t) = \eta e^{-\eta t}, \overline{A}(t) = e^{-\theta t}, \overline{B}(t) = e^{-\eta t}$$

[10.3-10.6]

For n=1 $g_i(t) = r_i e^{-r_i t}$, $\overline{G}_i(t) = e^{-r_i t}$ If i=1, 2, 3, 4

$$g_1(t) = r_1 e^{-r_1 t}, g_2(t) = r_2 e^{-r_2 t}, g_3(t) = r_3 e^{-r_3 t}, g_4(t) = r_4 e^{-r_4 t}$$

$$\overline{G}_1(t) = e^{-r_1 t}, \overline{G}_2(t) = e^{-r_2 t}, \overline{G}_3(t) = e^{-r_3 t}, \overline{G}_4(t) = e^{-r_4 t}$$

[10.7-10.14]

Also

$$p_{01} = \frac{\alpha}{x_1 + \theta}, p_{02} = \frac{\beta}{x_1 + \theta}, p_{03} = \frac{\gamma}{x_1 + \theta}, p_{07} = \frac{\theta}{x_1 + \theta},$$

$$p_{10} = \frac{r_1}{\beta + \gamma + r_1}, p_{14} = \frac{\beta}{\beta + \gamma + r_1}, p_{15} = \frac{\gamma}{\beta + \gamma + r_1},$$

$$p_{20}=\frac{r_2}{\alpha+r_2}, p_{24}=\frac{\alpha}{\alpha+r_2},\ p_{30}=\frac{r_3}{\alpha+r_3}, p_{35}=\frac{\alpha}{\alpha+r_3},\ p_{41}=\frac{r_2}{\gamma+r_2}, p_{46}=\frac{\gamma}{\gamma+r_2},$$

$$p_{51}=\frac{r_3}{\beta+r_3}, p_{46}=\frac{\beta}{\beta+r_3},$$

$$\mu_0=\frac{1}{x_1+\theta}, \mu_1=\frac{1}{\beta+\gamma+r_1}, \mu_2=\frac{1}{\alpha+r_2}, \mu_3=\frac{1}{\alpha+r_3}, \mu_4=\frac{1}{\gamma+r_2}, \mu_5=\frac{1}{\beta+r_3},\ \mu_6=\frac{1}{r_4}, \mu_7=\frac{1}{\eta}$$

where $x_1 = \alpha+\beta+\gamma$
[10.15-10.37]

MTSF= $=\dfrac{\mu_0L_0+\mu_1L_1+\mu_2L_2+\mu_3L_3+\mu_4L_4+\mu_5L_5}{(1-p_{02}p_{20}-p_{03}p_{30})(1-p_{14}p_{14}-p_{15}p_{51})-p_{01}p_{10}-p_{02}p_{01}p_{24}p_{41}-p_{03}p_{01}p_{35}p_{51}}$,

$$A_0(\infty)=\frac{\mu_0L_0+\mu_1L_1+\mu_2L_2+\mu_3L_3+\mu_4L_4+\mu_5L_5}{\sum_{i=0,1,2,3,4,5,6,7}\mu_iL_i},$$

$$B_0^{1^*}(\infty)=\frac{\sum_{i=1,2,3,4,5}\mu_iL_i}{\sum_{i=0,1,2,3,4,5,6,7}\mu_iL_i}, B_0^{2^*}(\infty)=\frac{\mu_7L_7}{\sum_{i=0,1,2,3,4,5,6,7}\mu_iL_i}, B_0^{3^*}(\infty)=\frac{\mu_6L_6}{\sum_{i=0,1,2,3,4,5,6,7}\mu_iL_i}$$

[10.38-10.42]

Where $L_0=(1-p_{14}p_{14}-p_{15}p_{51})$; $L_1=p_{01}+p_{02}p_{24}p_{41}+p_{03}p_{35}p_{51}$; $L_2=p_{02}(1-p_{14}p_{14}-p_{15}p_{51})$;

$L_3=p_{03}(1-p_{14}p_{14}-p_{15}p_{51})$; $L_4=[p_{02}p_{24}(1-p_{15}p_{51})+p_{03}p_{35}p_{14}p_{51}+p_{01}p_{14}]$;

$L_5=[p_{03}p_{35}(1-p_{14}p_{41})+p_{02}p_{24}p_{15}p_{41}+p_{01}p_{15}]$;

$$L_6=p_{01}(1-p_{10}-p_{14}p_{41}-p_{15}p_{51})+p_{02}p_{24}(1-p_{41}+p_{51}p_{41}-p_{15}p_{51}) + p_{03}p_{35}(1-p_{51}+p_{14}p_{51}-p_{14}p_{41}); L_7=p_{07}(1-p_{14}p_{14}-p_{15}p_{51})$$

[10.43-10.50]

11. Profit Analysis:-

The profit analysis of the system can be carried out by considering the expected busy period of the repairman in repair of the unit in (0,t].

Therefore, G(t) = Expected total revenue earned by the system in (0,t] -Expected repair cost of the failed units -Expected repair cost of the repairman in preventive maintenance -Expected repair cost of the Repairman in shut down

$$=C_1\mu_{up}(t)-C_2\mu_{b1}(t)-C_3\mu_{b2}(t)-C_4\mu_{b3}(t)$$

$$=C_1A_0-C_2B^1{}_0-C_3B^2{}_0-C_4B^3{}_0 \quad [11.1]$$

Where $\mu_{up}(t)=\int_0^t A_0(t)\,dt$; $\mu_{b1}(t)=\int_0^t B_0^1(t)\,dt$; $\mu_{b2}(t)=\int_0^t B_0^2(t)\,dt$; $\mu_{b3}(t)=\int_0^t B_0{}^3(t)dt$ [11.2-11.5]

C_1 is the revenue per unit time and C_2, C_3, C_4 are the cost per unit time for which the system is under simple repair, preventive maintenance and shut down repair respectively.

12. References:-

1. Barlow R.E., (1984), “ Computing k-out-of-n system reliability.”, IEEE Transactions on Reliability Vol.R-33, pp.322–323.
2. Li, W., Zuo, M.J., [2008], “ Reliability evaluation of multi-state weighted k-out-of-n systems.”, Reliability Engineering and System Safety , Vol.93, pp.160–167.
3. Gopalan, M.N.[1975]. Probabilistic analysis of a single server n-unit system with (n-1) warm standbys. Operation Research, Vol. 23, pp 591-595.
4. Arora, J.R. [1976]. Reliability of a two unit standby redundant system with constrained repair time. IEEE Transactions on Reliability, Vol. 25(3), pp 203-205.
5. Osaki, [1980]. A two unit parallel redundant system with bivariate exponential life. Microelectron & Reliability Vol. 20, pp 521-523.
6. Rander, M.C., Kumar Ashok and Suresh K [1994]. Cost analysis of two dissimilar cold standby system with preventive maintenance and replacement of standby unit. Microelectron & Reliability, Vol. 34, pp 171-174.
7. Chandrasekar, P. and Nataraja R. [1994]. Confidence limits for a steady state availability of a two units standby system. Microelectron & Reliability, Vol. 34(7), pp 1249-1251.
8. Gupta R ,Tyagi P.K. and Goel L.R.[1995]. A cold standby system with arrival time of server and correlated failures and repairs. Microelectron & Reliability, Vol. 35(4), pp 739-742.
9. Murari, K and Muruthachalan, C[1981]. Two unit system with periods of working and transactions on Reliability, R-3091,pp 108-116.
10. Pathak, V.K. [2013]. Profit analysis of a system having parallel one main unit and two supporting units. International Journal of Engineering and Computer Science. Vol. 2, pp 1-13.
11. Pathak V.K. [2014]. Comparative study of reliability parameter of a system under different types of distribution functions. African Journal of Mathematics and Computer Science Research, Vol. 7(4) pp 47-54.

ISSN 2347-1921

Figure 1: state transition diagram