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Computing the Omega polynomial of an infinite family of the linear parallelogram *P(n,m)*

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ABSTRACT

Let *G=(V,E)* be a molecular graph, where *V(G)* is a non-empty set of vertices and *E(G)* is a set of edges. The Omega polynomial *Ω(G,x)* was introduced by *Diudea* in 2006 and this defined as $\Omega(G,x)=\sum_c m(G,c)\,x^c$ where *m(G,c)* the number of *qoc* strips of length *c*. In this paper, we compute the omega polynomial of an infinite family of the linear parallelogram *P(n,n)* of benzenoid graph.

Indexing terms/Keywords

Molecular graph, linear parallelogram, Omega polynomial, qoc strip.

SUBJECT CLASSIFICATION

E.g., Mathematics Subject Classification; 05C05, 05C12

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INTRODUCTION

Let *G=(V,E)* be a simple connected molecular graph in chemical graph theory., where the vertex set and edge set of G denoted by *V(G)* and *E(G)* respectively and its vertices correspond to the atoms and the edges correspond to the bonds [1-3].

In graph theory, a topological index of a molecular graph G is a numeric quantity related to G. The oldest nontrivial topological index is the Wiener index which was introduced by Chemist *Harold Wiener* [3]. The *Wiener index* [3] is defined as

$$W(G)=\frac{1}{2}\sum_{u\in V(G)}\sum_{v\in V(G)} d(u,v)$$

where the distance *d(u,v)* between two vertices *u* and *v* is the number of edges in a shortest path connecting them.

There exits many topological indices in mathematical chemistry and several applications of them have been found in physical, chemical and pharmaceutical models and other properties of molecules. Mathematical chemistry is a branch of theoretical chemistry for discussion and prediction of the molecular structure using mathematical methods without necessarily referring to quantum mechanics. Chemical graph theory is a branch of mathematical chemistry which applies graph [1-3].

Let *G*(*V*, *E*), be a molecular graph, with the vertex set *V*(*G*), and edge set *E*(*G*), and $x, y\in V(G)$. Two edges *e=uv* and *f=xy* of *G* are called co-distant (briefly: *e co f*) if they obey the topologically parallel edges relation. For some edges of a connected graph *G* there are the following relations satisfied [4-7]

$$e\ co\ e$$

$$e\ co\ f \Leftrightarrow f\ co\ e$$

$$e\ co\ f\ \&\ f\ co\ h \Rightarrow e\ co\ h$$

though the last relation is not always valid.

Set $C(e):=\{f\in E(G),\ |\ e\ co\ f\}$. If the relation "co" is transitive on *C(e)* then C(e) is called an *orthogonal cut "oc"* of the graph *G*. The graph *G* is called co-graph if and only if the edge set *E(G)* is the union of disjoint orthogonal cuts.

$$E(G)=C_1\cup C_2\cup \ldots \cup C_{k-1}\cup C_k \text{ and } C_i\cap C_j=\emptyset,$$

for *i≠j and i, j=1, 2, …, k.*

The Omega polynomial *Ω(G,x)* for counting qoc strips in *G* was defined by *M.V. Diudea* in 2006 as

$$\Omega(G,x)=\sum_c m(G,c)\, x^c$$

where *m(G,c)* being the number of *qoc* strips of length *c* and the summation runs up to the maximum length of *qoc* strips in *G*. Also, first derivative of omega polynomial (in *x=1*), equals the number of edges in the graph *G*. For more study, see papers [8-16]:

$$\Omega'(G,1)=\sum_c m(G,c)\times c=|E(G)|$$

In this paper, we compute the omega polynomial of an infinite family of benzenoid system and called the *linear parallelogram P(n,n).* Reader can see general representation of this family in Figure1.

Herein, our notation is standard and taken from the standard book of graph theory [1-3] and for more study about Omega polynomial and other counting polynomilas see paper serices [4-20].

Main Results and Discussions

In this section is to compute the Omega polynomial for an infinite family of the linear parallelogram *P(n,m)* of benzenoid graph. In generally consider the linear parallelogram benzenoid graph *P(n,m)* depicted in Figure 1 and see [21-23]. This graph has *2mn+2m+2n* vertices, since *|V(P(n,m))|=(2n+2)(m+1)-2* and *|E(P(n,m))|=(3n+2)(m)+2n-1=3mn+2n+2m-1* $\forall n,m\in\mathbb{N}$. It is easy to see that *P(n,n)* has exactly *2n(n+2)* vertices and $3n^2+4n-1$ edges.

Theorem 1. The Omega polynomial of the linear parallelogram *P(n,m)* $\forall n,m\in\mathbb{N}$, is as follows:

$$\Omega(P(n,m),x) = \sum_{i=1}^{Min(n,m)-1} 2x^{i+1} + (|n-m|+1)x^{Min\{n,m\}+1} + mx^{2n+1} + nx^{2m+1}$$

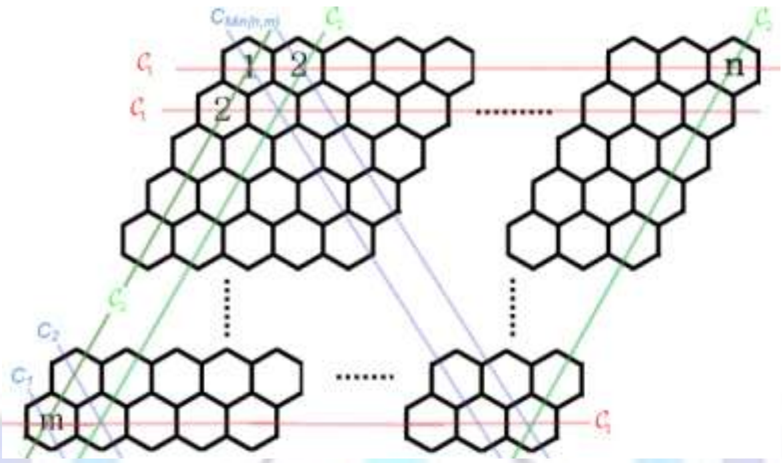

Fig 1. A general representation of the linear parallelogram *P(n,m)* $\forall n,m \in \mathbb{N}$.

Proof. Let *G* be the linear parallelogram *P(n,m)*, with *2mn+2m+2n* vertices and *3mn+2n+2m-1* edges. By using the following tables and figure of *P(n,m)*, the proof is easy. So, for all integer numbers *n≥m* and from Table 1 we have

$$\forall n \geq m \in \mathbb{N},\ \Omega(P(n,m),x) = \sum_c m(P(n,m),c)\,x^c$$

$$= mx^{2n+1} + nx^{2m+1} + \sum_{i=1}^{m-1} 2x^{i+1} + (n-m+1)x^{m+1}$$

$$= 2x^2 + 2x^3 + \ldots + 2x^m + (n-m+1)x^{m+1} + mx^{2n+1} + nx^{2m+1}$$

Table 1. The number of co-distant edges of the linear parallelogram *P(n,m)*.

quasi-orthogonal cuts	Number of co-distant edges	No
C_1	m	*2n+1*
C_2	n	*2m+1*
$C_i \forall$ *i=1,…,Min{n,m}-1*	2	*i+1*
$C_{Min\{n,m\}}$	*|n-m|+1*	*Min{n,m}+1*

On other hands, from Table 1, $\forall n \leq m \in \mathbb{N}$:

$$\Omega(P(n,m),x) = mx^{2n+1} + nx^{2m+1} + \sum_{i=1}^{n-1} 2x^{i+1} + (m-n+1)x^{n+1}$$

$$= 2x^2 + 2x^3 + \ldots + 2x^n + (m-n+1)x^{n+1} + mx^{2n+1} + nx^{2m+1}$$

and this completes the proof.■

Lemma 1. The Omega polynomial of the linear parallelogram *P(n,n)*, $\forall n \in \mathbb{N}$, is equal to

$$\Omega(P(n,n),x) = 2x^2 + 2x^3 + \ldots + 2x^n + x^{n+1} + 2nx^{2n+1}$$

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