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Multi G –cyclic Operators are G-cyclic

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ABSTRACT

Paris proved in 2001 Herrero's conjectured "that every multi-hypercyclic (multi- supercyclic) operator on a Hilbert space is in fact hypercyclic (supercyclic)".

In this paper we study this conjectured in many cases of operators between hypercyclic operators and supercyclic operators. And we proved that multi $\mathcal{G}$-cyclic operator is $\mathcal{G}$-cyclic operator.

Keywords

Hypercyclic operators; supercyclic operators.

SUBJECT CLASSIFICATION

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INTRODUCTION:

Let H be an infinite dimensional separable complex Hilbert space, and let $B(H)$ be the Banach algebra of all bounded linear operators on H. Let $T \in B(H)$, it is called hypercyclic if there is a vector x in H such that the set $orbt(T,x) := \{T^n x: n \geq 0\}$ is norm–dense in H [1]. An operator T is supercyclic if there exists an $x \in H$ such that $Sorbt(T,x) := \{\alpha T^n x: n \geq 0, \alpha \in \mathbb{C}\}$is norm–dense in H [1].

T is called multi-hypercyclic if there is a set of vectors $\{x_i\}_{i=1}^n$ in H such that the set $\bigcup_{i=1}^n orbt(T,x_i)$ is norm–dense in H [2]. An operator T is multi-supercyclic if there exists a set of vectors $\{x_i\}_{i=1}^n$ in H such that $\bigcup_{i=1}^n Sorbt(T,x_i)$is norm–dense in H [2].

In 1999, Herrero conjectured that every multi-hypercyclic (multi-supercyclic) operator is hypercyclic (supercyclic) [1]. Peris in 2001 settle this conjecture in the affirmative even for continuous linear operator [2].

In 2005, Naoum and Jamil introduced G-cyclic operator, an operator T is called G-cyclic over a multiplication semigroup of $\mathbb{C}$ with 1, if there exists a vector x such that $Gorbt(T,x) := \{\alpha T^n x: n \geq 0, \alpha \in S\}$ is norm dense in H, such a vector is called G-cyclic vector for T over S [3].

It is easy to see that every hypercyclic operator is G- cyclic operator, and every G- cyclic operator is supercyclic

In this paper we introduce the concept multi G-cyclic operator, T is called multi G-cyclic operator if there exists a finite set of vectors $\{x_i\}_{i=1}^n$ in H such that $\bigcup_{i=1}^n Gorbt(T,x_i)$is norm–dense in H. We prove that every multi G-cyclic operator is G-cyclic oprerator.

Main result:

First we need the following lemmas. Since every G-cyclic operator is supercyclic then we get:

Lemma 1:

If $T \in B(H)$ is multi G-cyclic, then T^* has at most one eigenvalue

Lemma 2:

Let $T \in B(H)$, then $int\left[\overline{Gorbt(T,x)}\right] \cap int\left[\overline{Gorbt(T,y)}\right] \neq \emptyset$ if and only if $int\left[\overline{Gorbt(T,x)}\right] = int\left[\overline{Gorbt(T,y)}\right]$.

Proof:

$\Leftarrow$) Clearly

$\Rightarrow$) Since $int\left[\overline{Gorbt(T,x)}\right] \cap int\left[\overline{Gorbt(T,y)}\right] \neq \emptyset$, we find $n \in \mathbb{N}$, $\alpha \in \mathbb{C}$ such that $\alpha T^n x \in int\left[\overline{Gorbt(T,x)}\right] \cap int\left[\overline{Gorbt(T,y)}\right] \subseteq \overline{Gorbt(T,y)}$. Since $\overline{Gorbt(T,y)}$ is$T$–invariant, then $\alpha T^k x \in \overline{Gorbt(T,y)}$, for all $k \geq n$. Thus $int\left[\overline{Gorbt(T,x)}\right] = int\left[\overline{\{\alpha T^k x | \alpha \in S,\ k \geq n\}}\right] \subseteq int\left[\overline{Gorbt(T,y)}\right]$ By the same argument we get the other inclusion.

Before we prove our main result, we list without proof some facts which will be frequently used in the proof.

Observations 3:

1. Let X be a topological space and $F_1, \cdots, F_n$ a finite family of closed subsets of X such that $X = \bigcup_{i=1}^n F_i$. If $int(F_1) = \emptyset$ then $X = \bigcup_{i=2}^n F_i$.
2. Let Xbe a topological space, $A \subset X$, and F a closet subset of X with $int(F) = \emptyset$. Then $int(\overline{A}) = int(\overline{A - F})$.
3. If D is an complex Housdorf local convex space A and us a proper subspace, then $D - A$ is connected.
4. Any subspace of H containing a non-empty open set must be all of H.
5. Let P be a complex polynomial, $P(T)$ has dense range if and only if $P(\lambda) \neq 0$ for every eigenvalue λ of T^*.

6. Now we are ready to state and prove the main theorem.

Theorem 4:

If $T \in B(H)$ is multi G-cyclic operator, then T is G-cyclic operator.

Proof

Since T is multi G-cyclic, then there exists a finite set $\{x_i\}_{i=1}^{n} \subset H$ such that $H = \bigcup_{i=1}^{n} \overline{Gorbt(T, x_i)}$ and n is minimal. We define $F_i = \overline{Gorbt(T, x_i)}; 1 \le i \le n$. If $n = 1$, T is G-cyclic. Now, if $n > 1$, then by ((3), part (1)) and the minimality of n, $int(F_i) \ne \phi$ for all i; $1 \le i \le n$. By (2) given any $u \in H$ with $int\left[\overline{Gotbt(T, y)}\right]$ is not empty.

$int\left[\overline{Gotbt(T, y)}\right] = int\, F_i$ for some i; $1 \le i \le n$. (1)

Moreover,

$int[Gotbt(T, y)] \subseteq int\left[\overline{Gotbt(T, y)}\right]$ (2)

Otherwise, there are $m \in \mathbb{N}$, $\alpha \in S$ such that $\alpha T^m y \in F_i$ for some $j \ne i$. And by ((3), part (2)), $intF_i = int\left[\overline{Gorbt(T, y)}\right] = int\left[\overline{\{\alpha T^k y | \alpha \in S,\ k \ge m\}}\right]$, thus $intF_i \subseteq intF_j$, a contradiction. Since T^* has one possible eigenvalue, say λ, then let $P \ne 0$ be a polynomial in T such that $P(\lambda) \ne 0$. By ((3), part (5)) $P(T)$ has dense range, thus $H = P(T)\overline{(H)} = \bigcup_{i=1}^{n} P(T)\overline{(F_i)}$. Since $P(T)\overline{(F_i)} = \overline{Gorbt(T, P(T)x_i)}$ and the minimality of n, then $int\left[\overline{Gorbt(T, P(T)x_i)}\right] \ne \phi$ for all i; $1 \le i \le n$. So by $\{T^n P(T) x_i | n \ge 0\} \subseteq int\left[\overline{Gorbt(T, P(T)x_i)}\right] \overset{(1)}{=} int\left[F_{j(i)}\right]$, with $j(i) = \{1, 2, \cdots, n\}$ and $i = 1, 2, \cdots, n$. Then $C := \bigcup_{P(\lambda) \ne 0} orbt(T, P(T)x_1) \subseteq \bigcup_{i=1}^{n} int(F_i)$. Now observe that $C = span\, orbt(T, x_1) - \overline{(T - \lambda I)(H)}$. Thus by ((3), part (3)) C is connected, hence $C \subseteq int(F_1)$. By ((3), part (4)) C is dense, then $H = \bar{C} \subseteq F_1$. Therefore T is G-cyclic operator.

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