ISSN 2347-1921

APPROXIMATION OF THEQUADRATIC DOUBLE CENTRALIZERS AND QUADRATIC MULTIPLIERS ON NON-ARCHIMEDEAN BANACH ALGEBRAS

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ABSTRACT

In this paper, we establish stability of quadratic double centralizers and quadratic multipliers on non-ArchimedeanBanach algebras. We also prove the superstability of quadraticdouble centralizers on non-ArchimedeanBanach algebras which are weakly commutative and weakly without order, and of quadratic multipliers on non-ArchimedeanBanach algebras which are weakly withoutorder.

Mathematics subject classification: 39B82, 47B47, 39B52, 46H25

Keywords: quadratic functional equation; non-ArchimedeanBanach algebras; multiplier; double centralizer; Stability; Superstability

Council for Innovative Research

Peer Review Research Publishing System

Journal: JOURNAL OF ADVANCES IN MATHEMATICS

Vol . 9, No. 6

www.cirjam.com , editorjam@gmail.com

1. INTRODUCTION

The stability problem of functional equations originated from a question of Ulam[11] in 1940, concerning the stability of group homomorphisms. Let (G_1, $*$) be a group and let (G_2, $*$) be a metric group with the metric d(.,.). Given $\varepsilon > 0$, does there exist a $\delta > 0$, such that if a mapping h:$G_1 \to G_2$satisfies the inequality $d(h(x, y),\ h(x) * h(y) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism H : $G_1 \to G_2$with $d(h(x),\ H(x)) < \varepsilon$ for all $x \in G_1$? In the other words, Under what condition does there exist a homomorphism near an approximate homomorphism? The concept of stability for functional equation arises when we replace the functional equation by an inequality which acts as a perturbation of the equation. In 1941, D.H. Hyers [6] gave a first affirmative answer to the question of Ulam for Banach space. Let $f : E \to E'$ be a mapping between Banach spaces such that

$$\left\| f(x+y) - f(x) - f(y) \right\| \le \delta$$

for all $x, y \in E$, and for some $\delta > 0$. Then there exists a unique additive mapping $T : E \to E'$ such that

$$\left\| f(x) - T(x) \right\| \le \delta$$

for all $x \in E$. Moreover, if $f(tx)$ is continuous in t for each fixed $x \in E$, then T is linear. In 1950, T. Aoki [1] was the second author to treat this problem for additive mapping. Finally in 1978, Th. M. Rassias[8] proved the following Theorem:
Theorem (Th. M. Rassias). Let $f : E \to E'$ be a mapping from a norm vector space E into a Banach space E' subject to the inequality

$$\left\| f(x+y) - f(x) - f(y) \right\| \le \varepsilon(\|x\|^P + \|y\|^P)$$

for all $x, y \in E$, where ε and p are constants with $\varepsilon > 0$ and $p < 1$. Then there exists a unique additive mapping $T : E \to E'$ such that

$$\left\| f(x) - T(x) \right\| \le \frac{2\varepsilon}{2 - 2^P} \|x\|^P$$

for all $x \in E$. *Also, if* the function $t \to f(tx)$ from R into E' *is* continuous for each fixed x *in* E, then T is linear.
This stability phenomenon *of* this kind is called the Hyers-Ulam-Rassias stability. In *1991, Z.* Gajda*[3]*answered the question for the case $p < 1$, which was raised by Rassias. In *1994,* a generalization *of* the Rassias' theorem was obtained by Gavruta as follows [4].
The functional equation is called stable *if* any function satisfying that functional equation "approximately" is near to a true solution of functional equation. We say that a functional equation is superstable if every approximately solution is an exact solution of it.
Suppose that A *is* a Banach algebra. Recall that $A_l(A) := \{a \in A : aA = \{0\}\}$ is the left annihilator ideal and $A_r(A) := \{a \in A : Aa = \{0\}\}$ *is* the right annihilator ideal on A. *A* Banach algebra A *is* said to be strongly without order if $A_l(A) = A_r(A) = \{0\}$. We say that a Banach algebra A *is* quartic without orderif $\left\{r \in A \left\{ra^2; a \in A\right\} = \{0\}\right\} = \{0\} = \left\{r \in A \left\{a^2 r; a \in A\right\} = \{0\}\right\}$. It is not hard to see that if A *is* weakly without order then A *is* strongly without order.
A linear mapping $L : A \to A$ *is* said to be left centralizer on A if $L(ab) = L(a)b$ for all $a, b \in A$. Similarly, a linear mapping $R : A \to A$ that $R(ab) = aR(b)$ for all $a, b \in A$ *is* called right centralized on A. *A* double centralizer on A *is* a pair (L, R), where L *is* a left centralizer, R is a right centralizer and $aL(b) = R(a)b$ for all $a, b \in A$. For example, (L_c, R_c) *is* a double centralizer, where $L_c(a) := ca$ and $R_c(a) := ac$. The set $D(A)$ of all double centralizers equipped with the multiplication $(L_1, R_1).(L_2, R_2) = (L_1 L_2).(R_1 R_2)$ *is* an algebra. The notion of double centralizer was introduced by Hochschild*[5]*and by Johnson *[7].* Johnson *[7]* proved that if A *is* an algebra satisfying $A_l(A) = A_r(A) = \{0\}$, and L, R are mappings on A fulfilling $aL(b) = R(a)b$, *(* $a, b \in A$*),* then (L, R) *is* a double centralizer. We can show that if $A^2 = A$ or $A_l(A) \cap A_r(A) = \{0\}$, then $L = R$ if and only if L and R are both left and right centralizer.
In particular, one of the important functional equations is the following functionalequation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \qquad (1.1)$$

which is called a quadratic functional equation. The function $f(x) = bx^2$ is a solution of this functional equation. Every solution of functional equation (1.1) is said to be a quadratic mapping. It is well known that a mapping f between real vector spaces is quadratic if and only if there exists a unique symmetric biadditive mapping B is
given by

$$B(x, y) = \frac{1}{4}(f(x+y) - f(x-y))$$

The stability of quadratic functional equation (1.1) was proved by skof [10] for mapping $f : E_1 \to E_2$ where E_1 is a normed space and E_2 is a Banach space. Cholewa[3] noticed that the theorem of Skof is still true if the relevant domain E_1 is replaced by an Abelian group.

A Banach algebra A *is* said to be *weakly commutative* if $(ab)^2 = a^2b^2$ for all $a, b \in A$. We can show that there is a Banach algebra weakly commutative that is not commutative (see Example *2.4* of the present paper).

Let K be a field. A non- Archimedean absolute value on K is a function $|.| : K \to R$ such that for any $a, b \in K$ we have

(i) $|a| \geq 0$ and equality holds if and only if $a = 0$,

(ii) $|ab| = |a||b|$,

(iii) $|a+b| \leq \max\{|a|, |b|\}$.

The condition (iii) is called the strict triangle inequality. By (ii), we have $|1| = |-1| = 1$. Thus, by induction, it follows from (iii) that $|n| \leq 1$ for each integern. We always assume in addition that $|.|$ is non trivial, i.e., that there is an $a_0 \in k$ such that $|a_0| \notin \{0,1\}$.

Let X be a linear space over a scalar field K with a non- Archimedean non- trivial Valuation $|.|$. A function $\|.\| : X \to R$ is a non- Archimedean norm (valuation) if it satisfies the following conditions:

(NA1) $\|x\| = 0$ if and only if $x = 0$;

(NA2) $\|rx\| = |r|\|x\|$ for all $r \in K$ and $x \in X$;

(NA3) the strong triangle inequality (ultrametrie); namely,

$$\|x+y\| \leq \max\{\|x\|, \|y\|\} \qquad (x, y \in X).$$

Then $(X, \|.\|)$ is called a non-Archimedean spase.

It follows from (NA3) that

$$\|x_m + x_l\| \leq \max\{\|x_{j+1} - x_j\| : l \leq j \leq m-1\} \qquad (m > l),$$

Therefore a sequence $\{x_m\}$ is Cauchy in X if and only if $\{x_{m+1}-x_m\}$ converges to zero in non-Archimedean space. By a complete non-Archimedean space we mean on in which every Cauchy sequence is convergent. A non-Archimedean Banach algebra is a complete non-Archimedean algebra A wich satisfies $\|ab\| \leq \|a\|\|b\|$ for all $a, b \in A$. For more detailed definitions of non-Archimedean Banach algebra, we can refer to [9].

2. MAIN RESULTS

In this section, let A be a non-ArchimedeanBanachalgebra. We establish the stability of quadraticdouble centralizers.

Definition 2.1. A mapping $L : A \to A$ is a quadratic left centralizer if L satisfies the following properties:

1) L is a quadratic mapping,

2) L is a quadratic homogeneous, that is, $L(\lambda a) = |\lambda|^2 L(a)$ for all $a \in A$ and $\lambda \in C$,

3) $L(ab) = L(a)b^2$ for all $a, b \in A$.

Definition 2.2. A mapping $R : A \to A$ is a quadratic right centralizer if R satisfies the following properties:

1) R is a quadratic mapping,

2) R is quadratic homogeneous, *that is,* $R(\lambda a) = |\lambda|^2 R(a)$ *for all* $a \in A$ *and* $\lambda \in C$,

3) $R(ab) = a^2R(b)$ for all $a, b \in A$.

Definition 2.3. A quadratic *double centralizer of an algebra* A is *a pair* (L, R), *where* L *is a* quadratic *left centralizer,* R *is a* quadratic *right centralizer and* $a^2L(b) = R(a)b^2$ *for all* $a, b \in A$.

The following example introduces a quadratic double centralizer.

Example2.4. Let $(A,\|.\|)$ be a unitalnon-ArchimedeanBanachalgebra. Let $B = A\times A\times A$ .We define $|||a||| = \|a_1\| + \|a_2\| + \|a_3\|$ for all $a = (a_1, a_2, a_3)$ in B. It is not hard to see that $(B,|||.|||)$ is a banach space. for arbitrarily elements $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ in B, we define $ab = (0, a_1b_3, 0)$. since A is a non-ArchimedeanBanach algebra, we conclude that B is a non-ArchimedeanBanach algebra.

It is easy to see that $B^3 = \{abc{:}a,b,c\in B\} = \{0\}$ But $B^2 = \{ab{:}a,b\in B\}$ is not zero. Now we consider the mapping $T : B \to B$ defined by

$$T(a) = a^2 (a \in B).$$

Then T *is a* quadratic *mapping and* quadratichomogeneous. *Since* $B^3 = \{0\}$, *we get*

$$T(ab) = (ab)^2 = 0 = a^2b^2 = T(a)b^2 = a^2T(b)$$

and

$$a^2T(b) = a^2b^2 = 0 = T(a)b^2$$

For all $a, b \in B$. Hence (T,T) is a quadratic double centralizer of B.

In the above example, B *is* a weakly commutative algebra, but it is not commutative.

Theorem 2.5. *Suppose that* $s \in \{-1,1\}$ *and that* $f : A \to A$ *is a mapping with* $f(0) = 0$ *for which there exist a mapping* $g : A \to A$ *with* $g(0) = 0$ *and functions* $\varphi_j : A\times A\times A\times A \to [0,\infty), \psi_i : A\times A \to [0,\infty)$ $(1 \le j \le 2, 1 \le i \le 3)$ *such that*

$$\tilde{\varphi}_j(a,b,c,d) := \sum_{k=0}^{\infty} \frac{\varphi_j(2^{sk}a, 2^{sk}b, 2^{sk}c, 2^{sk}d)}{|4|^{sk}} < \infty \qquad (1 \le j \le 2), \tag{2.1}$$

$$\lim_{n\to\infty} \frac{\psi_i(2^{sn}a,b)}{|4|^{sn}} = 0 = \lim_{n\to\infty} \frac{\psi_i(a,2^{sn}b)}{|4|^{sn}} \qquad (1 \le j \le 3),$$

$$\left\|f(\lambda a + \lambda b + \lambda c) + f(\lambda a - \lambda b - \lambda c) - 2\lambda^2 f(a) - 2\lambda^2 f(b) - 2\lambda^2 f(c)\right\| \le \varphi_1(a,b,c,d)$$

$$\left\|g(\lambda a + \lambda b + \lambda c) + g(\lambda a - \lambda b - \lambda c) - 2\lambda^2 g(a) - 2\lambda^2 g(b) - 2\lambda^2 g(c)\right\| \le \varphi_2(a,b,c,d) \tag{2.2}$$

$$\left\|f(ab) - f(a)b^2\right\| \le \psi_1(a,b) \tag{2.3}$$

$$\left\|g(ab) - a^2 g(b)\right\| \le \psi_2(a,b)$$

$$\left\|a^2 f(b) - g(a)b^2\right\| \le \psi_3(a,b) \tag{2.4}$$

for all $a, b \in A$ *and all* $\lambda \in T = \{\lambda \in C : |\lambda| = 1\}$ *.Also, if for each fixed* $a \in A$ *the mappings* $t \to f(ta)$ *and* $t \to g(ta)$ *from* R *to* A *are continuous, then there exists a unique* quadratic *double centralizer* (L,R) *on* A *satisfying*

$$\left\|f(a) - L(a)\right\| \le \frac{1}{|4|}\tilde{\varphi}_1(a,a,0,0), \tag{2.5}$$

$$\left\|g(a) - R(a)\right\| \le \frac{1}{|4|}\tilde{\varphi}_2(a,a,0,0), \tag{2.6}$$

for all $a \in A$.

Proof: Let $s = 1$. Putting $a = b, c = d = 0$ and $\lambda = 1$ in *(2.2)*, we have

$$\left\|f(2a) - 4f(a)\right\| \le \varphi_1(a,a,0,0)$$

for all $a \in A$. One can use induction to show that

$$\left\|\frac{f(2^n a)}{|4|^n} - \frac{f(2^m a)}{|4|^m}\right\| \le \frac{1}{|4|}\sum_{k=m}^{n-1} \frac{\varphi_1(2^k a, 2^k a, 0, 0)}{|4|^k} \tag{2.7}$$

for all $n > m \ge 0$ and all $a \in A$. It follows from *(2.7)* and *(2.1)* that sequence $\left\{\frac{f(2^n a)}{4^n}\right\}$ is Cauchy. Since A *is* a non-ArchimedeanBanach algebra, this sequence is convergent. Define

$$L(a) := \lim_{n\to\infty} \frac{f(2^n a)}{4^n}. \tag{2.8}$$

ISSN 2347-1921

Replacing *a* and *b* by $2^n a$ and $2^n b$, respectively, in *(2.2),* we get

$$\| \frac{f(2^n(\lambda a+\lambda b))}{4^n} + \frac{f(2^n(\lambda a-\lambda b))}{4^n} - 2\lambda^2 \frac{f(2^n a)}{4^n} - 2\lambda^2 \frac{f(2^n b)}{4^n} \| \le \frac{\varphi_1(2^n a, 2^n b, 0, 0)}{|4|^n}$$

Taking the limit as $n \to \infty$, we obtain

$$L(\lambda a+\lambda b)+L(\lambda a-\lambda b) = |2||\lambda|^2 L(a)+|2||\lambda|^2 L(b) \qquad (2.9)$$

for all $a, b \in A$ and all $\lambda \in T$. Putting $\lambda = 1$ in *(2.9), we* obtain that L *is* a quadratic mapping. Setting $b := a$ in *(2.9),* we get

$$L(2\lambda a) = |4||\lambda|^2 L(a)$$

for all $a \in A$, $\lambda \in T$. But L *is* a quadratic mapping. So

$$L(\lambda a) = |\lambda|^2 L(a)$$

for all $a \in A$ *and all* $\lambda \in T$. Under the assumption that $f(ta)$ *is* continuous in $t \in R$ for each fixed $a \in A$, by the same reasoning as in the proof of *[8],* $L(\lambda a) = \lambda^2 L(a)$ for all $a \in A$ and all $\lambda \in R$. we obtain

$$L(\lambda a) = |\lambda|^2 L(a)$$

for all $a \in A$ and $\lambda \in C(\lambda \ne 0)$. This means that L *is* quadratic homogeneous. It follows from *(2.3)* and *(2.8)* that

$$\left\|L(ab)-L(a)b^2\right\| = \lim_{n\to\infty} \frac{1}{|4|^n} \left\|f(2^n ab)-f(2^n a)b^2\right\| \le \lim_{n\to\infty} \frac{\psi_1(2^n a,b)}{|4|^n} = 0$$

for all $a, b \in A$. Hence L *is* a quadratic left centralizer on A. Applying *(2.7)* with $m = 0$,we get $\left\|L(a)-f(a)\right\| \le \frac{1}{|4|}\tilde{\varphi}_1(a,a,0,0)$ for all $a \in A$. It is well known that the quadraticmapping L satisfying *(2.5) is* unique. A similar argument gives us a unique quadratic right centralizer R defined by

$$R(a) := \lim_{n\to\infty} \frac{g(2^n a)}{4^n}$$

which satisfies *(2.6). Now* we let $a, b \in A$ arbitrarily. Since L *is* a quadratic homogeneous, it follows from *(2.4)* and *(2.5)* that

$$\begin{aligned}\left\|a^2L(b)-R(a)b^2\right\| &= \frac{1}{|4|^n}\left\|a^2L(2^n b)-4^n R(a)b^2\right\| \\ &\le \frac{1}{|4|^n}[\left\|a^2L(2^n b)-a^2f(2^n b)\right\| + \left\|a^2f(2^n b)-g(a)(4^n b^2)\right\| \\ &+ \left\|4^n g(a)b^2-4^n R(a)b^2\right\|] \\ &\le \frac{1}{|4|^{n+1}}\tilde{\varphi}_1(2^n b, 2^n a)\ \ \|a\|^2 + \frac{\psi_3(a, 2^n b)}{|4|^n} + \|g(a)-R(a)\|\ \ \|b\|^2 .\end{aligned}$$

The right hand side of the last inequality tends to $\|g(a)-R(a)\|\ \ \|b\|^2$ as $n \to \infty$.

By *(2.6), we* obtain

$$\left\|a^2L(b)-R(a)b^2\right\| = \frac{1}{|4|}\tilde{\varphi}_2(a,a,0,0)\|b\|^2 .$$

Since R *is* a quadratic mapping, we thus obtain

$$\begin{aligned}\left\|a^2L(b)-R(a)b^2\right\| &= \frac{1}{|4|^n}\left\|4^n a^2L(b)-R(2^n a)b^2\right\| \\ &\le \frac{1}{|4|}\tilde{\varphi}_2(2^n a, 2^n a, 0, 0)\ \ \|a\|^2 \\ &= \frac{1}{|4|}\sum_{k=n}^{\infty}\frac{\varphi_2(2^k a, 2^k a)}{|4|^k}\|b\|^2 .\end{aligned}$$

Passing to the limit as $n \to \infty$, we conclude $a^2L(b) = R(a)b^2$. *Thus* (L, R) is a quadraticdouble centralizer.

The proof for $s = -1$ is similar to $s = 1$.

Corollary 2.6. *Suppose that* $f: A \to A$ *is a mapping for which there exist a mapping* $g: A \to A$ *and constants* $\varepsilon > 0$ *and* $0 \le p \ne 2$ *such that*

$$\left\| f(\lambda a + \lambda b + \lambda c) + f(\lambda a - \lambda b - \lambda c) - 2\lambda^2 f(a) - 2\lambda^2 f(b) - 2\lambda^2 f(c) \right\| \le \varepsilon(\|a\|^p + \|b\|^p, \|c\|^p + \|d\|^p),$$

$$\left\| g(\lambda a + \lambda b + \lambda c) + g(\lambda a - \lambda b - \lambda c) - 2\lambda^2 g(a) - 2\lambda^2 g(b) - 2\lambda^2 g(c) \right\| \le \varepsilon(\|a\|^p + \|b\|^p, \|c\|^p + \|d\|^p),$$

$$\left\| f(ab) - f(a)b^2 \right\| \le \varepsilon \|a\|^p \|b\|^p,$$

$$\left\| g(ab) - a^2 g(b) \right\| \le \varepsilon \|a\|^p \|b\|^p,$$

$$\left\| a^2 f(b) - g(a)b^2 \right\| \le \varepsilon \|a\|^p \|b\|^p$$

for all $a, b \in A$ *and all* $\lambda \in T$ *.Also, if for each fixed* $a \in A$ *the mappings* $t \to f(ta)$ *and* $t \to g(ta)$ *from* R *to* A *are continuous, then there exists a unique* quadratic *double centralizer* (L, R) *on* A *satisfying*

$$\left\| f(a) - L(a) \right\| \le \frac{2\varepsilon}{|4| - |2|^p} \|a\|^p,$$

$$\left\| g(a) - R(a) \right\| \le \frac{2\varepsilon}{|4| - |2|^p} \|a\|^p$$

for all $a \in A$.

Proof: For $j = 1,2,$ *putting* $\varphi_j(a,b) = \varepsilon(\|a\|^p + \|b\|^p + \|c\|^p + \|d\|^p)$ and for $i = 1,2,3$ *putting* $\psi_i(a,b) = \varepsilon \|a\|^p \|b\|^p$ *in Theorem 2.5, we* get the desired results.

3. STABILITY OF QUADRATIC MULTIPLIERS

Throughout this section, assume that A is a non-ArchimedeanBanachalgebra.

Definition3.1. *We say that a mapping* $T: A \to A$ *is a* quadratic *multiplier if* T *satisfies the following properties:*

1) T *is a* quadratic *mapping,*

2) T *is*quadratic *homogeneous, that is,* $T(\lambda a) = \lambda^2 T(a)$ *for all* $a \in A$ *and* $\lambda \in C$,

3) $a^2 T(b) = T(a)b^2$ *for all* $a, b \in A$.

Example *2.4* introduces a quadratic multiplier. We investigate the stability of quadraticmultipliers.

Theorem 3.2. *Suppose that* $s \in \{-1,1\}$ *and that* $f: A \to A$ *is a mapping with* $f(0) = 0$ *for which there exist functions,* $\varphi: A \times A \times A \times A \to [0,\infty), \psi: A \times A \to [0,\infty)$ *such that*

$$\tilde{\varphi}(a,b,c,d) := \sum_{k=0}^{\infty} \frac{\varphi(2^{sk}a, 2^{sk}b, 2^{sk}c, 2^{sk}d)}{|4|^{sk}} < \infty, \tag{3.1}$$

$$\lim_{n\to\infty} \frac{\psi(2^{sn}a, b)}{|4|^{sn}} = 0 = \lim_{n\to\infty} \frac{\psi(a, 2^{sn}b)}{|4|^{sn}},$$

$$\left\| f(\lambda a + \lambda b + \lambda c) + f(\lambda a - \lambda b - \lambda c) - 2\lambda^2 f(a) - 2\lambda^2 f(b) - 2\lambda^2 f(c) \right\| \le \varphi(a,b,c,d),$$

$$\left\| a^2 f(b) - f(a)b^2 \right\| \le \psi(a,b) \tag{3.2}$$

for all $a, b \in A$ *and all* $\lambda \in T$ *.Also, if for* each *fixed* $a \in A$ *the mappings* $t \to f(ta)$ *from* R *to* A *are continuous, then there exists a unique* quadratic*multiplier* T *on* A *satisfying*

$$\left\| f(a) - T(a) \right\| \le \frac{1}{|4|} \tilde{\varphi}(a,a,0,0), \tag{3.3}$$

for all $a \in A$.

Proof. Let $s = 1$. Putting $c = d = 0$. By the same reasoning as in the proof of Theorem *2.5,* there existsa unique quadratic mapping $T: A \to A$ defined by

$$T(a) := \lim_{n\to\infty} \frac{f(2^n a)}{4^n}$$

with satisfying $T(\lambda a) = |\lambda|^2 T(a)$ for all $a \in A$ and all $\lambda \in C$ *. Also,* $\left\| f(a) - T(a) \right\| \le \frac{1}{|4|} \tilde{\varphi}(a,a,0,0)$ for all $a \in A$ .Let $a, b \in A$ be arbitrarily. Then T is quadratic homogeneous.

By using *(3.2)* and *(3.3)*, we have

$$\|a^2T(b)-T(a)b^2\| = \frac{1}{|4|^n}\|a^2T(2^n b)-4^nT(a)b^2\|$$
$$\leq \frac{1}{|4|^n}[\|a^2T(2^n b)-a^2f(2^n b)\| + \|a^2 f(2^n b)-f(a)(4^n b^2)\|$$
$$+\|4^n f(a)b^2-4^nT(a)b^2\|]$$
$$\leq \frac{1}{|4|^{n+1}}\tilde{\varphi}(2^n b, 2^n a, 0, 0)\ \ \|a\|^2 + \frac{\psi(a, 2^n b)}{|4|^n} + \frac{1}{|4|}\tilde{\varphi}(a, a, 0, 0)\ \ \|b\|^2.$$

It follows from *(3.1)* that

$$\|a^2T(b)-T(a)b^2\| = \frac{1}{|4|}\tilde{\varphi}(a, a, 0, 0)\ \ \|b\|^2.$$

Finally, we obtain

$$\|a^2T(b)-T(a)b^2\| = \frac{1}{|4|^n}\|4^n a^2T(b)-T(2^n a)b^2\|$$
$$\leq \frac{1}{|4|}\tilde{\varphi}_2(2^n a, 2^n a, 0, 0)\ \ \|b\|^2$$
$$= \frac{1}{|4|}\sum_{k=n}^{\infty}\frac{\varphi(2^k a, 2^k a, 0, 0)}{|4|^k}\|b\|^2$$
$$\to 0 \qquad as \quad n\to\infty.$$

So $a^2T(b) = T(a)b^2$. *Hence* T is a quadratic multiplier.
The proof for $s=-1$ is similar.

Corollary ***3.3.*** *Suppose that* $f : A \to A$ *is a mapping for which there exist nonnegative real numbers* ε *and* p *with* $p \neq 2$ *such that*

$$\|f(\lambda a+\lambda b+\lambda c)+f(\lambda a-\lambda b-\lambda c)-2\lambda^2 f(a)-2\lambda^2 f(b)-2\lambda^2 f(c)\| \leq \varepsilon(\|a\|^p+\|b\|^p),$$
$$\|a^2 f(b)-f(a)b^2\| \leq \varepsilon\|a\|^p\ \|b\|^p$$

for all $a, b \in A$ *and all* $\lambda \in T$ *.Also, if for* each *fixed* $a \in A$ *the mappings* $t \to f(ta)$ *from* R *to* A *are continuous, then there exists a unique* quadratic*multiplier* T *on* A *satisfying*

$$\|f(a)-T(a)\| \leq \frac{2\varepsilon}{|4|-|2|^p}\|a\|^p$$

for all $a \in A$.

Proof:*Putting* $\varphi(a,b)=\varepsilon(\|a\|^p+\|b\|^p+\|c\|^p+\|d\|^p)$ and $\psi(a,b)=\varepsilon\|a\|^p\|b\|^p$ *in Theorem 3.2, we* get the desired results.

4. SUPERSTABILITY OF QUADRATIC DOUBLE CENTRALIZERS

In this section, we prove the superstability of quadratic double centralizers on non-ArchimedeanBanach algebras which are weakly without order and weakly commutative.

Theorem ***4.1.*** *Suppose that* A *is a* non-ArchimedeanBanach*algebra* weakly *without order and* weakly *commutative and* $s \in \{-1,1\}$*.Let* $L, R : A \to A$ *are mappings for which there exists a function* $\psi : A\times A \to [0,\infty)$ *such that*

$$\lim_{n\to\infty} n^{-2s}\psi(n^s x, y) = 0 = \lim_{n\to\infty} n^{-2s}\psi(x, n^s y)$$
$$\|x^2 L(y)-R(y)^2\| \leq \psi(x,y)$$

for all $x, y \in A$. *Then* (L,R) *is a* quadratic *double centralizer.*

Proof: We first show that L *is* a quadratic homogeneous. To do this, pick $\lambda \in C$ and $x, y \in A$. We have

$$\left\|n^{2s}z^2(L(\lambda x)-\lambda^2L(x))\right\| = \left\|n^{2s}z^2L(\lambda x)-\lambda^2n^{2s}z^2L(x)\right\|$$
$$\le \left\|n^{2s}z^2L(\lambda x)-R(n^sz)(\lambda x)^2\right\| + \left\|\lambda^2R(n^sz)x^2-\lambda^2n^{2s}z^2L(x)\right\|$$
$$\le \psi(n^s z,\lambda x)+|\lambda|^2\psi(n^s z,x).$$

So

$$\left\|z^2(L(\lambda x)-\lambda^2L(x))\right\| \le |n|^{-2s}\psi(n^s z,\lambda x)+|\lambda|^2|n|^{-2s}\psi(n^s z,x).$$

Since A *is* weakly without order, we conclude that $L(\lambda x)=|\lambda|^2 L(x)$ Thequadraticity of L follows from

$$\left\|z^2(L(x+y)+L(x-y)-2L(x)-2L(y))\right\|$$
$$=|n|^{-2s}\left\|n^{2s}z^2L(x+y)+n^{2s}z^2L(x-y)-2n^{2s}z^2L(x)-2n^{2s}z^2L(y)\right\|$$
$$\le |n|^{-2s}[\left\|n^{2s}z^2L(x+y)-R(n^sz)(x+y)^2\right\|+\left\|n^{2s}z^2L(x-y)-R(n^sz)(x-y)^2\right\|$$
$$+|2|\left\|R(n^sz)x^2-n^{2s}z^2L(x)\right\|+|2|\left\|R(n^sz)y^2-n^{2s}z^2L(y)\right\|]$$
$$\le |n|^{-2s}[\psi(n^sz,x+y)+\psi(n^sz,x-y)+|2|\psi(n^sz,x)+|2|\psi(n^sz,y)]$$

for all $x,y\in A$.

Finally, since A is a quadratic commutative non-ArchimedeanBanach algebra, we have

$$\left\|z^2(L(xy)-L(x)y^2)\right\| = |n|^{-2s}\left\|n^{2s}z^2L(xy)-n^{2s}z^2L(x)y^2\right\|$$
$$\le |n|^{-2s}[\left\|n^{2s}z^2L(xy)-R(n^sz)(xy)^2\right\|$$
$$+\left\|R(n^sz)x^2y^2-n^{2s}z^2L(x)y^2\right\|]$$
$$\le |n|^{-2s}[\psi(n^sz,xy)+\psi(n^sz,x)\|y\|^2]$$

for all $x,y\in A$. So $L(xy)=L(x)y^2$. Thus L *is* a quadratic left centralizer. One can similarly prove that R *is* a quadratic right centralizer. Since L *is* quadratic homogeneous, $L(x)=|n|^{-2s}L(n^sx)$ for all $n\in N$ and $x\in A$. Thus

$$\left\|x^2(L(y)-R(x)y^2)\right\| = |n|^{-2s}\left\|x^2L(n^sy)-R(x)(n^{2s}y^2)\right\|$$
$$\le |n|^{-2s}\psi(x,n^sy)$$

and hence by (4. 1) we infer that $x^2L(y)=R(x)y^2$ for all $x,y\in A$. Thus (L,R) *is* a quadratic centralizer.

Corollary 4.2. *Suppose* A is *a* non-ArchimedeanBanach *algebra* weakly *without order and* weakly *commutative and* $L,R:A\to A$ *are mappings for which there exist a nonnegativereal number* ε *and a real number* p *either greater than* 2 *or less than* 2, *such that*

$$\left\|x^2T(y)-R(x)y^2\right\| \le \varepsilon\|x\|^p\|y\|^p$$

for all $x,y\in A$. *Then* (L,R) *is a* quadratic *double centralizer.*

Proof: Using Theorem 4.1 with $\psi(x,y)\le\varepsilon\|x\|^p\|y\|^p$ we get the desired result.

5. SUPERSTABILITY OF QUADRATIC MULTIPLIERS

In this section, we prove the superstability of quadratic multipliers on non-ArchimedeanBanach algebras which are weakly without order.

Theorem 5.1. *Suppose that* A is *a Banach algebra with* weakly *without order and* $s\in\{-1,1\}$. *Let* $T:A\to A$ *are mappings for which there exists a function* $\psi: A\times A\to[0,\infty)$ *such that*

$$\lim_{n\to\infty}|n|^{-2s}\psi(n^sx,y)=0=\lim_{n\to\infty}|n|^{-2s}\psi(x,n^sy)$$
$$\left\|x^2L(y)-R(y)^2\right\|\le\psi(x,y)$$

for all $x, y \in A$*. Then* (L, R) *is a* quadratic*multiplier.*

Proof: By the same reasoning as in the proof of Theorem 4.1, putting $L = R = T$, we can show that the mapping T is a quadratic multiplier.

Corollary 5.2.*Suppose that* A is *a* weakly *without order* non-ArchimedeanBanach *algebra and that* $T : A \to A$ is *a mapping for which there exist a nonnegative real number* ε *and a real number* p *either greater than* 2*or less than* 2, *such that*

$$\left\| x^2 T(y) - T(x) y^2 \right\| \le \varepsilon \|x\|^p \|y\|^p$$

for all $x, y \in A$*. Then* T *is a* quadratic*multiplier.*

Proof: Using Theorem 5.1 with $\psi(x, y) = \varepsilon \|x\|^p \|y\|^p$,we get the result.

ACKNOWLEDGEMENTS:

The author sincerely thank the anonymous reviewer for a careful reading, constructives to improve the quality of the paper and suggesting a related reference.

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