Heat Generation and Chemical Reaction Effects on MHD Boundary Layer Flow of a Moving Vertical plate with Suction and Dissipation

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ABSTRACT: In this paper, the study of the steady two-dimensional flow of an incompressible viscous fluid with heat and mass transfer and MHD heat generation past a moving vertical plate with suction in the presence of viscous dissipation and chemical reaction is investigated. Using similarity variables, the governing partial differential equations are transformed into non-linear ordinary differential equations. These equations are then solved numerically using fourth order Runge-Kutta method with shooting technique. The flow variables are presented graphically. The graphs showed that velocity rises for increasing Grashof number, mass Grashof number, Suction, Heat generation and Eckert number while reducing with increasing Magnetic parameter, Schmidt number, and Chemical reaction parameter and Prandtl number. Comparisons with previously published work are performed and are found to be in an excellent agreement.

Keywords: Chemical reaction parameter; free convection; heat generation; MHD; moving vertical plate.
1. INTRODUCTION

Convective flows with simultaneous heat and mass transfer under the influence of a magnetic field and chemical reaction arise in many transport processes both naturally and artificially in many branches of science and engineering applications. This phenomenon plays an important role in the chemical industry, power and cooling industry for dying, chemical vapour deposition on surfaces, cooling of nuclear reactors and petroleum industries. Natural convection flow occurs frequently in nature. It occurs due to temperature differences, as well as due to concentration differences or the combination of these two, for example in atmospheric flows, there exists differences in water concentration and hence the flow is influenced by such concentration difference. Changes in fluid density gradients may be caused by non-reversible chemical reaction in the system as well as by the differences in molecular weight between values of the reactants and the products. Chemical reaction can be modeled as either homogeneous or heterogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. A homogeneous reaction is one that occurs uniformly throughout a given phase. On the other hand, a heterogeneous reaction takes place in a restricted area or within the boundary of the phase. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration itself, Cussler [1]. For example, the formation of smog is a first order homogeneous reaction. Consider the emission of nitrogen dioxide from automobiles and other smoke-stacks. This nitrogen dioxide reacts chemically in the atmosphere with unburned hydrocarbons (aided by sunlight) and produces peroxymethyl nitrate, which forms an envelope which is termed photo-chemical smog.

The study of heat and mass transfer with chemical reaction is of great practical importance in many branches of science and engineering. Das et al. [2] studied the effects of mass transfer flow past an impulsively started infinite vertical plate with constant heat flux, and chemical reaction. Anjalidevi and Kandasamy [3] studied effects of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate. More authors intensive studies have been carried out to investigate effects of chemical reaction on different flow types. Seddeek et al. [4], Salem and Abd El-Aziz [5], Mohamed [6], Ibrahim et al. [7].

Boundary layer behavior over a moving continuous solid surface is an important type of flow occurring in a number of engineering processes. To be more specific, heat treated materials traveling between a feed roll and a wind-up roll, aerodynamic extrusion of plastic sheets, glass fiber and paper production, cooling of an infinite metallic plate in a cooling path, manufacturing of polymeric sheets and glass fiber and paper production, cooling of an infinite metallic plate in a cooling path, manufacturing of polymeric sheets are examples for practical applications of continuous moving flat surfaces. Since the pioneering work of Sakiadis [8] various aspects of the problem have been investigated by many authors. Mass transfer analysis at the stretched sheet were found in the studies by Erickson et al. [9] and relevant experimental results were reported by Tsou et al. [10] regarding several aspects for the flow and heat transfer boundary layer problems in a continuously moving sheet. Crane [11] and Grubka [12] have analyzed the stretching problem with constant surface temperature, while Soundalgekar [13] investigated the Stokes problem for a viscouselastic fluid wall temperature and heat flux. Raptis and Singh [14] studied flow past an impulsively started vertical plate in a porous medium by a finite difference method. The fluid considered in that paper is an optically dense viscous incompressible fluid of linearly varying temperature dependent viscosity. Ambethkar [15] studied numerical solutions of heat and mass transfer effects of an unsteady MHD free convective flow past an infinite vertical plate with constant suction.

Alam et al. [16] studied the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. They also investigated MHD free convective heat and mass transfer flow past an inclined surface with heat generation. Salem [17] discussed coupled heat and mass transfer in Darcy-Forchheimer mixed convection from a vertical flat plate embedded in a fluid saturated porous medium under the effects of radiation and viscous dissipation. Alam et al. [18] analyzed the effects of chemical reaction and thermo-photoreaction on MHD convective heat and mass transfer flow past an inclined plate in the presence of heat generation/absorption with viscous dissipation and Joule heating. PareshVyas and AshutoshRanjan [19] discussed the dissipative MHD boundary-layer flow in a porous medium over a sheet stretching nonlinearly in the presence of radiation. Muthuraj and Shrinivas [20] studied the influence of magnetic field and wall slip conditions on steady flow between parallel flat wall and a long wavy wall with Soret effect.

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Heat generation effects may alter the temperature distribution and this in turn can affect the particle deposition rate in nuclear reactors, electronic chips and semi-conductor wafers. Although exact modeling of internal heat generation or absorption is quite difficult, some simple mathematical models can be used to express its general behavior for most physical situations. Heat generation or absorption can be assumed to be constant, space-dependent or temperature-dependent. Tania et al. [21] has investigated the effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. Furthermore, Moslem [22] studied the effect of temperature dependent heat sources taking place in electrically heating on the heat transfer within a porous medium. Vajravelu and Nayfeh [23] were reported on the hydromagnetic convection at a cone and a wedge in the presence of temperature dependent heat generation or absorption effects. Moreover, Chamkha [24] studied the effect of heat generation or absorption on hydro magnetic three-dimensional free convection flow over a vertical stretching surface. Viscous dissipation changes the temperature distributions by playing a role like an energy source, which leads to affected heat transfer rates. The merits of the effect of viscous dissipation depend on whether the plate is being cooled or heated. Heat transfer analysis over porous surface is of much practical interest due to its abundant applications. To be more specific, heat-treated materials traveling between a feed roll and wind-up roll or materials manufactured by extrusion. Glass-fiber and paper production, cooling of metallic sheets or electronic chips, crystal growing are just some examples. In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the process of
stretching. The work of Sonthet al. [25] dealt with the effect of the viscous dissipation term along with temperature dependent heat source/sink on momentum, heat and mass transfer in a visco-elastic fluid flow over an accelerating surface. Chen [26] examined the effect of combined heat and mass transfer on MHD free convection from a vertical surface with ohmic heating and viscous dissipation. The effect of viscous dissipation and joule heating on MHD free convection flow past a semi-infinite vertical flat plate in the presence of the combined effect of Hall and non-slip currents for the case of the power-law variation of the wall temperature is analyzed by Abo-Eldahab and El-Aziz [27]. Gupta et al. [28] studied heat and mass transfer on a stretching sheet with suction or blowing. Ibrahim and Makinde [29] have investigated the effects of chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction.

This article aims to find numerical solutions of the coupled equations that govern the flow by using shooting technique with the forth order Range-Kutta method. In the problem formulation, the continuity, momentum, energy and concentration equations are reduced to some parameter problem by introducing suitable transformation variables. Pertinent results with respect to embedded parameters are displayed graphically for the velocity, temperature and concentration profiles and were discussed quantitatively. The local skin-friction coefficient and the heat and mass transfer results are obtained for representative values of the important parameters.

2. MATHEMATICAL ANALYSIS

Consider a two-dimensional free convective flow on the steady incompressible laminar MHD heat and mass transfer characteristics of a linearly started porous vertical plate, the velocity of the fluid far away from the plate surface is assumed zero for a quiescent state fluid. The flow configurations are linear. All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of the linear momentum equation. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field is neglected. The Hall effects and the joule heating terms are also neglected. Then under Boussinesq’s approximations, the governing boundary-layer equations that are based on the balance laws of mass, linear momentum, energy and concentration species for this investigation can be written as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\sigma B_0^2}{\rho} u + g \beta_1 (T - T_x) + g \beta_2 (C - C_\infty)
\]

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_x) + \frac{\nu}{C_p} \left( \frac{\partial u}{\partial y} \right)^2
\]

\[
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - k r' (C - C_\infty)
\]

The boundary conditions at the plate surface and for into the cold fluid may be written as

\[
v = V, u = Bx, T = T_w, T_x = T_\infty + \alpha x, C = C_w = C_\infty + bx, at y = 0, u \to 0, T \to T_\infty, C \to C_\infty as y \to \infty
\]

Where \( B \) is constant, ‘a’ and ‘b’ denotes the stratification rate of the gradient of ambient temperature and concentration profiles. We introduce the following non-dimensional variables:

\[
\eta = \sqrt{B \psi}, \phi(\eta) = \frac{T - T_x}{T_w - T_\infty}, M = \frac{\sigma B_0^2}{\rho B}, Ec = \frac{B^2}{C_p (T_w - T_\infty)}, Gr = \frac{g \beta_1 (T_w - T_\infty)}{xB^2}, Gc = \frac{g \beta_2 (C_w - C_\infty)}{xB^2}, Pr = \frac{\nu}{\alpha}, Sc = \frac{\nu}{D_m}, Q = \frac{Q_0}{\rho C_p B}, F = \frac{V}{\sqrt{Bv}}, k r' = \frac{k r B^2}{\nu}
\]

The velocity components \( u \) and \( v \) are respectively obtained as follows:
Applying the shooting technique, the missing initial condition at the initial point of the interval is worked out and is

\[ f''(0) = 1, \quad f(0) = -Fw, \quad \theta(0) = 1, \quad \phi(0) = 1 \]

\[ f''(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \]

The corresponding boundary conditions (2.5) then take the following form

where prime denotes partial differentiation with respect to \( t \)

### 3. NUMERICAL METHOD OF SOLUTION

The set of coupled non-linear governing boundary layer equations (2.8)-(2.10) together with the boundary conditions (2.11&12) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential Equations (2.8)-(2.10) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Alamet et al. [30]). In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial value problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy. For this type of iterative approach, one naturally inquires whether or not there is a systematic way of finding each succeeding (assumed) value of the missing initial condition. The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size =0.05 is used to obtain the numerical solution with decimal place accuracy as the criterion of convergence. The parameters of engineering interest for the present problem are the local skin friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively proportional to \( f''(0), -\theta'(0) \) and \(-\phi'(0)\) are worked out and their numerical values presented in a tabular form.

### 4. RESULTS AND DISCUSSION

The governing equations (2.8)-(2.10) subject to the boundary conditions (2.11)-(2.12) are integrated as described in section 3. The Prandtl number was taken to be \( Pr = 0.72 \) which corresponds to air, the value of Schmidt number(Sc) were chosen to be \( Sc = 0.24, 0.62, 0.78, 2.62 \), representing diffusing chemical species of most common interest in air like \( H_2, H_2O, NH_3 \) and Propol Benzene respectively. Paying attention on positive value of the buoyancy parameters that is local temperature Grashof number \( Gr > 0 \) and local concentration Grashof number \( Gc > 0 \). Throughout the calculations, the parametric values are fixed to be \( Gr = Gc = Ec = Q = M = 0.1, Sc = 0.62, Pr = 0.72, Kr = 0.5, \) and \( Fw = 0.1 \), unless otherwise indicated.

The effects of various parameters on velocity profiles in the boundary layer are shown in Figs. 1-8. It is noticed from Figs. 1-8, that the velocity is higher near the moving vertical plate surface and decrease to its zero value far away from the moving vertical plate surface satisfying the far field boundary condition for all parameter values.

In Fig. 1 the effect of increasing the magnetic field strength on the momentum boundary layer thickness is illustrated. It is now a well established fact that the magnetic field presents a damping effect on the velocity field by creating drag force that opposes the fluid motion, causing the velocity to decrease. However, in this case an increase in the \( M \) only slightly slows down the motion of the fluid away from the moving vertical plate surface towards the free stream velocity, while the fluid velocity near the moving vertical plate surface decreases.

Figs. 2, 3, 4 & 6 depict the variation of the boundary-layer velocity by the buoyancy forces parameters (Gr,Gc), magnitude of fluid suction (Fw) and heat source/sink parameter (Q). In both cases an upward acceleration of the fluid in the vicinity of the vertical wall is observed with increasing intensity of buoyancy forces. Further downstream of the fluid motion decelerates to the free stream velocity. Fig. 5 and Fig.9 shows that a slight decrease in the fluid velocity with an increase in the Schmidt number (Sc) and chemical reaction parameter. The effect of viscous dissipation parameter i.e., the Eckert number \( Ec \) on the velocity component is shown in Fig. 7. The positive Eckert number implies cooling of the plate.
i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a slightly increase in the velocity. Fig.8. Illustrates the velocity component for different values of the Prandtl number \( Pr \). The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity.

In general the fluid temperature attains its maximum value at the moving vertical plate surface and decreases exponentially to the free stream zero value away from the plate satisfying the boundary condition. This is observed in Figs. 9-17. From these figures, it is interesting to note that the thermal boundary layer thickness decreases with an increase in the intensity of the buoyancy forces \((Gr, Gc)\) and Prandtl number \((Pr)\). Moreover, the fluid temperature increases with an increase in the Schmidt number \((Sc)\) the chemical reaction parameter \((kr)\), Magnetic field \((M)\), heat source/sink parameter \((Q)\) magnitude of fluid suction \((Fw)\) and Eckert number \((Ec)\) leading to an increase in thermal boundary layer thickness.

Figs. 18-26 illustrate chemical species concentration profiles against span wise coordinate \( \eta \) for varying values physical parameters in the boundary layer. The species concentration is highest at the moving vertical plate surface and decrease to zero far away from the moving vertical plate satisfying the boundary condition. From these figures, it is important to reveal that the concentration boundary layer thickness decreases with an increase in, the buoyancy forces \((Gr, Gc)\), Schmidt number \((Sc)\) the chemical reaction parameter \((kr)\), heat source/sink parameter \((Q)\) and Eckert number \((Ec)\). Moreover, the fluid concentration increases with an increase in the magnetic field \((M)\), magnitude of fluid suction \((Fw)\) and Prandtl number \((Pr)\) leading to an increase in thermal boundary layer thickness.

![Graph](image-url)

**Fig.1:** Effect of the magnetic parameter \((M)\) on velocity profiles.
Fig. 2: Effect of the Grashof number (Gr) on velocity profiles.

Gr = 0.1, 0.5, 1.0, 3.0

Fig. 3: Effect of the mass Grashof number (Gc) on velocity profile.

Gc = Ec = Q = M = 0.1, Sc = 0.62, Pr = 0.72, Kr = 0.5, \( F_w = 0.1 \)
Fig. 4: Effect of the Suction ($F_w$) on velocity profiles.

Fig. 5: Effect of the Schmidt number ($Sc$) on velocity profiles.
Fig. 6: Effect of the heat generation parameter \( (Q) \) on velocity profiles.

\[ Gr = Ge = Ec = M = 0.1, \quad Sc = 0.62, \]
\[ Pr = 0.72, \quad Kr = 0.5, \quad F_w = 0.1 \]

Fig. 7: Effect of the Eckert number \( (Ec) \) on velocity profiles.

\[ Ec = 0.01, 0.1, 0.2, 0.5 \]
Fig. 8: Effect of the Prandtl number (Pr) on velocity profiles.

Fig. 9: Effect of the magnetic parameter (M) on temperature profiles.
Fig. 10: Effect of the Grashof parameter (Gr) on temperature profiles.

Fig. 11: Effect of the mass Grashof parameter (Gc) on temperature profiles.
Fig. 12: Effect of the suction parameter ($F_w$) on temperature profiles.

Fig. 13: Effect of the Schmidt number ($Sc$) on temperature profiles.
Fig. 14: Effect of the heat generation parameter \( Q \) on temperature profiles.

\[ \theta \]

\[ Q = 0.01, 0.1, 0.2, 0.5 \]

Fig. 15: Effect of the Eckert number \( Ec \) on temperature profiles.

\[ \theta \]

\[ Ec = 0.01, 0.1, 0.2, 0.5 \]

\[ Gc = Gr = Q = M = 0.1, Sc = 0.62, Pr = 0.72, Kr = 0.5, F_w = 0.1 \]
Fig. 16: Effect of the Prandtl number (Pr) on temperature profiles.

Fig. 17: Effect of the chemical reaction parameter (Kr) on temperature profiles.
Fig. 18: Effect of the magnetic parameter \((M)\) on concentration profiles.

Fig. 19: Effect of the Grashof number \((Gr)\) on concentration profiles.
Fig. 20: Effect of the mass Grashof number (Gc) on concentration profiles.

Fig. 21: Effect of the suction ($F_w$) on concentration profiles.
Fig. 22: Effect of the Schmidt number (Sc) on concentration profiles.

Fig. 23: Effect of the heat generation parameter (Q) on concentration profiles.
Fig. 24: Effect of the Eckert number (Ec) on concentration profile

Fig. 25: Effect of the Prandtl number (Pr) on concentration profiles.
Fig. 26: Effect of the chemical reaction parameter \((Kr)\) on concentration profiles.

Table 1: variation of \(f''(0), \theta'(0)\) and \(\phi'(0)\) at the plate with \(Gr, Gc, M, F_w, Sc\) for \(Pr = 0.72, Q = Ec = 0\).

| Gr | Gc | M  | Fw | Sc | Ibrahim and Makinde (2010) | Present work  
|----|----|----|----|----|----------------------------|----------------
| 0.1| 0.1| 0.1| 0.1| 0.62| 0.889971, 0.7965511, 0.7253922| 0.889085, 0.79653, 0.725477 
| 0.5| 0.1| 0.1| 0.1| 0.62| 0.695974, 0.8379008, 0.7658018| 0.696036, 0.837878, 0.765851 
| 1.0| 0.1| 0.1| 0.1| 0.62| 0.475058, 0.8752835, 0.8020042| 0.475093, 0.875269, 0.802026 
| 0.1| 0.5| 0.1| 0.1| 0.62| 0.686927, 0.8421370, 0.7701717| 0.687021, 0.842077, 0.770165 
| 0.1| 1.0| 0.1| 0.1| 0.62| 0.457723, 0.8818619, 0.8087332| 0.457782, 0.881824, 0.808717 
| 0.1| 1.0| 1.0| 0.1| 0.62| 1.264488, 0.7089150, 0.640051| 1.264045, 0.708798, 0.640369 
| 0.1| 3.0| 0.1| 0.1| 0.62| 1.868158, 0.5825119, 0.5204793| 1.867845, 0.582456, 1.866548 
| 0.1| 0.1| 1.0| 1.0| 0.62| 0.570663, 0.5601256, 0.5271504| 0.570745, 0.56011, 0.527309 
| 0.1| 0.1| 0.1| 3.0| 0.62| 0.275153, 0.2955702, 0.2902427| 0.276071, 0.299108, 0.296673 
| 0.1| 0.1| 0.1| 0.1| 0.78| 0.893454, 0.7936791, 0.8397779| 0.893518, 0.79374, 0.833984 
| 0.1| 0.1| 0.1| 0.1| 2.62| 0.912307, 0.7847840, 1.6504511| 0.91237, 0.784892, 1.65042
Table 2: Variation of $f''(0), \theta'(0)$ and $\phi'(0)$ at the plate with $Q$ and $Ec$. For $Gr= Gc= M= Fw= Kr= 0$, $Sc = 0.62$, $Pr= 0.72$.

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<th>$Q$</th>
<th>$Ec$</th>
<th>$Kr$</th>
<th>$f''(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\phi'(0)$</th>
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REFERENCES:


