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Didactic Strategy to Introduce the Concept of Punctual Continuity with Pre-University Students

Armando Morales Carballo[1], Angie Damián Mojica[1],

[1]Universidad Autónoma de Guerrero, México.

armando280@hotmail.com, adamian@uagro.mx

Abstract

This paper describes a didactic strategy to introduce the concept of continuous function in pre-university students, based on functions defined in pieces. The theoretical-methodological elements that made possible the exploration design fall in the records of semiotic representation, the formation, and definition of concepts and in the didactic functions of the class. As a result of the experimentation, it was found that the didactic strategy favored in the students the identification of the conditions that guarantee the continuity of a function in a point and under this scheme the step to the classical definition.

Keywords: Didactic Alternative, Punctual Continuity, Piecewise Function.

1. Introduction

The theme of continuous functions is a content framed within the Differential and Integral Calculation that is studied at the pre-university and university level (at least in Mexico), as well as the concept of limit, derivative, integral among other contents; Continuity is of the utmost importance for the development of mathematical analysis and more branches of advanced mathematics.

This paper describes a didactic strategy to introduce the concept of continuous function in pre-university teachers, based on functions defined in pieces.

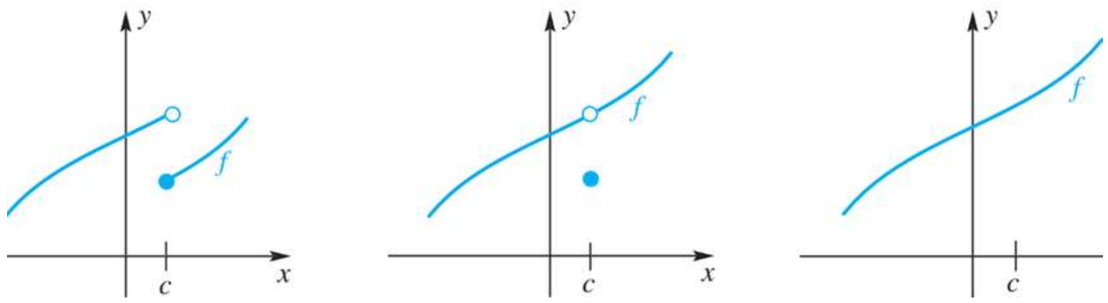
The theoretical-methodological elements that made possible the exploration design fall in the records of semiotic representation, the formation, and definition of concepts and in the didactic functions of the class.

Timely continuity in textbooks. At the upper and upper-middle level, the issue of punctual continuity is usually worked out after the limit content. The presentation of this content in the texts (Piskunov (2004); Spivak (2003); Stewart (2012); Swokowski (1988), Granville (2009); Purcell, Varberg & Rigdon (2007); Edwards & Penney (1997) and Anfossi & Flores (2003), among others) is based on an intuitive idea of punctual continuity, the definition is subsequently proposed, then the properties and examples are given, as described by the following examples:

For example, Granville (2009) begins the continuity treatment by studying the following functions: $y = x^2$ y $y = \frac{1}{x}$. It shows the graph of each of them, and from that representation identifies for which values of the independent variable the functions are continuous, continuity is justified from identifying the fields of existence (domain) of each of the functions (an approach intuitive). He then establishes the definition of a function's limit, its properties and its exemplification, then proposes to demonstrate the validity of the following limit: $\lim\limits_{x \to 2}(x^2 + 4x) = 12$. The test is based on the fact that x^2 like $4x$ are defined in $x = 2$, therefore, the limit is determined by substitution. In the exercise, it is observed that the limit coincides with $f(2) = 12$, therefore the author states that the function is continuous at the point $x = 2$.

With the particular analysis performed, introduce the definition of the continuous function at one point emphasizing the condition $\lim\limits_{x \to a} f(x) = f(a)$, that is, it states that if the condition is not met, then there is a discontinuity at the point. Finally, analyze discontinuities of the removable type.

Purcell, Varberg, and Rigdon (2007) initiate the issue of continuity through the analysis of the following graphs.



$\lim_{x \to c} f(x)$ don´t exist

$\lim_{x \to c} f(x)$ exist, but

$\lim_{x \to c} f(x) \neq f(c)$

$\lim_{x \to c} f(x) = f(c)$

Figure 1. Source: Purcell, E. J., Varberg, D., & Rigdon, S. E. (2007)

In these graphs, the authors analyze the meaning of the existence of the limit, and of the definition of the function in the value x = c. Subsequently, it establishes the definition of punctual continuity of a function and in said definition, identifies the following conditions: definition of the function in the study value, the existence of the limit when the independent variable has the study value, and the relationship of equality between the limit and the value of the function. It is established that if any of the conditions are not met, then there is no continuity, with the given observation, it analyzes cases of removable and non-removable discontinuity.

Stewart (2013) directly establishes the definition of continuity, and from it, highlights the fulfillment of three conditions: definition of the function at the point, existence of the limit, and equality of the limit value with the value of the function in the study value Subsequently, it exemplifies the concept, for this, it analyzes a graph with the purpose of justifying the discontinuity (from identifying that the three conditions given in the formulation of the definition of continuity of a function at a point are not met), see next figure.

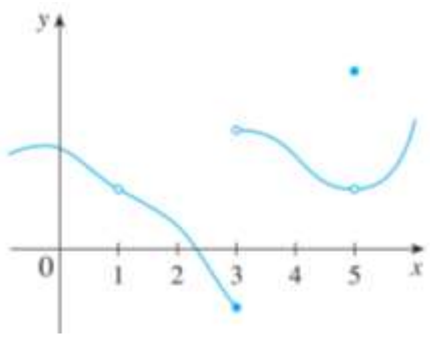
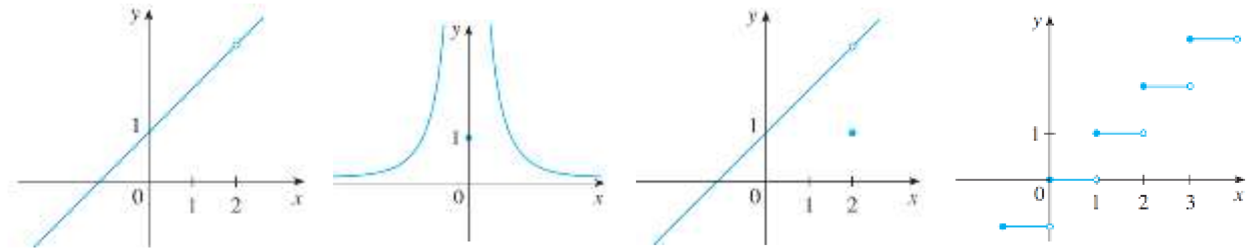


Figure 2. Source: Stewart, J. (2013).

Finally, he proposes to study four functions from the algebraic and graphic vision, with the aim of exemplifying the removable, jump, and infinite discontinuities. The following figure shows the graphs and associated functions.



a) $f(x)=\frac{x^2-x-2}{x-2}$ b) $f(x)=\begin{cases}\frac{1}{x^2}, & x\neq 0\\ 1, & x=0\end{cases}$ c) $f(x)=\begin{cases}\frac{x^2-x-2}{x-2}, & x\neq 2\\ 1, & x=2\end{cases}$ d) $f(x)=[\![x]\!]$

Figure 3. Source: Stewart, J. (2013).

In the presentation of the continuity identified in the texts that have been exemplified, the following order can be observed in its treatment: an intuitive idea of the concept, through graphic and analytical approaches with elementary functions, definition, exemplification and the study of some types of discontinuity It is clear that the presentations studied here, show that the objective that is generally raised in these texts is the presentation of knowledge as a finished product, the activity-oriented towards analysis, identification of conditions and properties associated with objects is scarce Study mathematicians, as is the case with continuity at one point.

Research in the field of Educational Mathematics. Sierra, González & López (2000) carry out an investigation with the purpose of contributing to the following objective: to discover the conceptions that students have about the concepts of limit and continuity, and find the possible relationships between them and the conceptions that are reported to the Over the years in other investigations. To do this, they carry out an exploratory study based on two questionnaires with specific questions about the concepts of interest. A pre-questionnaire answered by 80 students was previously prepared. The questionnaires were in different forms of representation and were answered in the exploration by 145 students.

The situations raised in the questionnaires were open so that students could freely express their ideas about both concepts. It should be noted that this questionnaire was applied weeks after the students received classes on the contents.

The results showed that there are difficulties in understanding the concepts even after their teaching, this was denoted by the wrong answers and justification. These activities proved the difference between the definition of concepts and the conceptions that students have. The analysis of the questionnaires was carried out using the SPSS statistical program, which they separated into two parts; a quantitative and qualitative analysis. Simultaneously, they studied the historical development of the concepts of limit and continuity. This analysis fulfilled several functions: It showed that the notions of limit and continuity are not developed in isolation but in connection with others, showing the context of problems in which both concepts have appeared, allowed to identify that the development has not been linear, but with advances, setbacks, indecisions.

Sierra, González & López (2003) carry out a historical study about the concept of continuity in high school and University Orientation manuals of the last 50 years, for this, they considered the dimensions: conceptual, didactic-cognitive and phenomenological. As main conclusions, the authors emphasize that the concept of continuity has undergone an evolution from considering it linked to the concepts of function and limit until reaching its own identity.

- In the first period, initially (until the fifties) definitions are given in which the concept of variable and function is mixed, and the language of increments is used to define continuity and, subsequently, with the clarification of the concept of limit, a precise definition of continuity is given.

- During the period of modern mathematics, almost all authors previously present an intuitive idea of continuity, and subsequently, it is defined from the limit; the emphasis is placed on a formal presentation and the topological definition is used.
- In the period 1975-1995 the previous trend is broken. Some authors begin with continuity and continue with the limit, and others mix both concepts. In addition, the metric definition of continuity is incorporated, in the last phase of this period, emphasis is placed on intuitive presentation relating it to situations of daily life and natural phenomena.
- Finally, in the last period, perhaps due to the reduction of the baccalaureate to two years (in the country of Spain), there is a certain “condensation” of the contents and, in particular, of the continuity, with a tendency on the part of most authors, to give a lot of information in a small space.

According to Hernández & Torres (2014), the classic, metric, topological, and geometric presentations appear in the description of the evolution of the concept of continuity. For example, they emphasize that in Apostle (1980), the following development is identified: classical and metric definition, in Spivak (2003) an intuitive idea of the concept, classical and metric definition.

Delgado (2013) studied the misconceptions in mathematics teachers regarding the concept of continuous function at one point based on inappropriate images. First, it analyzes the meanings about the continuity of functions that are usually raised in Mathematical Analysis: punctual, global, uniform continuity, among others.

The study indicates that in the set of erroneous situations that generate a didactic difficulty in students, the study of the teacher's conceptions cannot be ruled out. Some professors use a concept-bearing image that influences generating problems in students: That global continuity is identified in an environment of a point and a corresponding image, such as continuity in a point.

This identification justifies the way in which they present the content of continuity in the texts that are usually worked in the Baccalaureate and higher level, at least in Mexico. From the conceptual point of view, in the books that are currently used for work with the concept of continuous function, the intuitive vision, and its exemplification are prioritized at the pre-university level, although concepts such as the limit are identified in the treatments, function, field of existence, among others, only these concepts are incorporated to define continuity but there is no attention to the analysis of the role they play in guaranteeing it.

Gatica, Maz-Machado, May, Cosci, Echeverría & Renaudo (2010) carry out an exploratory and descriptive study on the difficulty presented by university students of Economic Sciences before the task of determining the continuity of a function when it is defined in pieces. For this, they used a written test with an algebraic record. The work used as representation the representation records.

One of the activities discussed consisted of: Given the function $f(x) = \begin{cases} x^2, x \leq 3 \\ 2x+1,\ x > 3 \end{cases}$

1. Perform the graphical representation of the function.
2. Indicate if the function is continuous.
3. If it is not continuous, the type of discontinuity it presents.
4. If possible, redefine the function to be continuous.

In this regard, students were asked to perform the graphical representation of the given function, this activity is in order to identify the type of difficulty they present when converting the algebraic record to the graph with this type of functions, those defined in pieces and, if any, corroborate what has been sustained in other investigations about the difficulties of making this change.

After carrying out the activity indicated in subparagraph a), it was identified that only 49.9% took into account the discontinuity of the function, although not all of them plotted correctly. Regarding the analysis of continuity b) 7.9% answered correctly in this subsection, which contrasts with the answer in subsection a), therefore there is an inconsistency between the graphic representation and the analytical interpretation.

Regarding the type of discontinuity, subsection c) 21% correctly indicate the type of discontinuity (essential), of that percentage, only 10.5% made the correct justification. In relation to subsection d), although it is an essential discontinuity, 7.6% of the students redefine it as if it were a function with avoidable discontinuity, this is evidence that the students have not understood the concept. With this study, it is identified that there is a low level of conceptual understanding of the continuity of a function, for the graphing and to identify the type of discontinuity, among others.

Pantoja & Ortega (2016) developed and applied a didactic proposal with activities supported in the use of software for learning the concepts of limit and continuity in students of Mathematics Degree at the Autonomous University of Nayarit (UAN). They used the clinical interview as an option to investigate such learning.

The proposal included situations aimed at highlighting the conceptual work on the operation, this in order to contribute in cognitive processes for the appropriation of the concepts under study. The activity was applied to fourteen students of the Basic Trunk of Area (TBA) of the first semester, who for the first time studied the subject of Differential Calculation, this experimental phase was structured by eight sessions.

After the experimental activities, the clinical interview was applied to four students (A1, A2, A3, and A4) who were the ones that most responded to the activities that were promoted, regarding the notion of continuity the following was obtained: A1 a From the graphing activity, he identified where discontinuities occur (it is observed that in this student the idea of continuity prevails as that action that consists in crossing the curve without taking off the pencil from the paper), A2 had difficulties in the algebraic reading of the functions and in its graphing, which prevented responding about the continuity or discontinuity of functions such as: $f(x) = \begin{cases} x-1, x>0 \\ -x-1, x<0 \end{cases}$, And of the type of discontinuity. A3 presented difficulties in reading functions with absolute value (only with the use of the software recognizes the type of functions). Regarding the notion of continuity, the following response was presented: there is continuity of function when the limit is not interrupted, when it is continuous, it is followed.

It can be seen that of the fourteen students, only two approach an intuitive idea of the notion of continuity, and the discussion about mathematical conditions to ensure continuity (punctual and global) is absent, both in the design and in the production of answers.

Millspaugh (2006) states that in most students, there is a disconnect between what they think is a continuous function and the formal definition given to them. The researcher emphasizes that students are left with the concern if the definition has something to do with the limit of a function at a particular point. To influence this problem, he proposed a proposal based on the fact that continuous functions satisfy the property of the intermediate value and the property of the extreme value, therefore, an approach towards understanding the definition was proposed based on the study of following problems:

1. The low-temperature yesterday was -17º and the high temperature was 6º, can you be sure that yesterday was exactly 0º at some point?

2. Our women's basketball team won 84 to 67 last night. Can you be sure that they had exactly 40 points at some time during the game (difference)?

From the discussion on the issues, the author concludes that the meaning of continuous function is favored from the fulfillment of the intermediate value theorem, which motivates to investigate later the graphic behavior of the theorem. Finally, it establishes that the theorems of average value and extreme value favor the connections of several relatively simple intuitive concepts, which are related to the concept of continuity.

Lynne (2013) states that both students and teachers are familiar with the difficulty of learning and teaching the concepts of limit and continuity. The research asked how students think about these concepts, including different conceptions and metaphors that students use when reasoning about the continuity and limit of a function at one point. As a result of the literature review, the researcher reports four potentially problematic conceptions (PPC) that students can use when reasoning about continuity and limit.

1. If the function is defined at one point, it is continuous at that point.

2. The limit value is equal to the value of the function.

3. If the limit exists at one point, the function is continuous at that point.

4. If the function takes limit at one point, the function must be discontinuous at that point.

Questionnaires were designed and applied to 861 students of the Brigham Young University-Provo (BYU) in several Mathematics courses to determine how prevalent and persist the PPCs among the students. In addition, in each course interviews of students of the first semester were conducted, in order to investigate how it influences if they use the PPC, the students showed evidence of having the four PPCs with a decrease in these conceptions after having an analysis course.

Students were able to reason properly using many different conceptions of continuity, when students use a dynamic conception of the limit they tend to improve their reasoning about continuity, students who used the dynamic conception of limit tended to use the PPC in general incorrectly, As a consequence of calculating the substitution limit, students have to apply this conception to functions with removable discontinuity. Another conception is that when studying a function defined by parts, observe that the values do not coincide when evaluating the function at a given point. Other conceptions that emerged are the following: To investigate continuity, students proceed first, to investigate discontinuity.

Pešić & Pešić (2015) point out that difficulties regarding the understanding of punctual continuity may be due to the abstraction of the concept or to the formal form of the presentation of the concept. These statements are the product of the elaboration and staging of the following proposal. First, they inquired about the initial ideas that students have about the concept, then they carried out the treatment of the concept and its definition through the $\varepsilon - \delta$ model, after the treatment of the definition they analyzed all the parties involved in the definition and of they highlight that those students have difficulties with the symbols ε y δ, with the quantifiers $\forall, \exists$, with the inequality: $|x - x_0| < \delta$ y $|f(x) - f(x_0)| < \varepsilon$.

The notion of function continues in pre-university students. In the application of a questionnaire on the continuity of a function a point to 15 students of the pre-university level of different educational subsystems of upper-middle level of the state of Guerrero, Mexico: Center for Industrial Technological and Services Baccalaureate (CBTis), National College of Technical Professional Education (CONALEP), College of Scientific and Technological Studies of the State (CECyTE), College of High School (COBACH) and Preparatory Schools of the Autonomous University of Guerrero (UAGRO) it was identified that in students only an intuitive idea of said concept, by asking them to describe the conditions that guarantee punctual continuity; the ignorance of such conditions and the misuse of those that they considered condition continuity were identified, but which are not such from the mathematical point of view. The difficulties encountered in the students have been identified in the researches that have been documented.

In order to contribute to the understanding of this concept, the following research problem was addressed: How to favor the introduction of the concept of punctual continuity through the study of the functions defined in strokes in pre-university students?

2. Theoretical - methodological elements

2.1. Semiotic Representation Records

According to Duval (1996), representations are fundamental to the understanding of mathematical concepts and mathematical properties, since their objects of study are constructions of the mind, and representations are required to interact with them. The author states that the representations are semiotic representations and these are defined as those in which production cannot be done without the mobilization of a semiotic system: thus semiotic representations can be discursive productions (in natural language, informal language) or not discursive (figures, graphs, schematics, etc.). Thus, for a semiotic system to be a representation register, it must allow the three cognitive activities of the apprehension or production of a semiotic representation: Formation, treatment and conversion.

The formation of an identifiable representation as a representation of a given record: statement of a sentence (understandable in a given language), elaboration of a text, design of a geometric figure, elaboration of a scheme, writing of a formula, among others.

The treatment of a representation is the transformation of this representation in the same register in which it has been formed. The treatment is an internal transformation to a registry. For example, the calculation is a form of treatment typical of symbolic writings (numerical calculation, algebraic calculation, propositional calculation, among others). Reconfiguration is a particular type of treatment for geometric figures: it is one of the many operations that give the register its heuristic role.

Conversion is the transformation of this representation into a representation of another record, retaining all or part of the content of the initial representation only. The conversion is an external transformation to the starting record (the representation record to be converted). Duval establishes that the student begins to understand a mathematical concept or content, when he is able to identify it in his different representation records and when through an appropriate transformation, he changes from one record to another.

2.2. Didactic functions of the class (FD)

The FD refers to the division of the class into partial didactic steps with very sharp functions that allow the teacher to provide a better teaching and the student a better learning:

Assurance of the level of departure. The assurance of the level of departure consists in the creation of the necessary prerequisites that students must possess in order to successfully face the new subject complex. This didactic step covers three aspects, the teacher must determine what knowledge the student needs, be convinced that they really have them, and reactivate certain necessary knowledge.

Goal orientation and motivation. The student (person who learns) should be directed to the anticipated affection of his activity. Regarding the orientation towards the objective, three important aspects must be taken into account: what is being done? What is it being done for? And why is it being done? For this, the teacher must have a good command of the objectives of the program.

The motivation of the student is understood as the intention of provoking in the student the conscious and desired realization of an activity and the motivation of the object as the motivation to deal exactly with this object, after having noticed that the knowledge they have in that At the moment they are not enough to solve this object.

I work with the new subject. It is important that students have the possibility to participate actively in the elaboration of the content. Therefore, three important aspects must be taken into account: Students must develop mental representations, concepts, judgments and conclusions must be formed, the capacity for the application of knowledge and power in practice must be developed.

Control. It is everything that provides information to the teacher and the student, about the teaching results. The control has a double function, provides the teacher with information on the successes and failures of his teaching work (important basis for the planning of teaching and a precondition for taking measures for individual work with students) and makes the students see students what level they have achieved in relation to the objectives to be achieved (willingness to learn is stimulated, discover existing difficulties, show how to increase efforts).

2.3. Development of concepts and definitions

The introduction of a concept presupposes that students know the characteristics of this concept, but not an implicit definition of it, however, the definition requires a thorough knowledge of the object to be defined. The elaboration of the concepts and their definition travels, basically, through the following phases (Ballester, 1992): Considerations and preparatory exercises. Through these students become familiar with corresponding phenomena and ways of working, and then be able to relate to the concept of the ideas that are acquired about the content. Concept formation. The part of the process is under. Assimilation of the concept. In this phase, the exercise, deepening, systematization and application are done, and the concept is reviewed, through mental and practical actions aimed at this objective.

Concept assimilation process. The student must perform the following actions: Approach, formalization, identification, and application of the concept (Morales, 2013). In the approach to the concept (recognize the concept from the intuitive point of view) in its different representations, it is necessary to operate with it in order to differentiate it with other concepts, establishing specific differences. Formalization of the concept. It consists of the whole process of formation to the definition of the concept, methodologically the didactic functions of the class are used and concrete activities are proposed, in each one of them; specific at the level of departure, motivation, orientation towards the objective, execution, control and assessment. Concept identification. In this phase, the concept is recognized based on its necessary and sufficient characteristics, in its different contexts (intra-mathematical and extra-mathematical). The activities that are worked on in this phase are: Identify other fields of application of the concept.

3. Method

The research is of a qualitative nature with a descriptive nature, and aims to provide elements for the planning of education of induction of the concept of punctual continuity in pre-university teachers based on the approach that allows the study of the functions to pieces.

3.1 The didactic strategy

It is composed of eight activities, which was validated with a group of seven pre-university students. Initially the design only contemplated five activities; which were enriched to favor the understanding of the concepts associated with punctual continuity and the conditions that guarantee this determination.

3.2 Activity system and experimental population.

The activities indicated below were experimented with a population of 30 students of different educational subsystems of the upper-middle level in the state of Guerrero, the selection of the population was carried out within the framework of the 2019 Olympiad process in which they were convened to second-year baccalaureate students interested in participating, in this process the best academic averages were not chosen, so the experimental population was heterogeneous.

3.3 Dynamics of experimentation

To carry out the activities, 15 teams of two members each were formed, each of the activities lasted 15 minutes, the development was carried out by activity, this in order not to influence the preparation of the responses.

Activity 1. Analyze the solution of the following limits and perform the activities indicated in each case:

1. Determine the following limit: $\lim_{x \to 3^-} \left(\frac{1}{3x+1}\right)$.

2. Be $f(x) = \begin{cases} 2x+1, x \leq 1 \\ \frac{1}{3}x^2 + \frac{8}{3}, \quad x > 1 \end{cases}$. The graph of the previous function is shown below.

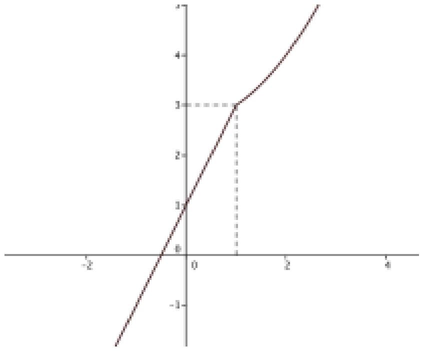


Figure 4. Source: Own elaboration

1. If you consider the values of $x \leq 1$, and increasingly closer to 1. How is the behavior of the expression $2x + 1$? Argue your answer.

2. If you consider the values of $x > 1$, and increasingly closer to 1. How does the expression $\frac{1}{3}x^2 + \frac{8}{3}$? Argue your answer.

3. Solve the following limits and compare the solutions: $\lim_{x \to 1^-}(2x+1)$, $\lim_{x \to 1^+}\left(\frac{1}{3}x^2 + \frac{8}{3}\right)$.

4. What can be said about the limit of a function, when the lateral limits exist as real numbers but are different?

5. If a given function f (x) is not defined in x = a, then can the $\lim_{x \to a} f(x)$? Argue your answer.

6. ¿What can be a condition to guarantee the existence of the limit of a function at a point?

Activity 2. Draw what is continuous for you.

Activity 3. Analyze the following graphs that correspond to the given functions and perform the activities indicated in each case.

$f(x) = \begin{cases} \sqrt[3]{x}, x \leq 0 \\ x^2, \quad x > 0. \end{cases}$

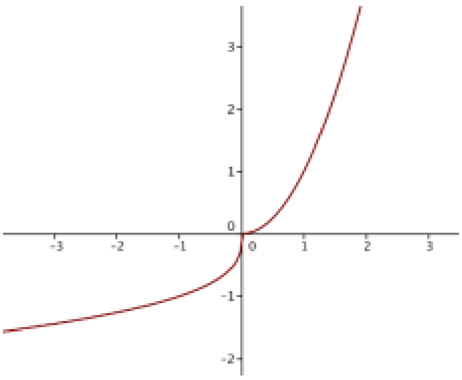


Figure 5. Source: Own elaboration.

1. Is it possible to traverse the graph without taking off the pencil from the paper? Argue your answer.
2. Is the given function defined in = 0? What is the value of the function at that point?
3. Determine the limits on the left and right of zero of the given function.
4. What can be said about the limit of the function at x = 0 and the value of the function at that point? Argue your answer

a) $f(x) = \begin{cases} \sqrt[3]{x}, x < 0 \\ x^2, \ \ x > 0. \end{cases}$

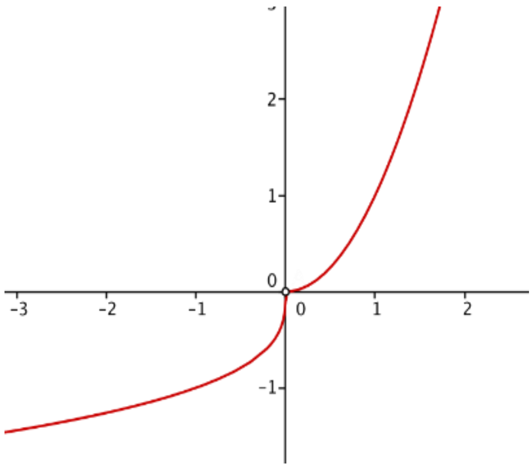


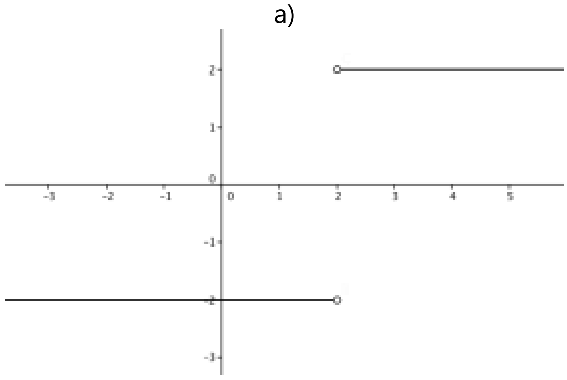
Figure 6. Source: Own elaboration.

5. Is it possible to traverse the graph without taking off the pencil from the paper? Argue your answer.
6. Is the given function defined in = 0? What is the value of the function at that point?
7. Determine the limits on the left and right of zero of the given function.
8. What can be said about the limit of the function at x = 0 and the value of the function at that point? Argue your answer.

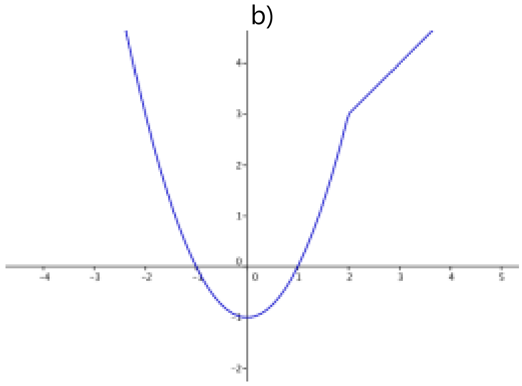
Activity 4. Describe in your own words what you understand by continuity of a function at one point.

Activity 5. Below is a table with different graphs and whose functions are indicated according to the paragraph, analyze these graphs, and mention in which cases there is continuity at the indicated points and in which cases there is no continuity. Argue your answer.

a)



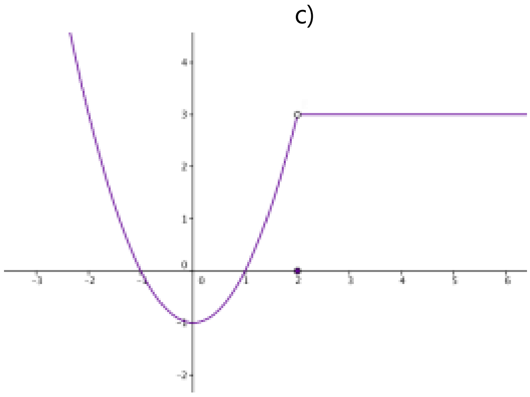
b)



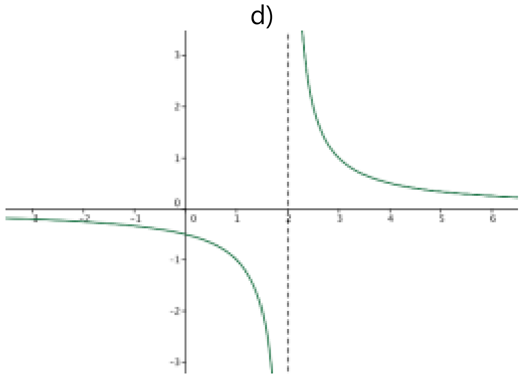
a) $f(x) = \begin{cases} 2, x > 2 \\ -2, x < 2, \end{cases}$

b) $g(x) = \begin{cases} x^2 - 1, x \leq 2 \\ x + 1, x > 2, \end{cases}$

c)



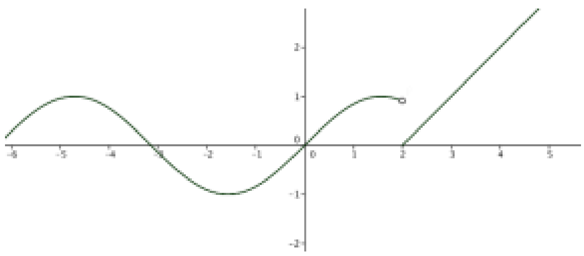
d)



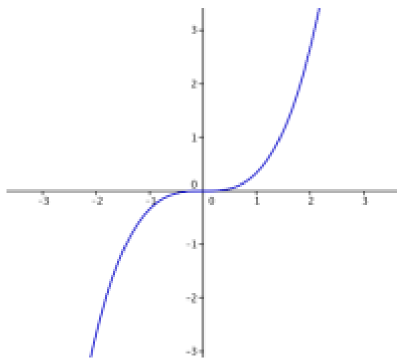
c) $f(x) = \begin{cases} x^2 - 1, x < 2, \\ 0, \quad x = 2, \\ 3, x > 2. \end{cases}$

d) $g(x) = \frac{1}{x-2}$.

e)



f)



e) $f(x) = \begin{cases} sen\ (x), x < 2 \\ x - 2, \quad x \geq 2. \end{cases}$

f) $g(x) = \frac{1}{3}x^3$.

Figure 7. Source: Own elaboration.

Activity 6. What conditions fulfill the functions that you determined to be continuous at the point x = 2?

Activity 7. Establish a definition of continuity of a function at one point.

Activity 8. Propose a piecewise function that meets any of the following conditions:

1.

a) Be defined in $x = \text{a}$.

b) There are the lateral limits as real numbers on the right and on the left of the indicated point, and that these are equal.

2.

a) Be defined in $x = a$.

b) That there is only one lateral limit.

c) That the value of that lateral limit that exists is equal to the value of the function at $x = a$.

Send the proposal to a colleague and ask him or her to determine whether or not there is punctual continuity.

4. Analysis and general conclusions

The following table shows the total correct answers to the activity system that structured the proposal.

<table>
<tr><td colspan="8">Activity 1</td></tr>
<tr><td rowspan="2"></td><td>P. 1.</td><td colspan="6">P. 2.</td></tr>
<tr><td></td><td>a)</td><td>b)</td><td>c)</td><td>d)</td><td>e)</td><td>f)</td></tr>
<tr><td>Number of correct answers per team</td><td>13</td><td>13</td><td>13</td><td>13</td><td>13</td><td>13</td><td>13</td></tr>
<tr><td colspan="8">Activity 2</td></tr>
<tr><td>Some answers</td><td colspan="7">The water cycle, straight and curved roads, time, space, earth movement around the sun, spirals, among other illustrations.</td></tr>
<tr><td>Number of correct answers per team</td><td colspan="7">15</td></tr>
<tr><td colspan="8">Activity 3</td></tr>
<tr><td></td><td colspan="3">a)</td><td colspan="3">b)</td></tr>
<tr><td>Number of correct answers per team</td><td>13</td><td>13</td><td>13</td><td>13</td><td>13</td><td>13</td></tr>
<tr><td colspan="8">Activity 4</td></tr>
<tr><td>Number of correct answers per team</td><td colspan="7">12</td></tr>
<tr><td>Some answers</td><td colspan="7">1. The function is defined at each point of its domain, therefore the graph may have no interruptions, in this case, there will be continuity.</td></tr>
</table>

	2. If there is continuity at one point, there is always image, and limit. 3. Walk a curve without lifting the paper-pencil.

Activity 5						
	a)	b)	c)	d)	e)	f)
Number of correct answers per team	14	14	14	12	14	14

Activity 6	
Number of correct answers per team	13
Some answers	1. The function is defined in the point, the lateral limits exist and are equal, the value of the limit coincides with the value of the function. 2. The function is defined at each point of its domain, therefore the graph may have no interruptions, in this case, there will be continuity.

Activity 7	
Number of correct answers per team	12
Some answers	1. A function $f\ (x)$ is continuous at the point $x\ =\ a$, if there are the lateral limits on the left and on the right of a, and they are equal. 2. A function$f(x)$ is continuous at a point $x = a$, if it is fulfilled that $\lim_{x \to a} f(x) = f(a)$.

Activity 8					
	i)		ii)		
Number of correct answers per team	a)	b)	a)	b)	c)
	13	13	12	12	12

Table 1. Analysis of the results

Activity 1 had the purpose of identifying whether students reconnect and use the notions of lateral limit, and if they establish that the existence and equality of these limits is a condition for continuity. In addition, the activity allowed us to identify whether the existence of the limit is clear in the students regardless of whether or not the function is defined at the indicated point. The items that have been proposed are intended to generate the minimum conditions to favor informal analyzes on the notion of continuity (a kind of starting level), in this regard it is observed that 13 teams achieved this goal, two more teams did not perform the full activity, and presented difficulties on the lateral limits.

Activity 2 was intended to identify which notion of continuity prevails or lives in students, the question was not restricted to the field of mathematics, since it was intended to know how the notion of continuity is conceived inside and outside it. This activity was proposed as a starting point for the search for conditions of continuity within mathematics. It was observed that the fifteen teams contributed the idea that on continuity tinen, it is interesting that their extra-mathematical notion, since it is considered that this notion influences the global conception of the concept to study punctual continuity and the difficulties that it entails.

Activity 3 sought to link the two previous activities, and focus on moving from the intuitive path of continuity (that of traveling a piece of the curve without lifting the pencil as a global conception of continuity) to the discussion about the mathematical elements and conditions that enable punctual continuity. From here, differentiate continuity at one point and continuity of a global type. In addition, the search for meanings was motivated: the function is defined at the point, it is not, the lateral limits exist or not, there are and are different, the relationship of existence of limit and that its value coincides with that of the function when the variable tends to the indicated point, among others. It was observed that 13 teams answered and argued correctly.

In activity 4, students are asked to establish a definition for the continuity of a function at one point, twelve answers that are considered correct were provided, in them, it was identified that in some teams the global conception prevails, and they use it to describe the sense of punctual continuity.

In activity 5 they tested that definition, in this case, different graphs associated with piecewise functions are presented in which it was requested to identify the punctual continuity, it was observed that 14 teams co-participated and argued correctly.

In activity 6, students were asked to establish what conditions are met to affirm that there is continuity at the indicated point. Here it was, that students recognize that only the existence of the limit, or only the definition of the function at the point of the study is not sufficient to establish continuity, in fact, it is from here that types can be generated of discontinuity and cases in which it can be avoided. In this regard, it was observed that 13 teams answered and argued correctly.

In activity 7, the student was required to rigorously formulate the concept of continuity, and it was identified that 12 teams provided an acceptable definition that guarantees the conditions of punctual continuity.

Finally, it was observed that in activity 8 the students in acceptable percentage put the definition they gave to the test, in the first activity 13 teams did it correctly, in the second 12 teams they provided a correct response

From the perspective of the authors of this work, it is considered that this proposal starts from the need to establish the conditions and give way to the formulation of the definition and not present the definition of the concept as a finished product.

The approach through the functions defined in pieces allows to identify the associated concepts such as limit, domain, graphic, among others and determine the conditions to guarantee punctual continuity.

This proposal breaks with the classic scheme of presentation of the content identified in the textbooks proposed at the pre-university level, and as a result of the validation and its experimentation, it is concluded that the presentation through the functions defined in pieces contributes to the student in expanding their possibilities of understanding the mathematical content of punctual continuity.

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