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Chaotic behavior of a coupled system of the Riccati map

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ABSTRACT

In this paper, We present the equivalent discrete system of coupled Riccati map. We study some the dynamic behavior such as (fixed points and their asymptotic stability, thelyapunov exponents, chaos and bifurcation) of the system. Numerical simulation is presented to ensure the analytical results.

KEY WORDS: Fixed point; stability; Lyapunov exponents; chaos, bifurcation; chaotic at-tractor.

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1- Introduction

Consider the coupled system of the discrete Riccati map

Our aim of this paper is to study the dynamic properties of the discrete dynamical system (1) with positive parameterρ.

2- Stability analyses

Here we study the stabilityof the coupled system (1):

2.1 Equilibrium points

A point x0 is anequilibrium points of the map f if f(x0) = x0 [4].

The fixed points of the system (1) is given by the solution of the equation

Solve these equations we obtain.

$x -$

$x -$

$x -$

Then we have

i.e.

Substituting in equation (1) we obtain

i.e.

which has the two solutions

Thus there are four fixed points:

$(i) - \quad \boldsymbol{fix}_1 = (\frac{1-\sqrt{4\rho-3}}{2\rho}, \frac{1+\sqrt{4\rho-3}}{2\rho}),$

$(ii) - \quad \boldsymbol{fix}_2 = (\frac{1+\sqrt{4\rho-3}}{2\rho}, \frac{1-\sqrt{4\rho-3}}{2\rho}),$

$(iii) - \quad \boldsymbol{fix}_3 = (\frac{1-\sqrt{4\rho-3}}{2\rho}, \frac{1+\sqrt{4\rho-3}}{2\rho}),$

$(iiii) - \quad \boldsymbol{fix}_4 = (\frac{1+\sqrt{4\rho-3}}{2\rho}, \frac{1-\sqrt{4\rho-3}}{2\rho}).$

To study the stability of these fixed points we take into account the proposition

Proposition: [1] Let $(\bar{x}, \bar{y})$ be a stationary state of the system

$x_{n+1} = f(x_n, y_n) \ \ , y_{n+1} = g(x_n, y_n)$

Let

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

be the Jacobian at the point $(\bar{x}, \bar{y})$ with eigenvalues λ_1, λ_2 Then:

$(i)|\lambda_{1,2}| < 1 \Rightarrow (\bar{x}, \bar{y})$ is locally stable.

$(ii)|\lambda_j| > 1$for one j$\in \{1,2\}$ respectively $\Rightarrow (\bar{x}, \bar{y})$ is unstable.

By considering a Jacobian matrix for one of these fixed points and calculating their eigen- values, we can investigate the stability of each fixed point based on the roots of the system Characteristic equation. The Jacobian matrix is given by

The jacobian matrix for system (1) at any point (x; y) takes the form ,

[1] – The jacobian matrix at the fixed point (fix1):

The eigenvalues of this matrix is given by

Thus the characteristic equation reads:

which have the solution

So the first fixed points (fix1) is asymptotical Stable if $|\lambda_i| < 1$,i = 1,2

i.e,

which implies that

This implies that the fixed point (fix1) is asymptotical Stable if$\rho < \frac{3}{4}$.

.[2] – For the second fixed point (fix2) we have

Thus the characteristic equation reads

which have the solution

This implies that the fixed point (fix2) is asymptotical Stable if$\rho < \frac{3}{4}$.

$[3] -$ For the third fixed point (fix3) we have

Thus the characteristic equation reads

which have the solution

This implies that the fixed point (fix3) is asymptotical Stable if $\rho < \frac{3}{4}$.

$[4] -$ For the fourth fixed point (fix4) we have

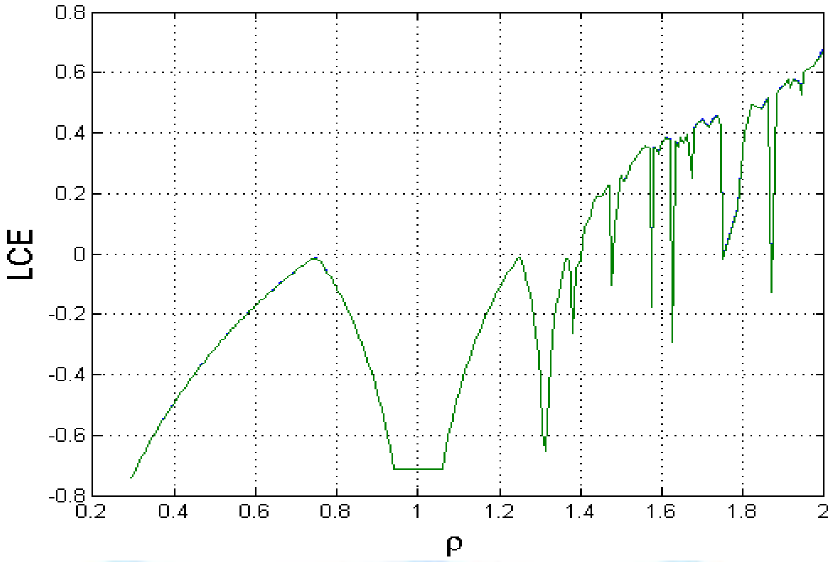
Thus the characteristic equation reads

which have the solution

This implies that the fixed point (fix4) is asymptotical Stable if $\rho < \frac{3}{4}$.

All fixed pointsfix1, fix2, fix3 and fix4are asymptotically stable if $\rho < \frac{3}{4}$.

2.2- TheLyapunov exponent



(**Figure 1**) Lyapunov exponent for the coupled system diag

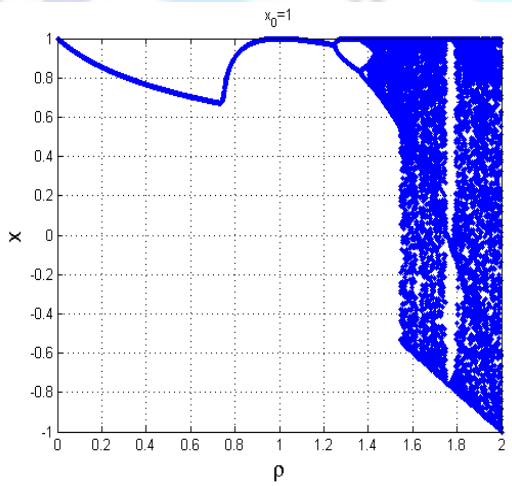
Since the Lyapunov exponent is a good indicator for existence of chaos[3], we compute theLyapunov Characteristic Exponents (LCEs) via the Householder QR Based Methods described in . LCEs play a key role in the study of nonlinear dynamical systems and they are a measure of the sensitivity of the solutions of a given dynamical system to small

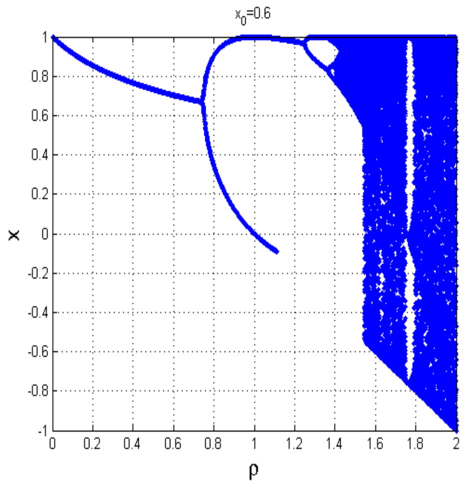
changes in the initial conditions. One feature of chaos is the sensitive dependence on initial conditions; for a chaotic dynamical system at least one LCE must be positive. Since for non-chaotic systems all LCEs are non-positive, the presence of a positive LCE has often been used to help determine if a system is chaotic or not. Figure (1) shows the LCEs for the coupled system (1) in the case $x_n \neq y_n$forn = 1,2…..with$\rho$is positive constant . we find that LCE1=0.6750 and LCE2=0.6664.

3 Numerical experiment

3.1 Bifurcation and chaos

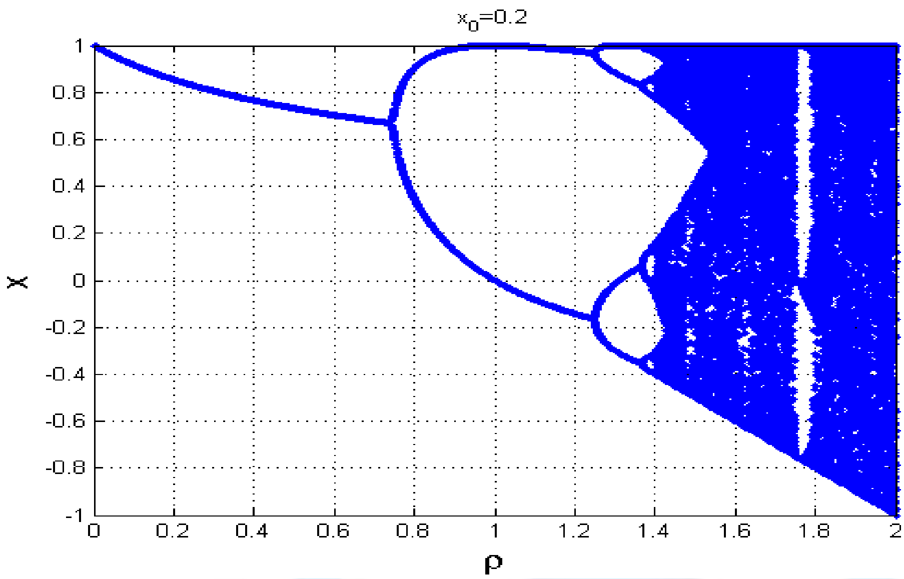
In this section we show by numerical experiments that the dynamical behavior of the dy- namical system (1) is sensitive to the change of initial conditions (x_0, y_0)[5].





(Figure 2) **(Figure 3)**

Bifurcation diagramwhen x_0= 1Bifurcation diagramwhen x_0= 0.6



(Figure4)

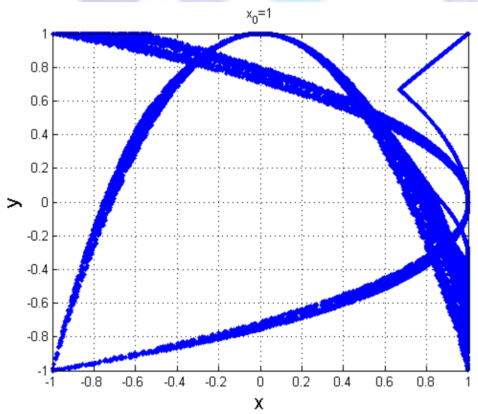
Bifurcation diagramwhen $x_0 = 0.2$

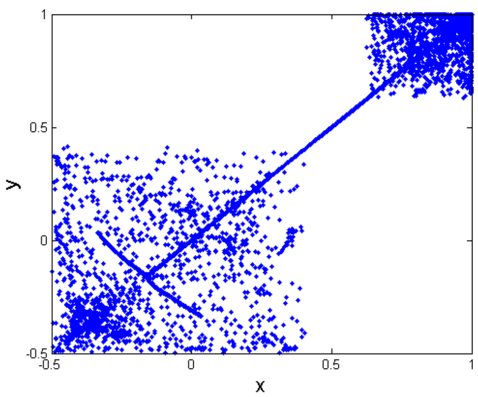
We see clearly in Figure (4) the bifurcation from a stable fixed point to a stable orbit of period 2 at$\rho \approx 0.7$, and then the bifurcation from period two to period four at ρbetween 1.2 and 1.3. The further period doubling occur at decreasing increments in ρ, and the orbit becomes chaotic for $\rho \approx 1.4$. Note the intriguing window just beyond 1.8. The same can

be said to Figures (2)-(3) .

3.2 Chaotic attractor

In this section, we show the chaotic attractor of system (1)[2], Figure (5) show the chaotic attractor of system (1) where$x_0 = 1$, while Figure (6) show the chaotic attractor of system (1) where$x_0 =$ any other values.





(Figure 5)

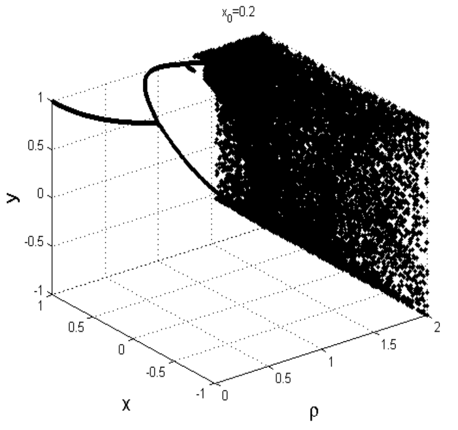
Attractor of the coupled system(1) when x0 = 1

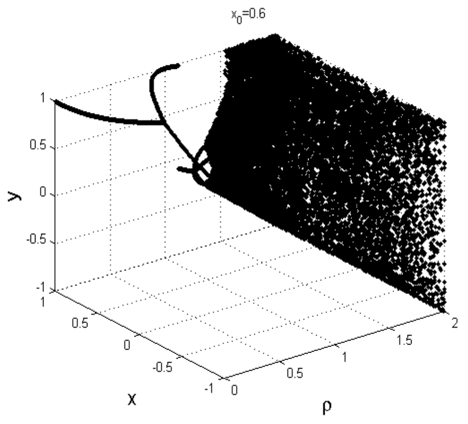
(Figure 6)

Attractor of the coupled system(1)when $x_0 = 1. x_0 =$any other values.

In this section, we show bifurcation diagram of the coupled system (1) in 3 D.

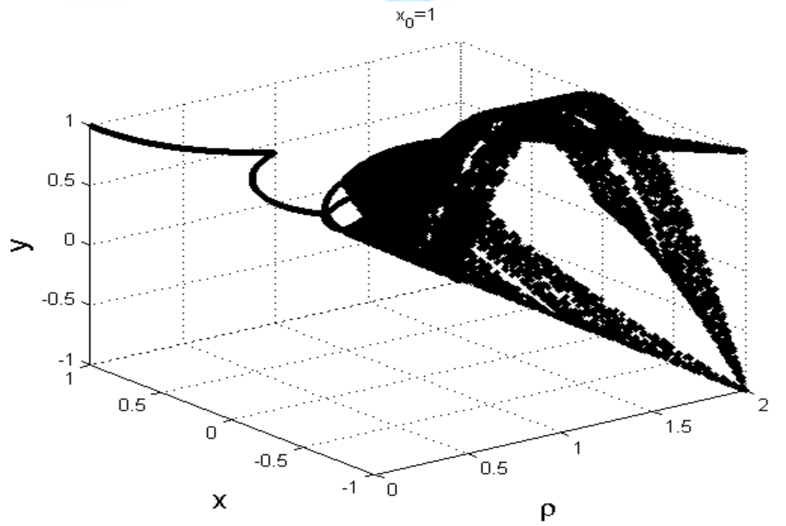
3.3 Bifurcation in 3D





(Figure 7) **(Figure8)**

Bifurcation diagram in 3Dwhen $x_0 = 0.2$ Bifurcation diagram in 3Dwhen $x_0 = 0.6$



(Figure9)

Bifurcation diagram in 3D of the coupled system(1) when $x_0 = 1$

4- Conclusion

In this paper we studied some dynamic properties of the coupled systems (1). Lyapunov exponent was numerically investigated to indicated chaos. All fixed points of the coupled system (1) are asymptotically stable if $\rho < \frac{3}{4}$. We have seen that route to chaos for the system is via period-doubling bifurcation.

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