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Methodology based on problem solving in the treatment of the concept of limit to infinity

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ABSTRACT

In this research, a methodological strategy it is raised for the assimilation of the concept of infinity limit in the teaching and learning of calculus at the University level. The strategy considers the contributions and theoretical elements of dialectical materialism and methodological elements and relies on the contributions of activity theory, problem solving and concept formation. It is wise to suggest that problem solving played central role in the process of assimilation of the concept of limit to infinity.

Keywords

Problem solving,activity theory, methodological proposal, limit, infinity

Academic Discipline And Sub-Disciplines

Mathematics, Mathematics Education

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INTRODUCTION

The teaching experience has enabled us to identify that the traditional practice of teaching in the middle and upper levels, math teachers tend to repeat verbatim the contents related to the area of the students, this means that, with that way of working the discussion of meaning, uses and identification of concepts, axioms, theorems and properties of relationship that are part of mathematical activity but necessary for learning axiomatic is not favored.

It has been identified that this way of treating education has implications for student learning, in particular, the teaching of the concept of limit through this pathway has implications for learning. Some of the difficulties associated with the concept that have been identified are: understand the limit as the value taken by the function at the point towards which the variable, difficult to interpret the terms of the concept, hard to use quantifiers in justification of the value limit of a function, spontaneous conceptions prevent the formal notion of the concept, among others.

Research on the concept of limit, to infinity and infinity, reporting Camacho and Aguirre (2001); Antibi (1996), Engler (2008), Orton (1980), Hitt and Paez (2005) show that there is a need for attention of the concept of *limit* although each pose different objectives, we have managed to identify around the boundary,

There are difficulties in its conception, to its teaching and learning activities that have been proposed, some difficulties have been identified in the students of middle and upper level such as: primitive ideas of the concept, idea and idea of approximation limit, meanings of different notations, conflicts with the idea of the concept of limit as a simple substitution, conflicts in reading graphs with respect to limit conflicts with the idea that a discontinuous function has no limit, meaning the "=" in the notation $\lim_{n \to \infty} a_n = L$, meaning of quantifiers, demonstration meanings in the context of the issue of boundaries, intuition and demonstration.

In the investigations mentioned, it is made clear that most of them are focused on the study of particle limit (finite) and research on the infinite limit and the limit to infinity are scarce. It is in this sense that we consider fundamental research on the concept of limit to infinity. Stewart (1991); Piskunov (2004); Hasser, et al, (2003); Alexandrov, Kolmogorov y Laurientiev (1985).

Moreover, by applying a design of activities on infinite processes related to a group of students from Bachelor of Mathematics from the Autonomous University of Guerrero. we identified that in the conception of the student is prevalent the notion of the potencional infinitive, and that in most cases there are difficulties associated with that extreme situation, it is not identified an infinite process that has an extreme situation only if it meets the principle of exahusción.

Plan and curriculum (2009) existing in the Bachelor of Mathematics from the Autonomous University of Guerrero, México, the topic of limits is proposed in the field of Calculus I, but to analyze the program, note that there is a proposal to address the types of limit, so we believe this is one reason why in practice there is discussed in depth limit types as borderline case infinity.

Supported on the above, the research question we ask ourselves is: Poor assimilation of the concept of limit to infinity in the teaching and learning of calculus in the senior students, the research object is: The process of teaching and learning Calculus in the senior students, the scope of this research is: The process of assimilation of the concept of limit to infinity at the university level, through problem solving. In order to influence the resolution of the problem of teaching and learning of a threshold at the top level, we propose the following research objective: Develop a methodological strategy to promote the adoption of the concept of limit to infinity in superior level students, in the teaching and learning of calculus through problem solving.

PHILOSOPHICAL FOUNDATIONS OF RESEARCH

The research is theoretical foundations contributions of Dialectical Materialism. This philosophical position studies the relationship between consciousness and the objective material world, the most general laws of motion and development of nature, society and knowledge. From this theoretical position assumes that part of the development of knowledge is the result of needs identified in the practical activity of man. The researchers state that this does not mean that in the early stages of teaching mathematics do not try to realize these abstractions for better initial understanding and although these stages are overcome, in the same school is not detached from the practical origin objects such as concepts, among others, and relationships that are studied.

From the point of view of the theory of knowledge of materialism, as noted by Rizo and Campistrous (2003), the path of knowledge begins in the product of a practical necessity, knowledge leads to an abstract level and again that knowledge takes practice to meet a need, in this sense knowledge begins and ends in practice, qualitatively superior conditions. This means that knowledge is an active reflection, oriented and appropriate reality in each man's consciousness and in social practice proven that this does.

THEORETICAL AND METHODOLOGICAL ELEMENTS

The research used as inputs theoretical elements of the historical-cultural approach of Vygotsky. In Hernandez (1997) reported that the fundamental problem that interested Vigotsky was the analysis of consciousness in all its dimensions.

From the Vygotskian position the development of knowledge occurs in two main stages: the first one occurs in the socio-cultural interaction and the second of which occurs in the process of individual internalization. For the specific case of research, we also support in the contributions of activity theory and mental actions, in order to guide the process methodologically individual internalization case of assimilation of the concept of limit to infinity.

Rizo and Campistrous (2003), think that the contributions of the cultural-historical approach of Vygotsky the teaching-learning process can be summarized to consider:

- Learning as a type of activity as opposed to the accumulation of reactions.
- Learning as a kind of creative activity towards learning as reproductive activity.

Zillmer (1981) states that Galperin identified as problem the following: The main deficiency of many teachers were limited in their classes to the oral description and intuitive demonstration of results and required actions, this means, insufficient management of the process of appropriation knowledge.

The conception of learning Galperin is based on the Marxist-Leninist philosophy, and in a way that all human qualities are developed in the activity that makes this philosophy. In this regard Rizo and Campistrous (2003) state that the actions are goal-directed,that means that has one goal and originate a reason. These actions are performed by a process whose fulfillment is a process, the process of implementing the action. This process of implementation of the action, also considered as stages of activity, can be divided into three phases: orientation, execution and control.

As methodological elements we reviewed the work on problem solving done by Sigarreta (2006); Sigarreta y Rosa (2003), Rizo y Campistrous (1996); Cruz (2002); Sigarreta y Laborde (2004); Sigarreta, et al (2011) in order to pursue the subject., after this analysis, we conclude using problem solving as a case study in our research. It also has been reviewed the various classifications of problems do Majmutov (1983), Polya (1945), González (1945), Palacios and Zambrano (1992) this review, we classify the problems to work on research: Problems closed: defined as one where the solution is deduced logically from the information contained in the problem and logically linked solutions are obtained and open problems defined as the ones where the solver needs to go beyond the information given, and from the change in some cases the meanings attributed to the elements of the problem reach conclusions.

Considering the above investigations and the ones that are reported by Sigarreta (2003) y Rizo y Campistrous (1996) in relation to the concept of problem, from the point of view of the author of this research we understand that a situation is a problem if it accomplish the following: there is one or more starting positions and one or more final standings, the way to move from the initial situation to the final must be unknown, the student must be willing to resolve the situation.

After reviewing contributions on the development and definition of concepts that make Jungk (1985) and Ballester (1992), in this paper we consider the adoption of the concept of limit to infinity will be achieved according to the following phases: Phase 1. Approaching the concept, Phase 2. Formalization of the concept, Phase 3. Id concept, Phase 4. Application of the concept, these phases together with the mathematical concept content are the central part of the strategy that is divided later.

CONCEPTION OF METHODOLOGICAL STRATEGY

In order to establish the concept of methodological strategy the contributions on learning strategy models that Escudero (1981), Santos (1991), Carrillo y Moy (2009), Rojas (2009), Escalona (2007), Sigarreta (2001) reported were reviewed; and about methodological strategies that provide the jobs of Méndez (2010) and Sigarreta (2011).

Under the above references, we understand as a methodological strategy: systemic structure in the process of learning and teaching, it allows systematizing a particular practice to advantage the development of certain skills, knowledge assimilation processes or about specific content of a discipline.

Shown below is the structure of the methodological approach to the treatment of the concept of limit to infinity, it will emphasize in it, both philosophical and psycho-pedagogical elements, which influence treatment planning and problem solving to promote the assimilation of the concept in question. Stages of assimilation and problem solving strategy indicated in each stage are explained.

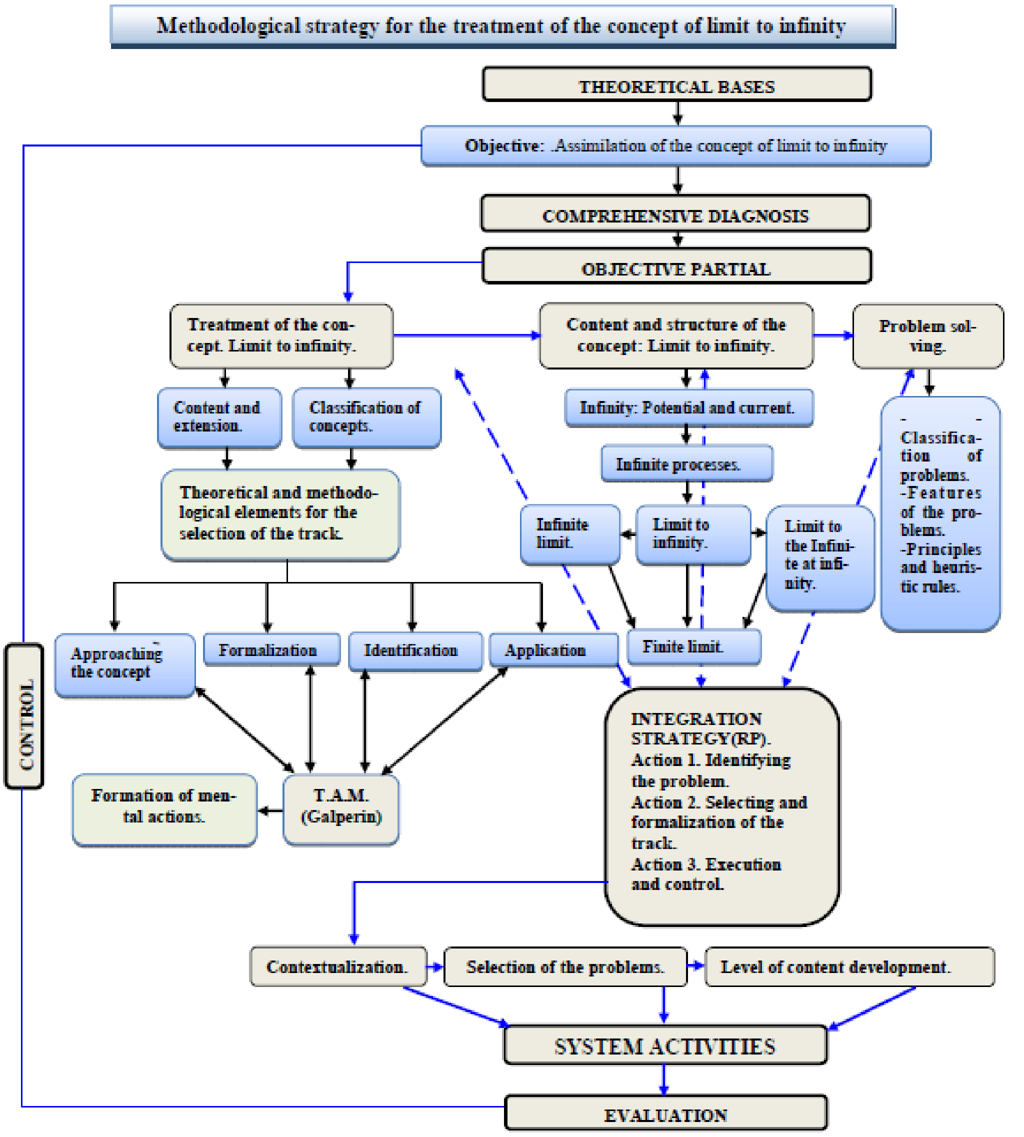

Figure 1

STAGES FOR THE ASSIMILATION OF THE CONCEPT OF LIMIT TO INFINITY

By assimilating a concept we understand that the student succeeds in making the approximation, formalization, identification and application of the concept. To advantage the concept approach (recognize the concept from the intuitive point of view) in its different representations, it is necessary to operate with the concept in order to differentiate it with other types of limits and establish specific differences, specifically the approximation of the infinite limit requires that the teacher:

- Analyze the concept in terms of its essential components. In this case; taking in to account the essential elements of the infinite limit, approach, vicinity, distance,standar, infinite process, etc.
- Use exercises (numerical and analytic, algebraic).
- Systematically Increase the degree of difficulty of the problems proposed, taking into account the cognitive interests of students.
- Analyze and discuss beliefs about the concept of limit.

The next phase in the process of assimilation is the formalization of the concept, to achieve this phase teaching functions of class starting level, motivation, goal orientation, implementation, monitoring and evaluation are used, and specific activities proposed in each one.

Application of the concept: the concept is applied to problems in different contexts and the ordering of the concept in a system of concepts is favored

Theoretical elements - methodology for the selection of the route, whereby it will effectively promoted the uptake the concept of infinite limit. The route is selected depending on the content and extent, depending on the classification of the concept as above described, *according to the historical evolution of the mathematical concept, from the relationship with other concepts, the possible theoretical or practical applications of the previous knowledge of the students, forms of thought.*

METHODOLOGICAL STRATEGY FOR PROBLEM SOLVING

The strategy for solving problems that favor the assimilation of the concept of limit to infinity consists of the following and they relate to operations under the theory of activity.

Step 1. *Identifying the problem.* This action provides guidance and identifies the requirement of the problem, given the conditions and the relationship of the concepts involved, etc.

Step 2. *Selecting and arranging the track.* This action can enrich orientation and determined by which way the Inductive, Deductive or Mixed conducting activities to promote the learning of some phase of assimilation.

Step 3. *Execution and control.* This action allows identify common and uncommon features of the concept of infinite limit is depending on the phase of assimilation to which you want to apply. For example, try this action allow the formal definition of the concept, or enables a specific application of the concept, or structure activities allows identification of the concept. The control allows to evaluate the process at each stage of development stages of assimilation, that is, to identify if it has achieved the approach to the concept, formalization of the concept, identifying the concept or application of the concept. Allowed in our case, knowing the characteristics of the concept of convergent sequence and infinite limit of a function, or to what extent assimilation has been achieved.

SYSTEM OF SPECIFIC ACTIVITIES OF MATHEMATICAL CONTENT FOR THE ASSIMILATION OF THE CONCEPT OF LIMIT TO INFINITY

Activities for the approach to the concept

In this part five activities were proposed, then shows one of them:

Let *C* be a circle with a given radius r, in such a circle inscribed circle regular polygons of 4, 6, 8 and 10 sides respectively.

- If a square is inscribed in the circle, and assume that l indicates the circumference measurement. Then the area of the square is an approximation to the circle area. The sum of the sides of the square, that is, 4l 'is an approximation of the length of the circumference.
- b) If a hexagon is inscribed in the circle, and assume that l indicates the circumference measurement. Then the area of the hexagon is an approximation to the circle area. The sum of the sides of the hexagon, ie 6l 'is an approximation of the length of the circumference.
- f a polygon with 8 sides is inscribed in the circle, and assume that l indicates the circumference measurement. Then the area of the polygon is an approximation to the circle area. The sum of the sides of the square, that is, 8l 'is an approximation of the length of the circumference l, the process is continued.

Students analyze and argue about the following questions:

- Following the above process, in how many more stages we cover the area of the circle?
- Is it possible to determine the exact number of polygons to be entered in the circle to determine the extent of area? In other words, the process is finite?

Activities to formalize the concept

In this part eight activities are proposed, the first of which students are oriented to the study of infinite processes, and then the activity is focused on those infinite processes associated with an limited situation, later to establish the concept of convergent sequence. Here are some suggested activities:

Activities for the identification of the concept

Four activities were proposed, students established the necessary and sufficient conditions of concept, formally establish the common and uncommon features of the concept, proposed some problems in the mathematical context and beyond involving the concept of limit to infinity. Where the primary task consists in identifying students that concept and to solve to determine the solution of the problem, they offers students a list of problem situations. With the aim to identify that they are linked to limit the concept of infinity.

Activities for the implementation of the concept

Students analyze and justify mathematical properties of the concept of limit to infinity, justify the convergence of an improper integral, justify the convergence of a function, identify and model the concept with him solving problems in contexts outside of mathematics, among others.

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VALIDATION OF METHODOLOGICAL STRATEGY

Consultation with experts

Five experts were consulted who work inUniversities de Carlos III de Madrid, UniversityPablo de Olavide, spain; Universityof Holguín, Cuba;Autonomous University of Guerrero, México. Such experts have strong expertise in research methodologies and mathematical analysis. Each expert was sent a summary of the research and an additional questionnaire, which was intended to know the valuation on methodological strategy.

The five experts agree that the methodological strategy has a well-argued theoretical and methodological approach, The five experts agree that the methodological strategy has a well-argued theoretical and methodological approach, specific activities are clearly supported as both took care of methodology for treatment as the mathematical aspect (content needed) to encourage the assimilation of be implemented, this strategy can yield results that benefit in understanding the limit concept, particularly in the assimilation limit to infinity, is relevant implementation at the university level.

Pedagogical experiment

We worked with two groups of first graders Degree in Mathematics from the Autonomous University of Guerrero, Mexico, one called the control group and the second experimental group. With the control group, we worked with content about limit,in a traditional way as is suggested in the curriculum of the bachelor in mathematics. With the experimental group we work with the methodological strategy.

SOME RESULTS OF THE PEDAGOGICAL EXPERIMENT

After treatment of the concept with the traditional methodology, the following results were identified in students:

There exist only intuitive ideas, they have difficulties on defining the concept using the model (ε-N), finding difficult to identify the geometric concept of the same mathematical problems, about 80% of students did not identify correctly the concept representation graph, finding difficult to give meaning the mathematical definition of the concept, among others.

After making the treatment of the concept of limit to infinity with the methodological strategy that reaches identified to better understand the concept, because at each stage of the assimilation process specific activities were worked. Some results are:

In step approach: 12 of 16 students in the experimental group identified certain numerical sequences that converge to a real number, and proceeded to determine the limit algebraically. Then they were able to identify that there are infinitely many processes geometric cut problems, and that these infinite processes tend to a real number. However, there are other infinite processes that do not have that feature. From the intuitive point of view, the students stated that possibly the infinite limit processes must be bounded, increasing or decreasing.

In the formalization stage: At this stage the work towards the resolution of problems that led to the discussion of the essential characteristics of those infinite processes associated with an extreme situation and explanation from the mathematics of these situations was oriented. 9 out of 16 students agreed: "... an infinite process has an associated limit situation, or means that there can be determined a number from which ensures that the infinite process is as close as desired to a real number, in this case the difference between the infinite process and the actual number can be made smaller than any given value. " These statements allow to reject the idea that the limit is only an approximation. Subsequently ideas were formalized and interest to reach the definition was raised.

In the identifying step: 12 of 16 students correctly identify situations involving the limit to infinity and tended to determine the extent and reasons, 4 of them are difficult activity. *In the implementation stage:* At this stage 12 students were successful in solving and justification of the proposed limits to infinity as situations within the same math, the variant was one of the examples demanded raise it in terms of the limit to infinity to work solution.

OVERALL CONCLUSIONS.

The main results achieved in this research was the development of a methodological approach to the treatment of the concept of limit to infinity in the process of teaching - learning of calculus at the top level. To do this, treatment is established from structured problems, supported by the theory of activity, in direct relation to the stages for the assimilation of concepts: approximation, formalization, identification and application. Our methodological strategy is structured around student learning, ie it the assimilation process is prioritized phased concept of limit to infinity taking into account the activities of the student and it is conceivable that it has assimilated the concept of limit to infinity if he can successfully pass the following stages: approach, formalization, identification and implementation of such a concept.

The completion of each stage allows processes promote internalization of knowledge on the concept of limit and the limit to infinity. Thus at each stage activities both correspond to the development stages of mental actions were proposed, and their development strategy to integrate problem solving that is based on the tenets of activity theory are used and conducting operations relating to the mental action.

Another success that characterizes the methodological strategy, is the attention to mathematical content of the concept, it is proposed and developed a new structure for the treatment of the concept of limit, considering the logical and conceptual relationships of concepts intertwined with the concept of limit to infinity, showing the theoretical and methodological validity in the proposed hierarchy: Treatment concept of infinity and their meanings: current and potential infinite processes, infinite limits, limits at infinity, infinite limits at infinity and on time limit. Other features to assert that the elaborate strategy fills a gap in the teaching and learning of the calculation are:

The methodological strategy allows integrating two typical situations in the teaching of mathematics, concept formation and its definition and problem solving, as a laudable theoretical element in the process of assimilation of the concept of limit to infinity.

Each of the stages of assimilation of the concept are structured according to the theory of mental actions, and each specific activity is carried out by problem solving addressing the three key actions: Identification of problem selection and formalization of the pathway, execution and control.

According to the provisions, we can state that do the treatment by the implementation of the strategy elements that guarantee encourage assimilation processes are thrown also the steps involved in this process allow evaluation during the process and if it is necessary redirect the activity. Thus we can say that by conducting pedagogical experiment, we found elements to conclude that the methodological strategy be implemented, can facilitate the process of assimilation of concepts, in particular the limit to infinity.

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