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MATHEMATICS TEACHERS' RESPONSES TO STUDENTS' MISCONCEPTIONS IN ALGEBRA

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ABSTRACT

The study investigated the responses of teachers of mathematics to students' misconceptions in algebra. Qualitative approach to the analysis of the data was employed. Eighty seven teachers took part in the study, and four questionnaires were designed to explore the responses of the teachers. The focus was on variables, algebraic fractions, equations and word problems. The study revealed that some of the teachers were successful in understanding students' thinking with regard to the algebraic concepts studied. The study also indicated that most teachers ask instructional questions instead of investigative questions. Most teachers were incapable of asking questions which would reveal students' source or cause of misconception. Another important finding was that some of the teachers had difficulties in understanding the problems, hence could not figure out the students' misconceptions or errors in the hypothetical solutions given in the questionnaires. The teachers' pedagogical content knowledge was generally inadequate.

Indexing terms/Keywords

Algebra; students' misconceptions; teachers' knowledge; mathematics teaching

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1. INTRODUCTION

The mathematics teachers' knowledge is necessary for effective and meaningful teaching. But 'what should teachers know?' asked Tanisli and Kose (2013). They pointed out that this can be explained by the concept of pedagogical content knowledge of the teachers. According to Shulman (1986) pedagogical content knowledge is defined as "the most useful form of representations, the most powerful analogies, illustrations, examples, explanation and demonstrations, the ways of representing and formulating the subject that makes it comprehensible to others". Pedagogical content knowledge is therefore, a kind of knowledge that shows teachers' meaningful and effective ways of teaching. Tamir (1988) categorized pedagogical content knowledge into four categories namely, knowledge of understanding students; knowledge of teaching methods, strategies and techniques; knowledge of measurement and evaluation; and knowledge of curriculum. Also, Grossman (1990) identified four components of pedagogical content knowledge: knowledge of strategies and representations for teaching particular topics; knowledge of students' understanding, conceptions and misconceptions of these topics; knowledge and beliefs about the purposes of teaching particular topics; and knowledge of curriculum materials available for teaching. Similarly, Marks (1990) divided pedagogical content knowledge into four: knowledge of understanding students; knowledge of teaching methods, strategies and techniques; knowledge of subject-matter; and knowledge of media.

Knowledge of understanding the students and teaching methods stands out prominently as the necessary and important component of the teacher's knowledge. The knowledge of the students, Shulman (1987) said is the teacher's knowledge of the students' thinking process, learning styles, difficulties and misconceptions in the process of learning a subject.

A limitation of much of the research regarding teachers' knowledge of student thinking is that the research has focused primarily on the domains of whole number and rational number in the elementary and early secondary schools. The concept of pedagogical content knowledge, which transcends knowledge of subject matter to the dimension of subject matter knowledge *for teaching*, according to Shulman (1986). has brought attention to the importance of teachers' knowledge of students' understandings, conceptions, and misconceptions of particular topics in a subject matter. And indeed, such attention to teachers' knowledge of students' thinking is reflected in the work of a number of scholars (e.g., Kazemi & Franke, 2004; Tanisli & Kose, 2013).

This study focused on knowledge of student, as one of the key components of pedagogical content knowledge of teachers. The study therefore, investigated teachers' responses to students' thinking process and misconceptions about some algebraic concepts. Given the need for integrating algebraic reasoning throughout the elementary and secondary mathematics curricular, researchers have begun to move toward teachers' knowledge of students'

thinking in the domain of algebra in elementary and secondary schools mathematics (e.g., Kaput et al., 2007; Stephens, 2006).

1.1 Students' Misconceptions in Algebra

Research on student thinking about variable has likewise shown that many students' conceptions are inadequate, particularly with respect to the use of literal symbols in algebra (e.g., Küchemann, 1978; Usiskin, 1988). Student misunderstandings include viewing variables as abbreviations or labels rather than as letters that stand for quantities, assigning values to letters based on their positions in the alphabet, and otherwise being unable to operate with algebraic letters as varying quantities rather than specific values (Küchemann, 1978).

According to Swan (2001), a misconception is not wrong thinking, but it is a concept in embryo or local generalization that the student has made. It may be a natural stage of development. He further stated that "although we can and should avoid activities and examples that might encourage them, misconceptions cannot simply be avoided". Therefore it is important to have strategies for remedying as well as for avoiding misconceptions. According to Akhtar and Stienle (2013), there has been considerable research into students' misconceptions in algebra. They pointed out that students who have incomplete understanding of the meaning of letters will have difficulty understanding this important branch in mathematics. Lins and Kaput (2004) noted that one of the sources of students' misconceptions in algebra could be attributed to the tradition of arithmetic then algebra. Stephens (2008) said students developing relational thinking with number sentences would assist their transition from arithmetic to algebra.

Radatz (1979) cited by Gunawandena (2011) identified four categories of misconceptions in algebra. These are (1) misconceptions due to processing iconic representations (2) misconceptions due to deficiencies of mastery prerequisite skills, facts and concepts (3) misconceptions due to incorrect associations or rigidity of thinking leading to inadequate flexibility in decoding and encoding new information and the exhibition of processing new information and (4) misconceptions due to the application of irrelevant rules or strategies. Barrera, Medina and Robaynal (2004) categorized misconceptions caused by a lack of meaning into three stages: algebra misconceptions originating from arithmetic, use of formula or procedural rule inadequately and misconceptions due to the properties themselves of algebraic language (or structured misconceptions). Gunawendena (2011) reported Kuchemann (1981) as classifying children's interpretation of letters into six categories namely: letter evaluated, letter ignored, letter as an object, letter as a specific unknown, letter as a generalized number and letter as a variable.

1.2 Statement of the Problem

The problem for this study was to examine mathematics teachers' responses to students' misconceptions in algebra. The focus of the study was specifically on teachers' responses to students' misconceptions in variables, algebraic fractions, equations and word problems. The study sought to find out whether mathematics teachers could predict and identify students' thinking process, and also offer useful suggestions in remedying the situation.

1.3 Purpose of the Study

The main purpose of the study was to investigate how mathematics teachers respond to students' misconceptions in algebra. In specific, the study aimed at:

a) Examining teachers' responses to students' misconceptions about variables.

b) Examining teachers' responses with respect to students' misconceptions about simplifying algebraic fractions.

c) Finding out how teachers respond to students' misconceptions about solving equations, and

d) Determining how teachers respond to students' inability to solve word problems.

1.4 Research Questions

The following research questions were asked to guide the study:

1. Do mathematics teachers understand students' thinking process with respect to the concept of variable?
2. How do mathematics teachers respond to students' misconceptions about algebraic fractions?
3. How successful are mathematics teachers in identifying students' source of misconceptions about solving equations?
4. How successful are mathematics teachers in identifying students' misconceptions about word problems?

2. METHODOLOGY

2.1 Research Design

The qualitative approach was employed in this study. In qualitative approach, the focus is on process rather than product. This research method was employed so as to examine how teachers explain their approaches in responding to students' thinking process and misconceptions.

2.2 Participants

Eighty seven Nigerian teachers took part in the study. At the time of the study these teachers were teaching at both Junior and Senior Secondary school levels. They had three to fifteen years of teaching experience, and majority of them were graduates of Mathematics Education. The participating teachers were randomly selected from five public secondary schools in Adamawa and Bauchi States in Nigeria.

2.3 Instrument for Data Collection

Four questionnaires were designed to explore how teachers respond to students' misconceptions, with a focus on variable, algebraic fraction, equation and word problem. Each questionnaire showed work from a hypothetical student, and requested teachers to indicate what they would do in response. The variable and algebraic fraction questionnaires have three items each, and are shown in Table 1. In the variable questionnaire, which is Problem 1, the hypothetical student regarded the cost of the items as objects and not variables, and teachers were asked to predict the thinking of the student. For the fraction questionnaire, Problem 2, the student correctly divided by the common factor, but did not understand that 1 is a factor of x. The teachers were asked about what they thought the student understood and what was not understood. Questionnaires on equation and word problem have three and one items respectively, and are shown in Table 2. In the equation problem, Problem 3, the hypothetical student did not consider the signs of the numbers and the equal signs. Teachers were asked about the perception of the student and how they would help the student. For the word problem, Problem 4, the hypothetical student did not understand the problem. Teachers were asked what they would do to help the student. These misconceptions were considered as typical examples of what teachers encounter in actual classroom practice.

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Table 1 Questionnaires on variable and algebraic fraction

Problem 1

*Apples cost **a** naira each, and bananas cost **b** naira each. If I buy 4 apples and 3 bananas, explain what **4a** + 3**b** represents.*

A student's explanation: *4a represents 4 apples and 3b represents 3 bananas.*

a) What might this student be thinking?

b) What questions would you ask the student in order to find out if your opinion about this student's thinking is correct?

c) How would you correct this student's misconception about variable?

Problem 2

A student was asked to simplify $\frac{xa+xb}{x+xd}$, and the student's solution is follows:

$$\frac{xa+xb}{x+xd}=\frac{a+b}{d}.$$

a) What does the student understand? What does he/she not understand?

b) How could you quickly convince the student that this answer is incorrect?

c) How would you help this student?

Table 2 Questionnaires on equation and word problem

Problem 3

John was asked to solve the equation*: 15-3x=6.*

John's solution*: 15-3x=6*

3x=6+15

3x=21

X=7

a) What might be the perception of John?

b) What questions or tasks would you ask John to find out if your opinion about his perception is correct?

c) How would you help John?

Problem 4

Mary was asked to solve the following word problem*: A piece of rope 3 metres long is cut into two pieces. One piece is x metres long. How long is the other piece?*

Mary's solution: *A piece of rope=3m*

One piece=x m

Other piece=?

$$\frac{3}{2}=x$$

Other piece=1.5m.

a) What would you do to help Mary?

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3. RESULTS

3.1 Responses of Teachers to Students' Misconceptions about the Concept of Variable

The data collected were analyzed qualitatively. The first questionnaire which was on teachers' knowledge of students about the concept of variable, focused on teachers' ability to predict students' thinking, identify and correct students' misconception about a variable. The first item in this questionnaire was: *What might be the thinking of this student*? Majority of the teachers responded by saying, '*The student might be thinking that the letter a represents apple and the letter b represents banana, and not cost per apple and banana respectively'*. The second item in this problem situation was directed at finding out if the teachers' prediction of students' thinking was correct. Specifically, the question was, '*What questions would you ask the student to find out if your opinion about this student's thinking is correct?'* The types of questions the teachers asked were more of instructional or teaching questions than questions that would evaluate the student's thinking process. However, some teachers asked questions that were investigative in nature. For instance, some teachers asked the following questions: '*What do you think **a** and **b** stand for in this expression?' 'What is the cost of one apple?' 'Or what is the cost of one banana?'* Some of the teachers said they would ask questions similar to the given question, but with different letters as variables. However, these teachers did not write down a sample of such questions.

On the third item focusing on correcting the student's misconception about the concept of variable, the teachers' employed mostly instructional or teaching strategy. For example, some said they would explain to the student, even though did not elaborate on the nature of the explanation. Others said they would give specific examples, but also did not write down any of such specific examples. Still others in correcting the misconception, simply said they would tell the student that ***a** and **b** stand for cost of apple and cost of banana respectively, and not letters*. It should be noted here that some of the teachers did not understand the difference between 'How' and 'What' questions, as their responses addressed 'What' question instead of 'How' question that was asked.

3.2 Responses of Teachers to Students' Misconceptions about Simplification of Algebraic Fractions

The second questionnaire which was on teachers' responses to students' misconception about simplification of algebraic fractions had three items. The first item expected the teachers to say what the student understands and what he does not understand. Responding to what the student understands, 47% of the teachers said the student understands division by common factors. 33% of the teachers said the student has the idea of 'cancellation'. The remaining 20% said the student understands that to simplify you need to factorize, if possible. And regarding what the student does not understand, few (25%) teachers responded by saying the student does not understand that 1 is a factor of x. Others simply said the student does not fully understand factorization. Majority of the teachers were successful in identifying what the student understands and what he does not understand.

The second item in this questionnaire required the teachers to say how they could quickly convince the student that the answer given was incorrect. This item was meant for a quick check, and not a step by step solution of the problem. However, 76% of the teachers responded by saying they would solve the problem explaining all necessary steps. This majority either ignored what the question demanded, or was incapable of suggesting a quick check to convince the student of his incorrect answer. 16% said they would ask the student to multiply his answer by x to see if he could get back the original fraction. Some said they would ask the student, *why a term in the denominator is missing from his answer*. One teacher simply said he would write down the correct answer for the student to see. Asking the student to multiply his answer by the divisor, as multiplication is reverse of division was one of the ways the incorrect answer could easily be verified. Writing down the correct answer is not a quick way of showing the student that his answer is incorrect

The third item required the teachers to respond to how they would help correct the misconception or error of this student. The teachers' responses were simply instructional. For instance, more than half of the teachers said they would *teach factorization and encourage the student to learn more about algebraic fractions,* while the rest said they would *solve the given problem, explaining each step*.

3.3 Responses of Teachers to Students' Knowledge about Solving Equations

Questionnaire number three required teachers' knowledge of students about solving equations (see Table 2). The first item in this problem situation required teachers' ability to predict the perception of John. Some teachers said '*John perceived 15 as a negative number'*; while others said '*John ignored the negative signs*'. Still others said, '*John perceived that positive or negative signs are written after and not before the numbers'*. The teachers' predictions of John's perception were virtually the same. Regarding what questions or tasks they would ask John to find out if their opinions about John's perception were correct, the teachers' responses were classified as either instructional or investigative. For example, about half (51%) asked the investigative question: *What happened to -3x to become +3x?* Others (33%) said they would give examples and counter examples to discover John's perception about number signs. However, none of them gave such examples and counter examples. Similarly, their responses with regard to helping John remove the misconception or error were purely instructional. For instance, while 67% of the teachers said they would solve the problem indicating actions taken at each step, the rest simply said they would *help him through teaching.*

3.4 Responses of Teachers to Students' Knowledge about Solving Word Problems

The fourth questionnaire was a problem situation in which Mary solved a word problem (Table 2). The teachers were requested to respond as to what they would do to help Mary in her misconception as indicated in her solution. About half of the teachers(52%) responded by saying they would draw a line segment to represent the rope and then demonstrate the cutting into two pieces. 37% of the teachers simply said they would explain to her that the rope is not to be cut into two equal pieces. However, these teachers did not write down their explanation. The responses of the remaining 11% indicated they themselves had misconceptions about the problem. For example, these teachers said they would draw a line segment and demonstrate how to cut it into two equal parts. This shows the teachers themselves did not understand the word problem, hence their inability to offer useful suggestions as to what they would do to help Mary. Generally, the teachers were not successful in identifying the student's misconception, because they themselves had misconceptions.

4. DISCUSSION

The study revealed that some of the teachers were able to understand students' thinking processes with regard to the algebraic concepts studied. The teachers were especially successful in understanding students' thinking processes with respect to concept of variable. However, majority of the teachers had difficulties in their art of questioning to unravel the source or cause of the students' misconceptions.

In mathematics teaching, asking effective questions is an important tool for better identifying students' thinking processes. It was revealed in this study that teachers asked instructional question types instead of asking questions that would evaluate students' thinking processes with a view to identifying the cause of misconceptions. This finding is in line with the findings by Moyer and Milewicz (2002) and Tanisli and Kose (2013). The question: how could you quickly convince the student that his answer is incorrect? was targeted at the use of quick strategy through effective questioning skill to reveal student's misconception or error. However, teachers' responses revealed their dependence on instructional strategy instead of investigative one. This shows their inability in evaluating students' common misconceptions, as found by Tanisli and Kose (2013) in their study with pre-service mathematics teachers.

Another important finding of the study was that some of the teachers themselves had difficulties understanding the problems. For example, regarding the item on word problem, which requested the teachers to say what they would do to help Mary, the responses of some of the teachers revealed misunderstanding of the problem. Some said they would use line segment to demonstrate how to cut the rope into two halves. Similarly, in response to the question: how would you help the student? Some simply said they would write down the correct answer for the student to see. This revealed inadequate pedagogical content knowledge on the part of these teachers. There is therefore the need for the improvement on the teachers' subject matter knowledge.

5. CONCLUSION

One basic finding revealed in this study, is the inability of most teachers ask investigative questions or competent questions that could evaluate students' thinking processes. Teachers who participated in this study did not exhibit effective questioning skills in identifying the source or cause of students' misconceptions. Knowing students' thinking can help teachers in addressing students' misconceptions, engaging students in mathematics learning, promoting students' thinking about mathematics and building on students' mathematics ideas. The teachers' responses were not informative enough to address any of these aspects of knowing students' thinking. Teachers need to improve in their skills of asking questions and in their knowledge of students about algebraic concepts.

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