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The study about determining the reliability of operations management, risk warning and opportunity of repair risk

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Abstract:
In this paper, how to transform operations management and supply chain management, industrial engineering problems, project management, and other issues into a complex system reliability problem has been provided. It become possible that reliability of users demand is determined first and opportunity of choosing the repair the risk. This paper can provide a theoretical basis for research on related issues, which has a certain value.

Keywords :reliability; repairable system; adjustable repair-rate; risk of early warning

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1. Introduction

Strategy scholars, especially those that have resource-based view assert that the competitive advantage of enterprises depends largely on the thoughtful deployment strategy ability.

Operations management and supply chain management, industrial engineering problems, project management, and other issues can be transformed into a complex system reliability problem to solve.

About the operations research and optimization problem in the planning, the process can be divided into three stages:

Natural operation stage, form a conditional function [past tense] → Planning and optimization stage, establish the objective function [present] → Stage of implem-entation of the model [future]

Figure one

If using state words to describe them, the natural operational phase is “past tense”, planning and optimization phase are “present tense”, the implementation phase of the model is “future tense”. In fact, planning problems and optimization problems in operations research is planning and optimization for "now" and "future", that is to say , we haven’t planed and optimized for the first stage(natural operational stage). In terms of the needs of users, we cannot say "the most optimal" or "very satisfying".

The view of the complex system reliability, reliability is not necessarily meet the users' needs. We should take the reliability range from [0,1],that can reach from "not satisfied" to "completely satisfied".

2. Determine the reliability of operation and management

The common objective of operations management and supply chain management, industrial engineering problems, project management, and other issues is through process control and process management to achieve the best possible results during the course, this problem can be transformed into through the input-output calculation, complex system reliability problems during the course of the prior artificial reliability (availability).

The modernization of equipment has been largely improved along with the development of science and technology，so the maintenance and support of equipment become more and more complex. Allen and D'esopo[1] proposed the idea that the spare parts should be classified before the 1960s. Cohen[2] divided needs into urgent needs and ordinary ones. Moore[3] did the classification according the functions of spare parts. Because of the influence of spare parts to manufacture and economy, many scholars have studied the amounts of the spare parts needed. P.Flint[4] provided the advice that we should develop the fellowship and the resource sharing to reduce the cycle time. Besides, Foote[5] studied stocks prediction, and Luxhoj and Rizzo[6] obtained the method of amounts of spare parts needed of the same set based on the set model. Kamath[7] used the Bayesian method to predict the amounts of spare parts needed.

The following describes the two prior calculation methods to determine the reliability of the model and its aircraft demand, on the basis give the calculation method of the Joint reserve aircraft demand to meet the reliability.

2.1 “Airplane repairing” model and calculation of its demand of airplanes

Assume that reserve of airplanes of a certain new civil aviation system starts from the zero moment, assessment will be done every $a \quad a>0$. If this system is normal, we will continue reserving; otherwise, we will do the repairing, that is, after the temporary repairing which cost time $b \quad 0<b<a$, civil aviation system recovers normal and proceeds reserving.

If this temporary repairing process equals to previous system which is normal absolutely, then this is called “airplane repairing model".

For briefly expressing, state random variable X_t is introduced.

$$X_t = \begin{cases} 1\ , & system is\ normal\ at\ time t \\ 0\ , & system\ fails at\ time t \end{cases}$$

Let $F\ \ t$ be the distribution function of random variable Z which is the first failure time of civil aviation system. From the assumption of the model we can know, when $b=0$, availability of system at time t (probability of system is in the normal state at time t) $A\ \ t\ = P\ \ X_t = 1\ = 1 - F\ \ t$; when $b>0$, note $a_k = k \cdot a\ \ k = 0,1,\ldots\ldots$, $b_k = k \cdot a + b\ \ k = 1,2,\ldots\ldots$, $A\ \ t$ satisfies these formulas according to the model assumption:

When $a_0 < t \le a_1$, $A(t) = 1 - F(t)$ (1)

When $a_k < t \le b_k, (k > 1)$

$$A(t) = P(X_t = 1 | X_{ak} = 1) P(X_{ak} = 1) = \frac{1 - F(t)}{1 - F(a_k)} A(a_k) \quad (2).$$

When $b_k < t \le a_{k+1}\ \ k \ge 1$

$$\begin{aligned} A(t) &= P(X_t = 1, X_{ak} = 0) + P\ \ X_t = 1, X_{ak} = 1 \\ &= P(X_t = 1 | X_{ak} = 0) P(X_{ak} = 0) + P(X_t = 1 | X_{ak} = 1) P(X_{ak} = 1) \\ &= P(X_t = 1 | X_{bk} = 1) P(X_{ak} = 0) + P(X_t = 1 | X_{ak} = 1) P(X_{ak} = 1) \\ &= \frac{1 - F(t)}{1 - F(b_k)} (1 - A(a_k)) + \frac{1 - F(t)}{1 - F(a_k)} A(a_k) \quad (3) \end{aligned}$$

From (2) and (3) we can see that $A\ \ t$ can be obtained once we calculate all the $A\ \ a_k\ \ , k = 1,2,\ldots,\ \ t/a\ ^*$. Let $t = a_{k+1}$ in (3), when $k \ge 1$, we have:

$$A(a_{k+1}) = \frac{1 - F(a_{k+1})}{1 - F(b_k)} + \left[\frac{1 - F(a_{k+1})}{1 - F(a_k)} - \frac{1 - F(a_{k+1})}{1 - F(b_k)} \right] A(a_k)$$

According to the formula above and $A\ \ a_1\ \ = 1 - F\ \ a_1$, we can calculate $A\ \ a_k\ \ , k = 1,2,\ldots,\ \ t/a\ ^*$, and then $A\ \ t$ can be obtained.

Suppose there are N systems starting the reserve of airplane repairing from time $t=0$ at the same time on the same condition, besides, the state of these N systems is independent each other. Then minimum of probability that there are at least N normal regional sub-systems at the given moment during the reserve period is not less than P_0 is:

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$$M = \min\left\{ m; \sum_{j=N}^{N+m} \binom{N+m}{j} A(t)^j (1-A(t))^{N+m-j} \geq P_0, \atop 0 \leq t \leq T_0 \right\} \tag{4}$$

M is just the amount of the reserve of airplanes we needed.

2.2 "Airplane purchasing" model

Assume that reserve of airplanes of a certain new civil aviation system starts from the zero moment, assessment will be done every $a \quad a>0$. If this system is normal, we will continue reserving; otherwise, if it fails and can't be repaired, we will purchase new airplanes, that is, this purchasing process equals to replacing a completely same single-airplane sub-system, then this is called "airplane purchasing" model.

Let random variable Z be the life expectancy of reserve of airplanes, and Z~F(t), $R \quad t = 1 - F \quad t$ .Let Z_1 be the life expectancy of reserve of airplanes from $t=0$, Y_1 be the interval between first failure and first completeness of temporary training, $Z_j \quad j>1$ be the life expectancy of system through the $j-1th$ updating, Y_j be the internal between the $j\,th$ failure and completeness of the $j\,th$ training. From the training rules above we know:

$$\begin{cases} Y_1 = ([Z_1/a]^* + 1)a - Z_1 + b \\ Y_n = ([(Z_n+b)/a]^* + 1)a - Z_n, \qquad n = 2,3,\ldots. \end{cases}$$

$$\begin{cases} T_1 = ([Z_1/a]^* + 1)a + b \\ T_n = ([(Z_n+b)/a]^* + 1)a, \qquad n = 2,3,\ldots. \end{cases}$$

Obviously, Tj= Zj+Yj,j=1,2,…because Z2,Z3,…~F(t) are independent random variables,T2,T3,…are independent and have same distribution function. Note

$$S_n = \sum_{j=1}^{n} T_j \qquad N_D(t) = \max(n : S_n \leq t)$$

Then $N_D \quad t \quad t \geq 0$ is a renewal process.

To be convenient, will sign of involved are defined as follows:

Z_j life expectancy of reserve of airplanes of system through the $j-1th$ updating;

Y_j training time after the jth failure;

T_j $T_j = Z_j + Y_j, \ j = 1, 2, .$

G distribution function of T_1;

H distribution function of $T_j \quad j = 2,3,\ldots$;

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F distribution function of $Z_j \ \ j=1,2,...$;

S_j the jth update time ;

$N_D(t)$ average update times of system during period $0,t$;

Besides, note $a_k = k\cdot a \ \ k=0,1,......$ $b_k = k\cdot a + b \ \ k=1,2,......$, let $T_1 \sim G \ \ t$, $T_2 \sim H \ \ t$, and

$G = \ g_j, j=1,2,...$, $H = \ h_j, j=1,2,...$ then

$$g_1 = F(a_1), \quad h_1 = F(a-b)$$

$$g_j = P\{T_1 = b_j\} = F(a_j) - F(a_{j-1}) \qquad j=2,3,...,$$
$$h_j = P\{T_2 = a_j\} = F(a_j - b) - F(a_{j-1} - b) \quad j=2,3,...,$$

Availability of time t is dependent on availability of the last update time after t in the renewal process, it is necessary to study the distribution of $S_{N_D \ t}$.

Next we will calculate $G*H_n$ ($*$ denotes for convolution, H_n denotes for n-fold convolution of H, $n\geq 1$) which is distribution of random variable S_{n+1}. We can see that $G*H_n$ is a discrete distribution, and value space of S_{n+1} is $b_k : k=n+1, n+2,...$ .From $g(s)=\sum_{i=1}^{\infty} g_i s^i$ and $h(s)=\sum_{l=1}^{\infty} h_l s^l$ which are the probability generating function of G and H, respectively, we know that probability generating function of $G*H_n$ is $g(s)h^n(s)$ .Besides, from $G*H_n = \{v_j^{(n)}; j=n+1, n+2, \cdots\}$,we can obtain that probability generating function of $G*H_n$ is $\sum_{j=n+1}^{\infty} v_j^{(n)} s^j$, so $\sum_{j=n+1}^{\infty} v_j^{(n)} s^i = g(s)h^n(s)$

Compare terms coefficients about s above, the following iterative formula can be obtained:

$$\begin{cases} v_j^{(0)} = g_j, & j=1,2,\cdots, \\ v_j^{(1)} = \sum_{k=1}^{j-1} v_k^{(0)} h_{j=k}, & j=2,3,\cdots, \\ \vdots & \\ v_j^{(n)} = \sum_{k=n}^{j-1} v_k^{(n-1)} h_{j=k}, & j=n+1, n+2, \cdots, \end{cases}$$

From the assumption above we can see $Z_n, Y_n \ \ , n=1,2,...$ are a series of independent two-dimension random vectors, and they obey same distribution, then $Z_n, Y_n \ , n=1,2,...$ are delayed renewal process. Let $b_m < t \le b_{m+1}$,then $S_{N_D\ t}$ is the last update time during $0,t$ in the delayed renewal process From total probability

$$A(t) = P(X_t = 1, S_{N_D(t)} = 0) + \sum_{k=1}^{m} P(X_t = 1, S_{N_D(t)} = b_k)$$

$$\begin{aligned}
&Since \ \ P(X_t = 1, S_{N_D(t)} = 0) \\
&\quad = P(Z_1 > t, T_t > t) = P(Z_1 > t, Z_1 + Y_1 > t) \\
&\quad = P(Z_1 > t) = 1 - F(t) \\
&When\, k \ge 1\ P(X_t = 1, S_{N_D(t)} = b_k) \\
&\qquad = \sum_{j=1}^{k} P(X_t = 1, S_{N_D(t)} = b_k, N_D(t) = j) \\
&\qquad = \sum_{j=1}^{k} P(Z_{j+1} > t - b_k, T_{j+1} > t - b_k, S_j = b_k) \\
&\qquad = \sum_{j=1}^{k} P(Z_{j+1} > t - b_k, T_{j+1} > t - b_k) P(S_j = b_k) \\
&\qquad = \sum_{j=1}^{k} P(Z_{j+1} > t - b_k) P(S_j = b_k) \\
&\qquad = \sum_{j=1}^{k} (1 - F(t - b_k)) v_k^{(j-1)}
\end{aligned}$$

formula:

So $$A(t) = 1 - F(t) + \sum_{k=1}^{m} \sum_{j=1}^{k} v_k^{(j-1)} (1 - F(t - b_k))$$

2.3 Calculation of amounts of joint reserve of airplanes needed

Generally, an airplane repairing of civil aviations in different regions is carried out independently. However, considering the problem of reducing the costs as possible as we can on the condition that the joint reserve of airplanes model is satisfied, we propose a new training back way – joint reserve of airplanes, that is the method that amounts of joint reserve of airplanes needed are calculated uniformly after determining sum of civil aviations system in each company.

In the “airplane repairing” model, suppose there are $N_1 + M_1$ （M_1 is the amounts of airplanes needed） systems in company A, starting the reserve of airplane repairing from time $t=0$ at the same time on the same condition, besides, the state of these $N_1 + M_1$ systems is independent each other. Then minimum of probability that there are at least N_1 normal systems during the reserve period isn’t less than P_0 . That prior to determine the reliability of M minimum is:

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$$M_1 = \min\left\{m_1; \sum_{k=N_1}^{N_1+m_1}\binom{N_1+m_1}{k}A(t)^k(1-A(t))^{N_1+m_1-k} \geq P_0, 0 \leq t \leq T_0\right\}$$

At the same time, there are $N_2 + M_2$ systems meeting "airplane repairing", the conditions are the same as region A, then amounts of reserve of airplanes needed M_2 is:

$$M_2 = \min\left\{m_2; \sum_{k=N_2}^{N_2+m_2}\binom{N_2+m_2}{k}A(t)^k(1-A(t))^{N_2+m_2-k} \geq P_0, 0 \leq t \leq T_0\right\}$$

According the idea of joint reserve of airplanes, sum of these $N_1 + N_2$ systems can be treated as N joint reserve of airplanes systems, then amounts of airplanes needed M is:

$$M = \min\left\{m; \sum_{k=N}^{N+m}\binom{N+m}{k}A(t)^k(1-A(t))^{N+m-k} \geq P_o, 0 \leq t \leq T_o\right\}$$

3. Risk warning

The repairable system model we studied has three separate identical functional modules, one works, the other two is in a hot standby, when the working module fails, the hot standby module immediately into the work of the state. Two failures may occur in the work of the module: function module itself causes failure in the operation, routine failure (such as fires, earthquakes, etc.). The waiting module may also fails, when the waiting module fails, only one repair unit to repair the module, and the repair time is arbitrarily distributed. We study the numerical analysis about the warning systems and early warning system's reliability (availability). This system state's transition flow chart (Figure 2) as follows:

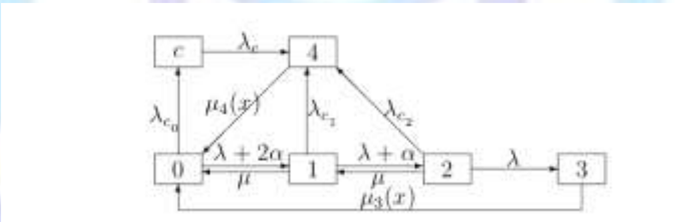

Figure two

Figure 2 symbols represent physical meaning

$i = 0$: A job, two hot standby state

$i = 1$: A work, a failure, a thermal reserve status

$i = 2$: A job, the two faulty state

$i = 3$: Three failure state

$i = 4$: The conventional fault of state

$i = c$: System lead to system failure due to the conventional fault, but the system can continue to run

λ : The rate damage caused by the running machine by itself reason

λ_c : General failure rate in the state i system

α : The thermal reserves machine damage rate

μ : Constant repair rate of the running machine

$\mu_j(x)$: Repair rate when the x moment the system is in state j

$p_j(t)$: The probability of t moment the system is in state j

$p_j(x,t)$: The probability of the t moment the system is in state j and repair time x

Fixes the corresponding mathematical model of the system with this (I) can integral - differential equations said

$$\frac{dp_0(t)}{dt} = -(\lambda + 2\alpha + \lambda_{c_0})p_0(t) + \mu p_0(t) + \sum_{j=3}^{4}\int_0^\infty p_j(x,t)\mu_j(x)dx$$

$$\frac{dp_1(t)}{dt} = -(\lambda + \alpha + \lambda_{c_1} + \mu)p_1(t) + (\lambda + 2\alpha)p_0(t) + \mu p_2(t)$$

$$\frac{dp_2(t)}{dt} = -(\lambda + \lambda_{c_1} + \mu)p_2(t) + (\lambda + \alpha)p_1(t)$$

$$\frac{dp_c(t)}{dt} = -\lambda_c p_c(t) + \lambda_{c_0} p_0(t)$$

$$\frac{\partial p_j(x,t)}{\partial x} + \frac{\partial p_j(x,t)}{\partial x} = -\mu_j(x)p_j(x,t) \qquad (j = 3,4)$$

Model initial and boundary conditions:

$$p_3(0,t) = \lambda p_2(t)$$

$$p_4(0,t) = \lambda_c p_c(t) + \sum_{i=0}^{2}\lambda_{c_i} p_i(t)$$

$$p_0(0) = 1, p_i(0) = p_c(0) = 0 (i = 1,2); p_j(x,0) = 0 (j = 3,4)$$

It is easy to show that the model is the problem about boundary value compatible with the initial conditions, and that proof system solution uniqueness, stability. [8]

We make $\lambda_c = \gamma\lambda$, then, that an early warning system to alert, repairing personnel not to repair or stop the system continues to work, that is, that the early warning system failed to play a role. Then the system will approach a new model, we call this model a non-warning system, as shown below. Here we discuss the early warning system and the reliability of the non-early warning system (availability).

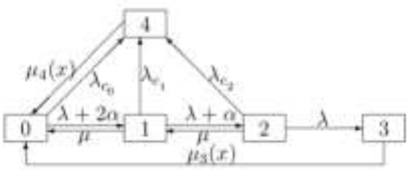

Figure 3

Make $a_1 = \lambda + \alpha + \lambda_{c_1} + \mu = 2\lambda + \mu + \alpha, a_2 = \lambda + \lambda_{c_2} + \mu = 2\lambda + \mu, a_3 = \lambda + 2\alpha$

$a_4 = \lambda + \alpha$

According to another papers[8] to get warning system reliability(availability)

$$\hat{A}_1 = \frac{\hat{p}_0 + \hat{p}_1 + \hat{p}_2 + \hat{p}_c + \hat{p}_3 + \hat{p}_4}{\hat{p}_0 + \hat{p}_1 + \hat{p}_2 + \hat{p}_c + \int_0^\infty \hat{p}_3(x)dx + \int_0^\infty \hat{p}_4(x)dx}$$

$$= \frac{1 + S + Q + \frac{1}{\gamma}}{1 + S + Q + \frac{1}{\gamma} + (2\lambda S + \lambda + \lambda Q)\frac{1}{\mu}}$$

Similarly by calculating the non-reliability of the early warning system (availability)

$$\hat{A}_2 = \frac{\hat{p}_0 + \hat{p}_1 + \hat{p}_2 + \hat{p}_c + \hat{p}_3 + \hat{p}_4}{\hat{p}_0 + \hat{p}_1 + \hat{p}_2 + \hat{p}_c + \int_0^\infty \hat{p}_3(x)dx + \int_0^\infty \hat{p}_4(x)dx}$$

$$= \frac{1 + S + Q}{1 + S + Q + (2\lambda S + \lambda + \lambda Q)\frac{1}{\mu}}$$

Among: $S = \frac{(2\lambda + \mu)(\lambda + 2\alpha)}{\mu(\lambda + \alpha) - (2\lambda + \mu + \alpha)(2\lambda + \mu)}$ $Q = \frac{(\lambda + 2\alpha)(\lambda + \alpha)}{\mu(\lambda + \alpha) - (2\lambda + \mu + \alpha)(2\lambda + \mu)}$

In order to discuss issues in convenience, make $\alpha = \lambda$,so the warning system reliability (availability)

$$\hat{A}_1 = \frac{1}{1 + \frac{1}{\mu}\frac{12\lambda^3 + 12\lambda^2 + 3\lambda^2\mu + \lambda\mu^2 + 3\lambda\mu}{18\lambda^2 + 6\lambda\mu + \mu^2 + \frac{1}{\gamma}(6\lambda^2 + 3\lambda\mu + \mu^2)}}$$

Non-warning system reliability (availability)

$$\hat{A}_2 = \frac{1}{1 + \frac{1}{\mu}\frac{12\lambda^3 + 12\lambda^2 + 3\lambda^2\mu + \lambda\mu^2 + 3\lambda\mu}{18\lambda^2 + 6\lambda\mu + \mu^2}}$$

To this end, we have the numerical analysis of the warning system and the non-reliability of the early warning system

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Table one

	λ	μ	γ	$\hat{A}_1$	$\hat{A}_2$	$Error$
1	0.1	1	0.01	0.996	0.759	0.3123
2	0.1	1	0.1	0.964	0.759	0.27
3	0.1	1	1	0.848	0.759	0.1173
4	0.1	1	10	0.773	0.759	0.0184
5	0.1	1	100	0.762	0.759	0.004
6	0.1	1	1000	0.76	0.759	0.0013
7	0.1	1	10000	0.759	0.759	0.000
8	0.1	1	100000	0.759	0.759	0.000
9	0.1	1	1000000	0.759	0.759	0.000
10	0.1	1	10000000	0.759	0.759	0.000

Table two

	λ	μ	γ	$\hat{A}_1$	$\hat{A}_2$	$Error$
1	0.5	1	0.01	0.952	0.539	0.8219
2	0.5	1	0.1	0.87	0.539	0.6141
3	0.5	1	1	0.633	0.539	0.1744
4	0.5	1	10	0.551	0.539	0.0223
5	0.5	1	100	0.541	0.539	0.0037
6	0.5	1	1000	0.539	0.539	0.000
7	0.5	1	10000	0.539	0.539	0.000
8	0.5	1	100000	0.539	0.539	0.000
9	0.5	1	1000000	0.539	0.539	0.000
10	0.5	1	10000000	0.539	0.539	0.000

According to the above numerical analysis shows, the reliability of the warning system and the reliability of the non-early warning system, when $\gamma \rightarrow \infty$, $\hat{A}_1 = \hat{A}_2$

Table three

	λ	μ	γ	$\hat{A}_1$	$\hat{A}_2$	$Error$
1	0.1	2	0.01	0.995	0.692	0.4379
2	0.1	2	0.1	0.956	0.692	0.3815
3	0.1	2	1	0.808	0.692	0.1676
4	0.1	2	10	0.709	0.692	0.246
5	0.1	2	100	0.693	0.692	0.0015
6	0.1	2	1000	0.693	0.692	0.000
7	0.1	2	10000	0.693	0.692	0.000
8	0.1	2	100000	0.693	0.692	0.000
9	0.1	2	1000000	0.693	0.692	0.000
10	0.1	2	10000000	0.693	0.692	0.000

These numerical simulation that when other parameters constant, with the $\gamma \to \infty$, two systems stability and reliability relative error tend to 0.

4. Opportunity of risk repair

The repairable system refers to the system that after the system failure, repair or replace one or more components to make system recover, select a four-state repairable operating system [9], this system's work flow chart and related symbolic significance in figure IV:

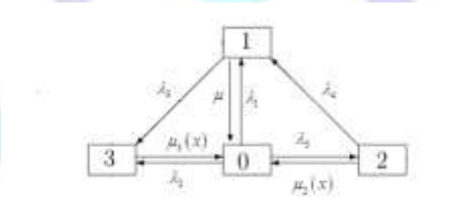

Figure four

$i=0$ Normal state: the normal function of the system; normal state: the normal function of the system;

$i=1$ Weak state: the functionality of the system to maintain (due to partial failure led to more than 70% reliability = 0.7);

$i=2$ Complete failure of the state: the system cannot run;

$i=3$ The catastrophic failure of the state: The system can not complete the operation;

λ_i The system constant failure rate $i=1,2,3,4,5;$

μ The weak state of constant repair rate;

μ_i x Repair rate in the state i of repair time $i=2,3;$

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$P_i(t)$ At time t, the system is in state i the probability (reliability) $i=0,1;$

$P_i(x,t)$ The system at time t in state i repair time x probability (reliability) $i=2,3;$

This model is described by differential - integral equation:

$$\frac{dp_0(t)}{dt}=-(\lambda_1+\lambda_2+\lambda_5)p_0(t)+\mu p_1(t)+\sum_{i=2}^{3}\int_0^\infty p_i(x,t)\mu_i(x)dx \qquad (4.1)$$

$$\frac{dp_1(t)}{dt}=-(\lambda_3+\lambda_4+\mu)p_1(t)+\lambda_1 p_0(t) \qquad (4.2)$$

$$\frac{\partial P_i(x,t)}{\partial x}+\frac{\partial P_i(x,t)}{\partial t}=-\mu_i(x)P_i(x,t) \quad (i=2,3 \qquad (4.3)$$

$$P_2(0,t)=\lambda_4 p_1(t)+\lambda_5 p_0(t) \qquad (4.4)$$

$$P_3(0,t)=\lambda_3 p_1(t)+\lambda_2 p_0(t) \qquad (4.5)$$

$$p_0(0)=1, \quad p_i(0)=0 \qquad (4.6)$$

In order to facilitate the calculation, make $a_0=\lambda_1+\lambda_2+\lambda_5$, $a_1=\lambda_3+\lambda_4+\mu$ 。

Suppose $0\le\mu_i(x)<\infty, \int_0^\infty \mu_i(\xi)d\xi=\infty,(i=2,3)$

References [10] has used elementary methods and C_0 - semigroup theory to prove the existence and uniqueness of a non-negative solution.

Reference [11] has proved the relationship between the transient reliability and firmly reliability of $P(t)\ge P_0^*=\lim_{t\to\infty}P_0(t)$, As long as further proof $P_0(t)$ is a monotone decreasing function of the reliability of the system is proved. Predecessors by the constant value assigned with the failure and repair rate draw $p_0(t)$ image obtained $p_0(t)$ is a monotonically decreasing function, $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda_5=\lambda$; $t\in$ $[0,t_0]$ suppose $\mu_2(x)=\mu_3(x)=\mu_1(\text{cont ent})$, if $\int_0^\infty p_i(x,t)dx=p_i(t)$, $i=2,3$;then $\sum_{i=0}^{3}p_i(t)=1$

The original differential - integral equation can be reduced to:

$$\frac{dp_0(t)}{dt}=-(a_0+\mu_1)p_0(t)+(\mu-\mu_1)p_1(t)+\mu_1 \qquad (4.7)$$

$$\frac{dp_1(t)}{dt}=-a_1 p_1(t)+\lambda p_0(t) \qquad (4.8)$$

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Abstract:

$$\begin{cases} \dfrac{d\overrightarrow{P(t)}}{dt} = A\overrightarrow{P(t)} + \overrightarrow{\mu_1} \\ \overrightarrow{P(0)} = (1,0)^T \end{cases} \tag{4.9}$$

Among $\overrightarrow{P(t)} = (p_0(t), p_1(t))^T$, $\overrightarrow{\mu_1} = (\mu_1, 0)^T$

$$A = \begin{pmatrix} -(a_0+\mu_1) & \mu-\mu_1 \\ \lambda & -a_1 \end{pmatrix}$$

Solution of the following requirements (4.9) , According to the literature [11],

$$\overrightarrow{P(t)} = e^{At}\overrightarrow{P(0)} + \int_0^{t_0} e^{A(t-s)}\overrightarrow{\mu_1}ds = e^{At}\overrightarrow{P(0)} - A^{-1}\overrightarrow{\mu_1} + A^{-1}e^{At}\overrightarrow{\mu_1} \tag{4.10}$$

First matrix A all eigenvalues γ_1, γ_2

$$\det(\gamma I - A) = \begin{vmatrix} \gamma+3\lambda+\mu_1 & \mu_1-\mu \\ -\lambda & \gamma+2\lambda+\mu \end{vmatrix} = 0$$

Solution was $\gamma_1 = -2\lambda-\mu_1$, $\gamma_2 = -3\lambda-\mu$

Literature [12],make $Q_0 = I$, $Q_1 = A-\gamma_1 I = \begin{pmatrix} -(a_0+\mu_1+\gamma_1) & \mu-\mu_1 \\ \lambda & -(a_1+\gamma_1) \end{pmatrix}$

$$q_1(t) = e^{\gamma_1 t} , \quad q_2(t) = \frac{e^{\gamma_1 t}-e^{\gamma_2 t}}{\gamma_1-\gamma_2}$$

then

$$e^{At} = q_1(t)Q_0 + q_2(t)Q_1 = \begin{pmatrix} e^{\gamma_1 t} & 0 \\ 0 & e^{\gamma_1 t} \end{pmatrix} + \frac{e^{\gamma_1 t}-e^{\gamma_2 t}}{\gamma_1-\gamma_2}\begin{pmatrix} -(a_0+\mu_1+\gamma_1) & \mu-\mu_1 \\ \lambda & -(a_1+\gamma_1) \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{e^{\gamma_2 t}(a_0+\mu_1+\gamma_1) - e^{\gamma_1 t}(a_0+\mu_1+\gamma_2)}{\lambda_1-\lambda_2} & \dfrac{(e^{\gamma_1 t}-e^{\gamma_2 t})(\mu-\mu_1)}{\lambda_1-\lambda_2} \\ \dfrac{\lambda(e^{\gamma_1 t}-e^{\gamma_2 t})}{\gamma_1-\gamma_2} & \dfrac{e^{\gamma_2 t}(a_1+\gamma_1) - e^{\gamma_1 t}(a_1+\gamma_2)}{\gamma_1-\gamma_2} \end{pmatrix}$$

And

$$A^{-1} = \frac{A^*}{|A|} = \frac{1}{(3\lambda+\mu)(2\lambda+\mu_1)}\begin{pmatrix} -(2\lambda+\mu) & \mu_1-\mu \\ -\lambda & -(3\lambda+\mu_1) \end{pmatrix}$$

Substituting (4.10), finishing

$$\vec{P} = \begin{pmatrix} \dfrac{2\lambda(\mu-\mu_1)}{(2\lambda+\mu_1)(\gamma_1-\gamma_2)}e^{\gamma_1 t} + \dfrac{\lambda(3\lambda+\mu-\mu_1)}{(3\lambda+\mu)(\gamma_1-\gamma_2)}e^{\gamma_2 t} + \dfrac{a_1\mu_1}{(3\lambda+\mu)(2\lambda+\mu_1)} \\ \dfrac{2\lambda^2}{(2\lambda+\mu_1)(\gamma_1-\gamma_2)}e^{\gamma_1 t} - \dfrac{\lambda(3\lambda+\mu-\mu_1)}{(3\lambda+\mu)(\gamma_1-\gamma_2)}e^{\gamma_2 t} + \dfrac{\lambda\mu_1}{(3\lambda+\mu)(2\lambda+\mu_1)} \end{pmatrix}$$

$$p_0(t)=\frac{2\lambda(\mu-\mu_1)}{(2\lambda+\mu_1)(\gamma_1-\gamma_2)}e^{\gamma_1 t}+\frac{\lambda(3\lambda+\mu-\mu_1)}{(3\lambda+\mu)(\gamma_1-\gamma_2)}e^{\gamma_2 t}+\frac{a_1\mu_1}{(3\lambda+\mu)(2\lambda+\mu_1)} \quad (4.11)$$

Under normal circumstances, because with the increase of input to the system, the repair rate of the system will be improved. The same warning state is assumed constant repair rate μ, examine how it will change in a time lapsed state of repair rate, system reliability.

suppose $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda_5=\lambda$;take $t_0>0$ when $t\le t_0$, $\mu_2(x)=\mu_3(x)=\mu_1$, Where μ_1 is the normal number

when $t>t_0$ suppose $\mu_2(x)=\mu_3(x)=\mu_2=\mu_1+h$ （$h>0$;obvious when $h<0$ system reliability）, make $\int_0^\infty p_i(x,t)dx=p_i(t)$, $i=2,3$;then $\sum_{i=0}^{3}p_i(t)=1$

The original differential - integral equation can be reduced to:

$$\frac{dp_0(t)}{dt}=-(a_0+\mu_2)p_0(t)+(\mu-\mu_2)p_1(t)+\mu_2 \quad (4.12)$$

$$\frac{dp_1(t)}{dt}=-a_1p_1(t)+\lambda p_0(t) \quad (4.13)$$

Abstract:

$$\begin{cases}\dfrac{d\overrightarrow{P(t)}}{dt}=A_1\overrightarrow{P(t)}+\overrightarrow{\mu_2}\\ \overrightarrow{P(0)}=(p_0(t_0),p_1(t_0))^T\end{cases} \quad (4.14)$$

among $\overrightarrow{P(t)}=(p_0(t),p_1(t))^T$, $\overrightarrow{\mu_2}=(\mu_2,0)^T$

$$A_1=\begin{pmatrix}-(a_0+\mu_2) & \mu-\mu_2\\ \lambda & -a_1\end{pmatrix}$$

Eigenvalues of the matrix A_1 : γ_1',γ_2'

$$\det(\gamma^* I-A_1)=\begin{vmatrix}\gamma^*+3\lambda+\mu_2 & \mu_2-\mu\\ -\lambda & \gamma^*+2\lambda+\mu\end{vmatrix}=0$$

Solution was $\gamma_1'=-2\lambda-\mu_2$, $\gamma_2'=-3\lambda-\mu$

Literature [12],

$$\overrightarrow{P(t)}=e^{A_1 t}\left(\int_{t_0}^{t}\mu_2 e^{-A_1\xi}d\xi+C\right)=-A_1^{-1}\overrightarrow{\mu_2}+e^{A_1 t}A_1^{-1}e^{-A_1 t_0}\overrightarrow{\mu_2}+e^{A_1 t}C \quad (4.15)$$

Substituting the initial value $\overrightarrow{P(t_0)}$, Solving C

$$\begin{pmatrix} p_0(t_0) \\ p_1(t_0) \end{pmatrix} = \begin{pmatrix} \frac{e^{\gamma_2 t_0}(a_0+\mu_1+\gamma_1)-e^{\gamma_1 t_0}(a_0+\mu_1+\gamma_2)}{\gamma_1-\gamma_2} & \frac{(e^{\gamma_1 t_0}-e^{\gamma_2 t_0})(\mu-\mu_1)}{\gamma_1-\gamma_2} \\ \frac{\lambda(e^{\gamma_1 t_0}-e^{\gamma_2 t_0})}{\gamma_1-\gamma_2} & \frac{e^{\gamma_2 t_0}(a_1+\gamma_1)-e^{\gamma_1 t_0}(a_1+\gamma_2)}{\gamma_1-\gamma_2} \end{pmatrix} C$$

Finishing ,

$$C = \begin{pmatrix} \frac{\mu_1(\mu-\mu_1)(3\lambda+\mu)e^{-\gamma_1 t_0}+\lambda\mu_1(2\lambda+\mu_1)e^{-\gamma_2 t_0}+\lambda(\gamma_1-\gamma_2)(6\lambda+2\mu+\mu_1)}{(2\lambda+\mu_1)(3\lambda+\mu)(\gamma_1-\gamma_2)} \\ \frac{\lambda\mu_1(3\lambda+\mu)e^{-\gamma_1 t_0}-\lambda\mu_1(2\lambda+\mu_1)e^{-\gamma_2 t_0}-\lambda\mu_1(\gamma_1-\gamma_2)}{(2\lambda+\mu_1)(3\lambda+\mu)(\gamma_1-\gamma_2)} \end{pmatrix}$$

and

$$e^{A_1 t} = \begin{pmatrix} \frac{e^{\gamma_2' t}(a_0+\mu_2+\gamma_1')-e^{\gamma_1' t}(a_0+\mu_2+\gamma_2')}{\gamma_1'-\gamma_2'} & \frac{(e^{\gamma_1' t}-e^{\gamma_2' t})(\mu-\mu_2)}{\gamma_1'-\gamma_2'} \\ \frac{\lambda(e^{\gamma_1' t}-e^{\gamma_2' t})}{\gamma_1'-\gamma_2'} & \frac{e^{\gamma_2' t}(a_1+\gamma_1')-e^{\lambda_1' t}(a_1+\gamma_2')}{\gamma_1'-\gamma_2'} \end{pmatrix}$$

$$A_1^{-1} = \frac{1}{(3\lambda+\mu)(2\lambda+\mu_2)} \begin{pmatrix} -(2\lambda+\mu) & \mu_2-\mu \\ -\lambda & -(3\lambda+\mu_2) \end{pmatrix}$$

$$e^{-At_0} = \begin{pmatrix} \frac{e^{-\gamma_2 t_0}(a_0+\mu_1+\gamma_1)-e^{-\gamma_1 t_0}(a_0+\mu_1+\gamma_2)}{\gamma_1-\gamma_2} & \frac{(e^{-\gamma_1 t_0}-e^{-\gamma_2 t_0})(\mu-\mu_1)}{\gamma_1-\gamma_2} \\ \frac{\lambda(e^{-\gamma_1 t_0}-e^{-\gamma_2 t_0})}{\gamma_1-\gamma_2} & \frac{e^{-\gamma_2 t_0}(a_1+\gamma_1)-e^{-\gamma_1 t_0}(a_1+\gamma_2)}{\gamma_1-\gamma_2} \end{pmatrix}$$

Finishing

$$e^{A_1 t}A_1^{-1}e^{-At_0}\overrightarrow{\mu_2} = \begin{pmatrix} -\frac{\lambda\mu_2(2\lambda+\mu_2)(\gamma_1'-\gamma_2')e^{\gamma_2(t-t_0)}+\mu_2(\mu-\mu_2)(3\lambda+\mu)(\gamma_1-\gamma_2)e^{\gamma_1' t-\gamma_1 t_0}+\lambda h\mu_2(2\lambda+\mu_2)e^{\gamma_2' t-\gamma_1 t_0}}{(\gamma_1'-\gamma_2')(2\lambda+\mu_2)(3\lambda+\mu)(\gamma_1-\gamma_2)} \\ \frac{\lambda\mu_2(\gamma_1'-\gamma_2')(2\lambda+\mu_2)e^{\gamma_2(t-t_0)}-\lambda\mu_2(3\lambda+\mu)(\gamma_1-\gamma_2)e^{\gamma_1' t-\gamma_1 t_0}+\lambda h\mu_2(2\lambda+\mu_2)e^{\gamma_2' t-\gamma_1 t_0}}{(\gamma_1'-\gamma_2')(2\lambda+\mu_2)(3\lambda+\mu)(\gamma_1-\gamma_2)} \end{pmatrix}$$

$$p_0(t) = \frac{-\lambda h}{(3\lambda+\mu)(\gamma_1-\gamma_2)e^{\gamma_2 t_0}}e^{\gamma_2 t} + \frac{\lambda h\mu_1(\gamma_1'-\gamma_2')-2\gamma^2h^2}{(\gamma_1'-\gamma_2')(2\lambda+\mu_1)(3\lambda+\mu)(\gamma_1-\gamma_2)e^{\gamma_1 t_0}}e^{\gamma_2 t} + \frac{\lambda(\mu_1(\mu-\mu_2)+\lambda(6\lambda+2\mu+\mu_1))}{(\gamma_1'-\gamma_2')(2\lambda+\mu_1)(3\lambda+\mu)}e^{\gamma_2 t}$$

$$+\frac{2\lambda h(\mu_2-\mu)}{(\gamma_1'-\gamma_2')(2\lambda+\mu_1)(2\lambda+\mu_2)e^{\gamma_1 t_0}}e^{\gamma_1' t} + \frac{2\lambda(\mu-\mu_2)}{(\gamma_1'-\gamma_2')(2\lambda+\mu_1)}e^{\gamma_1' t} + \frac{\mu_2(2\lambda+\mu)}{(3\lambda+\mu)(2\lambda+\mu_2)}$$

That steady-state reliability $p^* = \frac{u_2 a_1}{(3\lambda+u)(2\lambda+u_2)}$ 。Obvious The larger h , The larger p^* 。But to make the system reliable, make $p_0'(t) < 0$, get it

$$t_0 < \frac{1}{\gamma^*}\ln\frac{2a_1\lambda h}{6\lambda^3+\lambda^2(8\mu-5\mu_1)+\lambda(2\mu+\mu_1)(\mu-\mu_1)}$$, among $\gamma^* = \min\{\gamma_1,\gamma_2\}$ 。

For this system to be ascertained to achieve a desired value by the above discussion, we must first examine the repair rate system repair, the repair rate of the reliability of the system to achieve the desired, then according to the expected reliability of size within a predetermined time, i.e. in the range of t_0 to strengthen the repair of the system, the corresponding increase in the repair rate. If you exceed the prescribed time t_0 to strengthen the maintenance, the system is no longer reliable.

5. Conclusions

Through the introduction of the calculation of the reserve requirement of the civil aviation system under the model of the aircraft repair and aircraft purchase, give the operations' management and supply chain management, industrial engineering problems, project management, and other issues that can be transformed into not only reliable complex systems issues, and also based on users' demands give the reliability of probability in advance, that the reliability (availability). Put forward the risk early warning theoretical basis of the complex system reliability issues and early warning risk method, the effective time of the processing risk and the length of time to deal with the risk. Provide the operations management and supply chain management, industrial engineering problems, project management, and other issues a theoretical basis and practical significance.

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